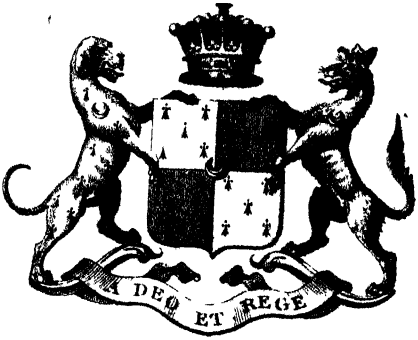
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Agricultural Research Institute

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PHILOSOPHICAL

TRANSACTIONS,

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXX.

PART I.

LONDON,

PRINTED BY W. BULMER AND W. NICOL, CLEVELAND ROW, ST. JAMES'S;
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MDCCCXX.

ADVERTISEMENT.

The Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the council-books and journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries, till the Forty-seventh Volume: the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions;* which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shewn to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public news-papers, that they have met with the highest applause and approbation. And therefore it is hoped, that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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Meteorological Journal Kept at the Apartments of the Royal Society, by Order of the President and Council.

PHILOSOPHICAL TRANSACTIONS.

I. *The Croonian Lecture. A farther investigation of the component parts of the Blood.* By *Sir* EVERARD HOME, *Bart. V.P.R.S.*

Read November 4, 1819.

As the capacity of one individual is rarely able to bring to perfection different branches of science, it is no wonder that, in the investigation of so complex a subject as the component parts of the blood, HEWSON failed, HUNTER fell short in the attempt, and Dr. YOUNG, after the interesting discovery of the colouring matter being readily separated from the red globules, made no farther advance. This view of the subject, led me, at the time I ventured upon this enquiry, to engage as my associates, Mr. BAUER, whose microscopical observations have been too long put to the proof, to admit of their being disputed; and Professor BRANDE, whose chemical researches, and the judgment with which they have been pursued, are fully established.

In two former Lectures, assisted by these skilful coadjutors, I have been enabled to prove, that the human blood in the act of coagulation, evolves aeriform matter, so as to pervade the coagulum in every direction; and that such currents, passing through the serum, form permanent tubes, which are immediately afterwards filled with red blood, when the circumstances in which the coagulum is placed, admit of their being so.

In the present Lecture, I trust that I am enabled to make out the greater number, if not the whole, of the component parts of the blood.

My former experiments were made upon coagula recently formed from the blood, whether out of the body, or in the interstices of parts possessed of life. Upon the present occasion, I have taken an opposite course, and have examined the coagula formed in aneurismal tumors. It is to be understood, that in this disease of the arteries, the coats at the part diseased, yield to the impulse of the heart, and admit of being permanently dilated, so as to form a pouch, in which the blood that remains at rest, coagulates. This dilatation is gradual; and as the pouch enlarges, the coagulum is found to be made up of a succession of layers, affording an opportunity of observing the changes coagulated blood undergoes under such circumstances at different periods of time.

In the examination of the section of an aneurismal tumor in the microscope, Mr. BAUER found that the layer of the coagulum, in contact with the blood in circulation, was red in its colour, loose in its texture, and principally consisted of red globules from which the colour had been discharged, and remained diffused through the mass. Besides these globules, he saw others of a smaller size, which he had never met with in fluid blood, however frequently he had examined it; there was also a small quantity of a transparent elastic mucus insoluble in water. The newly discovered globules were $\frac{1}{2800}$ part of an inch in diameter, and their number in proportion of one to four of the large ones. The other layers, in proportion to the length of time coagulation had taken place, were become paler in colour, denser in texture, and what is most particularly deserving of notice in the present investigation, the proportion of the number of the small globules to the large ones gradually increased, and in the layer longest coagulated, they were in the proportion of four to one. In this layer there was also the largest proportion of the transparent elastic mucus.

The coat of the artery forming the pouch, appeared to be made up of zig-zag, or serpentine fibres, connected by the elastic mucus, in which many of the small globules were detected.

The condensed cellular membrane on the outside of the pouch, consisted of thin membranes or films, easily separated, and between them were found many small globules.

In the section of a large aneurismal tumor a deposit of crystals was met with. This uncommon appearance is represented in the annexed drawings. These salts, in the absence of Professor BRANDE, were analyzed by Mr. FARADAY, Assistant in the Laboratory of the Royal Institution: they are sulphate of lime with muriate and phosphate of soda; salts usually met with in the blood, but probably never before seen in the form of crystals.

The discovery of small globules in aneurismal coagula, and the increase of their number in proportion to the duration of the coagulum, throws great light upon the materials of which the blood is composed.

Till this discovery was made, we knew of no globules in the blood but the red globules, either enclosed in their colouring matter, or deprived of it: indeed these smaller ones being held in solution in the serum, are only brought to view by the act of coagulation, and under the same circumstances we find the salts crystallize.

To ascertain whether these small globules constitute the substance thrown out in inflammation, Mr. Bauer examined a small portion of a mass of coagulable lymph taken from the vagina of an ass, where it had been deposited by a violent attack of inflammation; and another portion from the internal surface of an inflamed vein.* He found both substances made up of the small globules just discovered, mixed with a few red globules, deprived of their colouring matter. The globules which in a former Lecture were stated to have been produced in the serum, are now found to be similar to these, and had been held in solution in the serum when put into the tube.

The globules found by Basilius in the serum, after filtration through paper, must have been of the same kind.

In the prosecution of this enquiry, I procured the coagulum of some highly inflamed blood, as it is termed. The buff was very thick and firm, the lower portion loose in its texture. Mr. Bauer found the buff to consist almost wholly of the small globules, which I shall now call those of lymph, and the lower portion principally of red globules; so that the buffy appearance occurs when the lymph is so unusually slow in coagulating, that the red globules, which are so much larger and heavier, sink before that process has taken place.

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* Both of these preparations are described in Hunter's Work upon the Blood, Inflammation, and Gun-shot Wounds.

In the absence of Professor BRANDE, Mr. FARADAY analyzed a portion of the buff, and of the part made up of red globules: having previously washed away the colouring matter, their chemical properties were in all respects the same.

That I might compare the structure of tumors, with that of the layers in aneurismal coagula, I got Mr. BAUER to examine in the microscope the structure of a tumor in the prostate gland, made up of rounded nodules; the last formed of these was produced by the bursting of a small artery in the substance of the gland, so short a time before death, that the rupture of the vessel was distinctly seen when the parts were examined. He found the texture of the tumor soft and spongy; it was made up almost wholly of red globules free from colour, very few of those of lymph, and some of the transparent elastic jelly; the bands by which the nodules were separated, were composed of three-fourths of lymph, one-fourth of red globules from which the colour had been discharged, and a considerable proportion of the transparent jelly.

A tumor in the breast of long standing, of which the first formed part was hard and colourless, the last less compact and full of vessels or tubes, is shown in the annexed drawings. When its structure was examined in the microscope, the hard part was made up almost wholly of lymph globules, and elastic jelly; the last made one-fourth of the whole. The soft part consisted only of about one-fourth of lymph globules, the rest being red globules which had lost their colour.

The structure of such tumors is nearly allied to that of the layers in an aneurism. That these layers never become vascular, arises from the aeriform matter, evolved at the time that the blood coagulates, readily escaping into the circulating blood with which it is in contact.

To ascertain whether the proportion of aeriform matter in the blood is liable to vary, as well as to determine its nature, a very buffy coagulum was placed in the receiver of an air pump, with a syphon passing from the vessel containing it into a bottle filled with barytes water. The pump was worked, and the gas only came over in single bubbles, which occasioned a precipitation of carbonate of barytes. From a less buffy coagulum the gas came over in several bubbles at a time. When there was no buff, the gas was abundant, and the precipitation copious. To ascertain whether this gas is produced in the process of digestion, a pauper from one of our work-houses, an hour after eating a hearty dinner and drinking a pint of porter, was bled at the arm to six ounces. The coagulum was tested in the same manner as the others in the air pump, and at the same distance of time from that at which the blood was drawn, the gas passed through the syphon in a torrent, and there was a proportionate precipitation.

Carbonic acid gas, Professor Brande finds to be commonly met with in the urine; but in greater quantity immediately after a full meal.

The source from whence the carbonic acid gas is supplied, having been thus determined, an attempt was made to trace the lymphatic and red globules to their origin. The pyloric portion of the stomach, and the duodenum, are filled with a glary mucus. Mr. Bauer found in this mucus a great number of lymph globules and a smaller number of red globules without colour, so that such globules appear to be produced in the earliest stage of digestion.

In the human species the produce of the process of digestion becomes white, and therefore is readily distinguished from any other fluids; in general it is first met with in this state in the beginning of the jejunum, but sometimes in the whole course of the duodenum; the readiness with which this substance is absorbed, and the velocity with which it is carried along the lacteal vessels, make it difficult to procure enough for examination till it reaches the glands in the mesentery, through which it must pass before it arrives at the thoracic duct.

An opportunity occurred of making this examination upon the contents of the mesenteric glands, of a man who died in a fit an hour after having eaten his dinner. Mr. Bauer having extricated some of the glands of the mesentery from the surrounding fat and cellular membrane, divided them transversely, when a quantity of milk-white fluid issued out. On examination, the white colour was found to depend upon an infinity of white globules floating in a clear, perfectly colourless fluid, in the same manner as the red globules do in the serum. About eight-tenths of these globules varied in size from the smallest speck to the size of the lymph globules, about one-tenth were of the size of red globules deprived of the colouring matter, and about one-twentieth were of the size of the red globules enclosed in their colouring matter. When this fluid was left a few minutes on the glass, not only many new globules were formed, but the original small ones visibly increased on the field of the microscope, not by several globules uniting, but by accession of substance; and he watched several that enlarged to the full size of blood globules enclosed in their colouring matter: in that state they appeared more opaque; and when the glass was laid upon black paper, they appeared as distinctly to be milk-white, as the globules of the blood when the glass is laid upon white paper appeared to be bright red. When the fluid is diluted with water, no additional globules are produced, and the large ones are reduced in size, in the same manner as the red globules are, when their colouring matter is dissolved, and is leaving them. When there is a sufficient quantity of the fluid left to evaporate, ramifications are formed in every respect as distinct as those shown in a former Lecture, in the magnified drawing of a drop of human blood in the state of coagulation.

From the observations Mr. BAUER has had the opportunity of making upon the contents of the lacteal glands, he is satisfied that the full sized globules acquire that form in these glands, and that afterwards, so far as respects their external appearance, no change is necessary for their conversion into red globules, but their becoming red.

As the exposure of the blood to the air in its passage through the lungs, restores the brilliancy of colour that is lost in the circulation through the body, we can have no doubt that it is in the vessels of the lungs the blood receives its original hue.

EXPLANATION OF THE PLATES.

PLATE I.

Contains three figures; the first exhibits a transverse section of an aneurismal coagulum of the natural size; the other two represent the salts of the blood in a crystallized state, magnified five diameters.

Fig. 1. This section represents that part of the coagulum which lay in the bottom of the sac. It shows the different shades of colour of the layers according to their length of standing, and the crystallized salts in different parts of the coagulum.

Figs. 2 and 3. Different views of these crystals.

PLATE II.

Represents sections of a tumor taken from a patient in St. George's Hospital: there are seven figures.*

Fig. 1. The surface of the section of the natural size; one part is colourless; another appears to be very vascular.

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* The following is an account of the case. The tumor was perceived when of the size of a pea: in six months it increased to that of an orange: in fourteen days more it doubled that size. Arsenic, 48 grs. to ℥i. of water, was applied once in twenty-four hours for two days. The pain was too great to allow it to be renewed In ten days the application was taken off, and the tumor was black and shrunk, but not dead to any depth. Equal parts of white arsenic and sulphur were applied once in twenty-four hours for two days: the pain was excessive. In fourteen days, one half of the tumor came away. The remaining surface resembled a coagulum of blood, soft and dark coloured. This came away next day, eight months and a half from the first appearance of the tumor. This last is the tumor of which a section is represented. The drawing was made twenty-four hours after it was removed, in which time it had undergone no change.

Figs. 2 and 3. Portions of the vascular part, magnified five diameters.

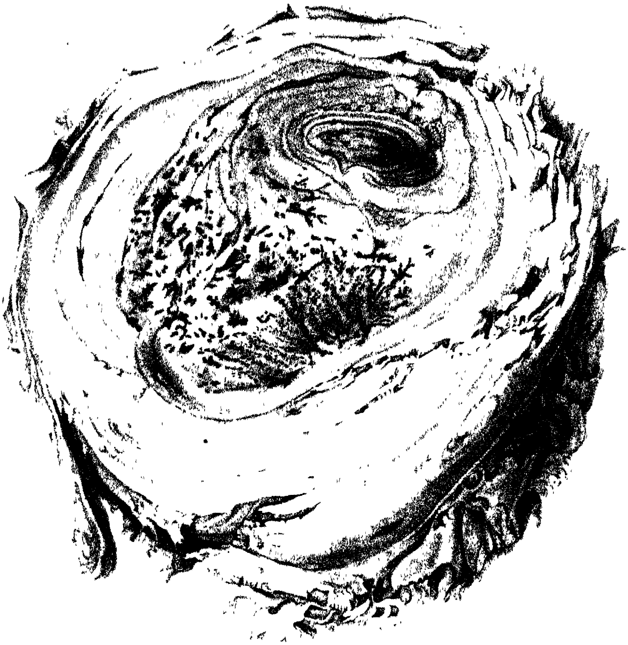
Fig. 4. A small portion of the colourless part, magnified 200 diameters, to show the lymph globules, of which it is made up.

Fig. 5. A similar portion of the vascular part, magnified in the same degree.

Fig. 6. A row of lymph globules, magnified 400 diameters.

Fig. 7. A row of blood globules deprived of their red colour; magnified 400 diameters.

Fig. 1



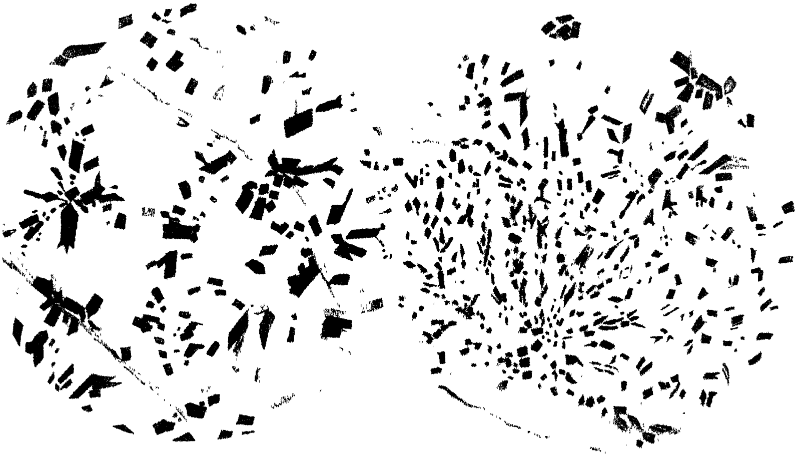


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Fig 1

Natural size

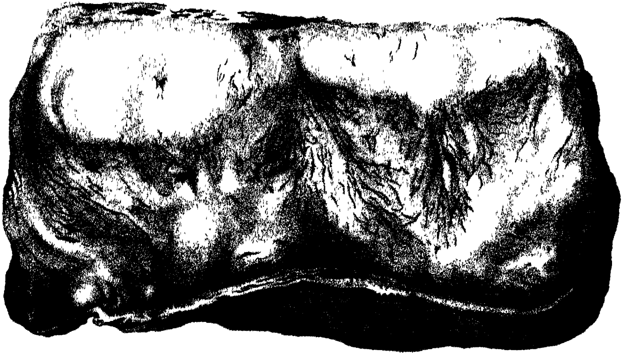


Fig 2

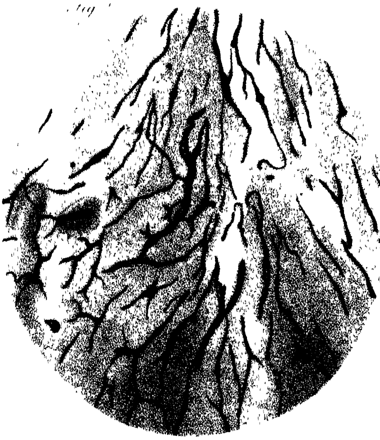


Fig 3

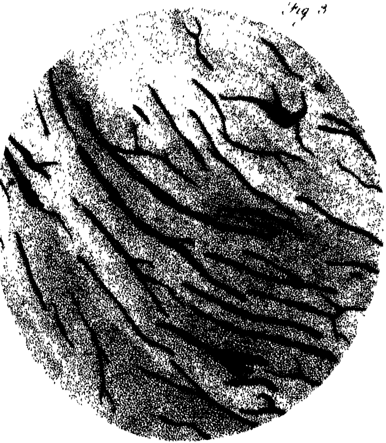


Fig 6

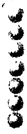


Fig 4

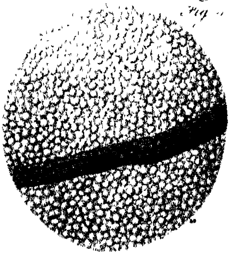


Fig 5

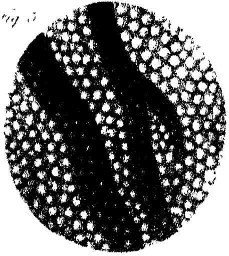
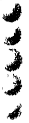


Fig 7



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II. *The Bakerian Lecture. On the composition and analysis of the inflammable gaseous compounds resulting from the destructive distillation of coal and oil, with some remarks on their relative heating and illuminating powers.* By WILLIAM THOMAS BRANDE, *Esq. Sec. R. S. Prof. Chem. R. I.*

Read November 18, 1819.

THE experiments detailed in the following pages, were originally undertaken with a view of ascertaining the relative fitness of the gases obtained by the decomposition of coal and oil for the purposes of illumination, and of elucidating some apparent anomalies in their economical applications. Merely as such, however, I should not have deemed them of sufficient novelty or importance to form the subject of the Bakerian Lecture; but during the progress of the inquiry, some new views relative to the constitution of these gaseous mixtures, suggested themselves, and some properties of terrestrial radiant matter became apparent, which I trust will be thought worthy the attention of this Society.

SECTION I.

On the inflammable gases afforded by the destructive distillation of pit coal and of oil.

THE gases used in the following experiments, except where it is otherwise expressly stated, were those employed for the common purposes of illumination; the coal gas being that supplied from the Company's works in Westminster, and the oil gas furnished by the decomposition of common whale oil, in an apparatus erected for that purpose by Messrs. TAYLORS and MARTINEAU, at Apothecaries' Hall.* These gases have been submitted to analysis by different chemists of eminence; and we are more especially indebted to Dr. HENRY for a series of valuable researches respecting their production and composition.† It is therefore with considerable diffidence that I venture to propose views relating to them in many respects different from those of my predecessors in this important branch of chemical inquiry.

It is generally admitted, that there are two definite compounds of carbon and hydrogen; the one, usually termed *olefiant gas*, consisting of *one* proportional of carbon and *one* of hydrogen; and the other called *light hydrocarburet*, composed of *one* proportional of carbon and *two* of hydrogen: the former of these gases appears to have been discovered in 1796, by the associated Dutch chemists, Messrs. BONDT, DIEMAN, VAN TROOSTWICK, and LAWERENBOURG,‡ and the other first examined by Mr. DALTON.§ Assuming hydrogen as 1, the specific gravity of olefiant gas is 13,4; and it contains 1 proportional of carbon $= 5{,}7 + 1$ proportional of hydrogen $= 1$. Light hydrocarburet has generally been considered as consisting of 1 proportional of carbon = 5,7 + 2 proportionals of hydrogen = 2, and its specific gravity has been stated as 7,7 compared with hydrogen; or as 57365, assuming atmospheric air as 1.

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* A description and plate of this apparatus is given in the Quarterly Journal of Sciences, &c. Vol VIII. p. 120.

† NICHOLSON's Journal, Vol. XI. p. 65. Philos. Trans. 1808. Manchester Memoirs, Vol. III. New Series.

‡ Journal de Physique, XIV. § New System of Chemical Philosophy.

My first object in the examination of coal gas was to ascertain its specific gravity; and I was surprised to find the first that I examined so low as ,4430. There was some variation in different specimens; and the specific gravity of that prepared in the laboratory of the Royal Institution, and purified in the usual way by condensation in cold vessels, and passing through lime water, was as high as ,4940, which is the heaviest that I have yet met with.

Having been led to consider coal gas as consisting essentially of the two varieties of carburetted hydrogen, I imagined that the specific gravity of the light hydrocarburet must have been estimated too high; I therefore prepared light hydrocarburet from acetate of potash, and having separated its carbonic acid by lime, found its specific gravity ,687; the specific gravity of the gas from stagnant water, according to Mr. DALTON,* is ,600, and that from moistened charcoal when purified is ,480.† It became evident, therefore, that coal gas could not consist principally of the two hydrocarburets; nor could the presence of carbonic oxide be suspected, its specific gravity being ,9834. Hence it occurred to me, that the only mode of explaining these apparent anomalies, was to consider coal gas as a mixture of olefiant and hydrogen gases; and the following experiments were undertaken with a view to determine this point.

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* New System of Chemical Philosophy.

† HENRY's Elements, p. 320.

1. One hundred volumes of coal gas were detonated by the electric spark over mercury, with 200 of oxygen; the carbonic acid was absorbed by liquid potassa, and 36 volumes of pure oxygen remained in the tube. Whence it appears that 100 volumes of the coal gas under examination required for its perfect combustion 164 parts of oxygen; consequently, as 100 parts of olefiant gas require 300 of oxygen, and 100 of hydrogen 50, for their respective combustion, it might be concluded from the above experiments, supposing no foreign gases present, that the 100 of coal gas consisted of about 55 parts of hydrogen and 48 of olefiant gas; a mixture, of which 100 cubical inches would weigh nearly 15 grains, and which closely corresponds with the specific gravity of the coal gas.

2. One hundred measures of coal gas were introduced into a small bent glass tube containing a little sulphur, and inverted in mercury; a red heat was applied until the inclosed gas underwent no further dilatation; and on examining its volume when cold, it was found to occupy 140 measures. If we consider the increase of bulk as resulting from the decomposition of olefiant gas, this experiment gives the composition of coal gas 60 hydrogen and 40 olefiant by volume.

3. One hundred measures of coal gas were introduced into a mercurial gasometer, connected with a second gasometer by means of a platinum tube, in the manner described by Messrs. Allen and Pepys, in their Essay on the Combustion of Carbon.* Some small quartz crystals previously heated red hot were introduced into the platinum tube, which was heated bright red; the gas was then passed through it from

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* Phil. Trans. 1807.

one gasometer to the other for about a quarter of an hour. The apparatus having cooled, the gas was found to have sustained an increase of volume = 40 parts; it burned with the pale flame of hydrogen; and when detonated over mercury required scarcely more than half its volume of oxygen, and afforded a very minute portion of carbonic acid. The interior of the platinum tube was lined with charcoal, the crystals were covered with it, and some had assumed a beautiful brown tint.

4. The conclusions drawn from the last experiment are founded upon the supposition, that olefiant gas is decomposed by the simple operation of a high temperature, and that one volume is resolved into two volumes of hydrogen, losing at the same time its carbon. The importance of this fact, as connected with these researches, induced me to repeat with every requisite precaution, the beautiful experiment of M. BERTHOLLET, which consists in decomposing this gas by passing it repeatedly through a red hot earthen tube; instead of which, however, I employed a tube of platinum, arranged as in the last experiment, increasing the heated surface by the introduction of quartz crystals. One hundred measures of olefiant gas,* obtained by distilling alcohol with sulphuric acid, were passed and repassed through the tube heated to high redness, until they ceased to dilate: when the apparatus was cool, the volume of gas was almost exactly doubled; there was a copious deposition of charcoal in the part of the tube that had been ignited, and the evolved hydrogen was so

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* This gas was washed with solution of potassa to separate a little carbonic acid, and was then ascertained to be pure by the action of chlorine, with the precautions afterwards described.

free from carbon, that when detonated with its volume of oxygen, half a volume of the latter remained, which scarcely rendered lime water turbid, and underwent no perceptible diminution by exposure to liquid potassa.

It may be supposed, that in consequence of the dilution of the last portions of olefiant by the hydrogen evolved, the perfect decomposition of the gas is a matter of difficulty, and a trace of carbon will, I believe, always remain in the hydrogen evolved, since the decomposition is progressive. I cannot, however, on this account see reason to believe, with M. BERTHOLLET,* that carbon and hydrogen are capable of forming several definite compounds; the data are, on the contrary, such as to warrant an opposite conclusion.

In making this experiment in the manner just described, and more especially when the tube is only dull red, the first portions of gas that reach the receiving gasometer, are obscured by a considerable quantity of vapour, which, however, afterwards disappeared. To examine more particularly the cause of this phenomenon, I passed some pure olefiant gas, very slowly, through a red hot glass tube, about two feet in length, and containing in the heated part some pure and well burned charcoal: the gas was collected in a cold receiver, the sides of which became lined with a brown viscid substance of an agreeably fragrant odour, perfectly soluble in alcohol, and precipitated from this solution by water, which rendered it turbid, and of a whitish green hue. This peculiar resinous matter appears to be a compound of hydrogen and carbon; its vapour is perfectly decomposed by passing it through a highly heated platinum tube, hydrogen being evolved and carbon deposited.

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* THENARD, Traité de Chimie, Tom. I. p. 293.

5. Mr. Faraday, whose accuracy as an operator is not inferior to his assiduity as my Assistant in the Laboratory of the Royal Institution, has shown in a paper published in the Quarterly Journal of Science, that the supposed distinction between olefiant and light hydrocarburet, by means of the action of chlorine, has no foundation; and that at common temperature, all varieties of carburetted hydrogen are condensed by, and combine with, chlorine.

To ascertain how far the action of chlorine could be depended upon as a means of analyzing mixtures of olefiant and hydrogen gases, I mixed equal volumes of chlorine and hydrogen, over water at the temperature of 55°, in a tube of half an inch diameter, and exposed to ordinary daylight, but carefully excluded from direct sunshine. After twenty-four hours, the whole of the chlorine had been absorbed by the water, and the original volume of hydrogen remained unaltered.

One volume of hydrogen mixed with one of olefiant gas and two of chlorine, was reduced under the same circumstances to very little more than one volume, the whole of the olefiant having been absorbed.

In these cases it is convenient to use considerable excess of chlorine, and in this way the purity of olefiant gas may be ascertained; it will be found, even when obtained with every caution, to afford a small residue of hydrogen; but as this is sometimes as little as one per cent, it may, generally speaking, be disregarded.

6. The analysis of a mixture of hydrogen with carburetted

hydrogen, carbonic oxide, and carbonic acid, presents peculiar difficulties in the ordinary mode of proceeding; and as it often requires to be performed in investigations relating to the gases used for illumination, it became an object to facilitate the process, for which I have used the following plan.

A hundred measures of the gas are introduced into a graduated tube, and the carbonic acid absorbed by a solution of potassa; the remaining gas is then transferred to thrice its volume of chlorine of known purity, standing over water in a tube of about half an inch diameter, and exposed to daylight, but carefully excluded from the direct solar rays; after twenty-four hours the carburetted hydrogen and the excess of chlorine will have been absorbed, and the remaining gas, consisting of carbonic oxide and hydrogen, may be analysed by detonation with oxygen in excess; the measure of carbonic acid formed being the equivalent of that of the original carbonic oxide.

This proceeding depends upon the non-formation of chloro-carbonic acid in a mixture of carbonic oxide and chlorine in the contact of water, and out of the direct agency of the solar rays. Such mixture I have kept several days, occasionally renewing the chlorine as it became absorbed by the water, and have not observed any diminution in the bulk of the carbonic oxide. In all these cases it is necessary to ascertain the purity of the chlorine by its absorption by water, and to be aware of the evolution of common air from water during that process.

7. I repeated many of the above experiments, substituting for coal gas a mixture of six volumes of hydrogen with five of olefiant gas. The specific gravity of this mixture was,

,4700; one hundred cubical inches weighing 14,2 grains. The flame with which this mixture burned was of the same colour and intensity as that of common coal gas; its dilatation by heat was similar, and it underwent an analogous increase of bulk when heated with sulphur.

The readiness with which carburetted hydrogen is decomposed, when passed through red hot tubes, appears to me to offer a solid objection to a mode of purifying coal gas, which has been proposed by Mr. G. H. PALMER,* since it would deposit carbon, and consequently sustain great loss in illuminating power. The object in view was probably to get rid of the sulphuretted hydrogen; but neither is this so to be attained. In examining coal gas, I have often been struck with the formation of sulphurous acid during its combustion; though when passed through solution of acetate of lead, it occasioned no blackening, a circumstance which led me to suspect the presence of some other sulphureous compound; and I have often thought, in passing the open gas pipes in the streets, that I perceived the smell of sulphuret of carbon. When sulphurous acid or sulphuretted hydrogen are passed with carburetted hydrogen through a red hot tube, a portion of carburet of sulphur is always formed, and the vapour of that highly volatile compound may well exist in the gas employed for illumination, which is always hurried through the condensers and gasometer.

8. Most of the above experiments were now repeated upon the gas obtained by the decomposition of whale oil; its specific gravity was ,7690; so that 100 cubical inches weighed rather more than 23 grains. Deducing the compo-

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* PECKSTON, on the Theory and Practice of Gas-lighting, p. 213.

sition of this gas, considered as a mixture of hydrogen and olefiant, from its specific gravity, we should conclude that it is composed of 1 volume of hydrogen and 3 of olefiant, upon the presumption that 100 cubical inches of hydrogen weigh 2,25 grains, and 100 of olefiant 30,15.

Such a mixture, when submitted to the action of heat, of sulphur, and of chlorine, and when detonated with oxygen, afforded results similar to those obtained by experiments upon the original oil gas, and it burned with the same degree of brilliancy.

9. I have also submitted to similar experiments the inflammable gases obtained by the decomposition of acetate of potash, of alcohol, and ether, and by passing water over red hot charcoal. All these contain a considerable portion of carbonic acid, which, when abstracted by potassa, leaves a mixture of carburetted hydrogen, hydrogen, and carbonic oxide, in proportions liable to much variation, according to the materials employed, and to the circumstances under which their decomposition has been effected. The specific gravity of these products is of course liable to corresponding variations.

10. The inference which, I think, may be drawn from the preceding experiments and observations, is, that there exists no definite compound of carbon and hydrogen, except that usually called *olefiant gas;* that the various inflammable compounds employed for the purpose of illumination, and produced by the destructive distillation of coal, oil, &c. consist essentially of a mixture of olefiant gas and hydrogen; that the gas procured from acetate of potash and from moist charcoal contains the same elements, with carbonic oxide and

carbonic acid; and that no other definite compound of carbon and hydrogen can be recognised in them, except olefiant gas.

SECTION II.

Comparative experiments on the illuminating and heating powers of olefiant, coal, and oil gases, and on some general properties of radiant matter.

1. In the following experiments I employed a gasometer with counterpoise weights acting over regulating pullies, and capable of containing about 5000 cubical inches, or about 2,89 cubical feet: the different jets were attached to it in the usual way, and the pressure was measured by the difference in the level of the water within and without the bell, to which was attached an accurately graduated scale sliding through the frame of support.

2. Having filled this gasometer with pure olefiant gas, it was allowed to issue from a brass jet having a single perforation of $\frac{1}{60}$ of an inch diameter, under a pressure of a half inch column of water; it was then inflamed, and regulated by means of a stopcock, so as to produce a light equal to that of a wax candle burning with full brilliancy; the relative intensity of the light of these flames was ascertained by a comparison of shadows. Under these circumstances, the consumption of gas was found $=640$ cubical inches per hour, or 0,37 cubical feet. When the same burner was used with oil gas, it consumed 800 cubical inches per hour, or $=0,47$ cubical feet.

3. I now employed an Argand burner, with a cylindrical glass, constructed in the usual way, with 12 holes each of the

same dimensions as that of the single jet, and forming a circle 0,7 inch diameter. The pressure being 0,5 inch, the flame was so regulated as to burn with its full intensity without producing smoke, and its light being measured by a comparison of shadows, it was found equal to ten wax candles. The consumption of gas amounted to 2600 cubical inches, or about a cubical foot and a half per hour.

If the result of this experiment be compared with the above, in which a single jet was used, it will appear, that the proportion of light from a given quantity of gas is increased in a very high ratio by employing many flames near each other, the consumption of the single jet giving a light of one candle, being $=640$ cubical inches, whereas the Argand burner gave a light of ten candles, with a consumption not of 640×10 cubical inches, but of 2600 cubical inches. It will be remembered, that in the latter, the combustion is perfected by a central current of air, rendered more rapid by the glass tube which surrounds the flame. Count Rumford showed some time ago, " that the quantity of light emitted by a given portion of inflammable matter in combustion, is proportional in some high ratio to the elevation of temperature, and that a lamp having many wicks very near each other, so as to communicate heat, burns with infinitely more brilliancy than the Argand's lamps in common use."* The construction of the gas Argand burner is particularly calculated to produce an effect of this kind; and to such a cause the great increase of light relative to the consumption of gas may probably be attributed.†

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* Davy's Elements of Chemical Philosophy, p. 224.

† In the annexed drawing (Plate III.) is represented an Argand burner for oil

4. The gasometer being filled with oil gas, an Argand burner, giving the light of eight wax candles, was found to consume 3900 cubical inches per hour; and the same intensity of light was produced by the same quantity of artificial oil gas; that is, of a mixture of three parts of olefiant and one of hydrogen.

5. The apertures of burners for coal gas require to be considerably larger than those for olefiant or oil gas. In the burner employed in the following experiments, each hole was $\frac{1}{30}$ inch diameter, and the circle upon the circumference of which they were placed, was 0,9 inch diameter. The light of the flame was found equal to five wax candles only, and the consumption of gas per hour amounted to 6560 cubical inches.

With a mixture of six parts by measure of hydrogen with five of olefiant gas, the light of the flame was somewhat more intense; and the quantity of gas consumed by the same burner, so adjusted as not to smoke, was 6000 cubical inches.

6. It appears from the above data, that to produce the light of ten wax candles for one hour, there will be required,

2600 cubical inches of olefiant gas.
4875 - - - oil gas.
13120 - - - coal gas;

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gas, upon what I believe to be the most economical construction. The bevilling of the perforated edge contributes greatly to the perfection of the light, as shown in the section, Fig. 1. The diameter of the circle of holes is 0,7 inch, and the holes should not be more than $\frac{1}{30}$ of an inch in diameter. Consuming at the highest average 4000 cubical inches per hour, it gives the light of between eight and nine wax candles of four to the pound.

Fig. 3. is a sketch of what is technically termed a *rose burner*; it has six holes of the same dimensions as those of the Argand, and when so regulated as to produce a light equal to that of six wax candles, its greatest average consumption of gas amounts to 4800 cubical inches per hour.

and that the quantity of oxygen consumed

by the olefiant gas will be = 7800 cubical inches.
by the oil gas - = 11578.
by the coal gas - = 21516.

Olefiant gas cannot of course be employed for any economical purposes, and is only here adverted to for the sake of comparison. The relation of the quantity of oil gas to that of coal gas, furnishes a datum that may be practically useful, especially as indicating the relative sizes of gasometers required for the supply of establishments. It may, I think, be stated with sufficient accuracy for practical purposes, that a gasometer containing 1000 cubical feet of oil gas, is adequate to furnish the same quantity of light as one of 3000 cubical feet of coal gas, provided due attention be paid to the construction of the burners, and to the distribution of the lights.

7. For the ordinary purposes of illumination by oil gas, I consider ten hole Argand burners, each consuming about a cubical foot and a half per hour, and giving the light of seven wax candles, or nearly two oil Argands, as the most economical and generally useful. Single jet burners, or those in which the flames do not coalesce, consume, as has been above shown, a very much larger quantity of gas for the production of an equal quantity of light; and for the same reason, Argand burners, in which the flames do not coalesce, consume more gas for an equal production of light, than those in which the apertures are more numerous, but sufficiently near each other to allow of the union of the separate flames.

8. To ascertain the relative heating powers of the flames of olefiant, oil, and coal gases, I employed the twelve hole Argand burners mentioned above, and placed over each, as near to the lamp glass as was consistent with a clear flame, a

clean copper boiler, 2,5 inches deep and 5 inches diameter, slightly concave at bottom, capable of holding rather more than a quart of water, with an immersed thermometer, and a small vent for steam. It contained two pounds of distilled water, which was raised to the boiling point in similar times, namely, 20′ by each of the flames; so that it would appear, that to raise a quart of water from 50° to 212,° at 30 inches barometrical pressure, requires

870 cubical inches of olefiant gas,
1300 - - oil gas,
2190 - - coal gas.

From this experiment it may be inferred, that the air of a room equally lighted by oil and coal gas, will be much less heated by the former than the latter; but that the actual heating power of the flames is in the direct ratio of the quantity of olefiant gas.

9. Having occasion in some of the foregoing experiments to produce light of great brilliancy by the combustion of olefiant gas, and finding it very difficult to measure its intensity by a comparison of shadows, in the manner pointed out by Count Rumford, I endeavoured to avail myself of Mr. Leslie's photometer; for this purpose I concentrated the light by a plano-convex lens, and placed the blackened ball of the instrument in the focus. I found the effect, however, so great, as to lead me to believe that I had obtained a focus of considerable heating power, and on substituting a delicate mercurial thermometer, it rose 4°,5 in 5′. In the focus thus obtained from the light of a large Argand burner supplied with olefiant gas, the elevation of temperature was very sen-

sible to the hand, and in depressing and elevating the flame by means of a regulating stopcock, corresponding effects were produced upon the thermometer: the lens itself, which was a thick one, did not become heated.

These experiments coincide in result with those of Dr. MAYCOCK, and of M. DELAROCHE,* and show that the calorific rays emanating from common combustibles, are capable of passing through transparent media like those of the sun.

10. There are certain substances, the chemical relations of which are singularly affected by the influence of direct solar rays. Among these, the mixture of chlorine and hydrogen is most remarkable: if kept in common daylight, but out of direct sunshine, the gases do not act upon each other; but the moment the mixture is placed in the sunshine, the muriatic acid begins to be formed. I therefore hoped that this property might be applicable in certain photometrical experiments. I exposed a mixture of equal volumes of chlorine and hydrogen, in a tube inverted over water, capable of holding about four cubical inches, and blown into a thin bulb at its upper extremity, to the brilliant focus produced by a large olefiant gas flame; it was exposed for 15′, but underwent no other change than a slight increase of bulk, acting as an air thermometer.

11. It now occurred to me to try how far any effect would be produced by the more intense light of the Voltaic battery, and I placed the tube containing the mixed gases in a darkened room, within about an inch of the charcoal points connected with an apparatus of one hundred pairs of plates highly

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* MURRAY's System of Chemistry, Vol. I. p. 336. 4th. Edition.

charged: upon making the contact, the effect of the light upon the mixed gases was very remarkable; fumes of muriatic vapour were instantly produced, the water rose in the tube in consequence of the production of muriatic acid, and in about five minutes the absorption was entire; but the most curious circumstance was, that in two instances an explosion of the gases took place the moment they felt the impulse of the electric light.

12. As I have in no case been able to produce an analogous effect by any other terrestrial light, however intense, I cannot but consider the phenomenon as dependent upon some peculiar property belonging to the rays of solar and electric light.

The lunar rays produce no effect upon mixed chlorine and hydrogen, nor upon chloride of silver; neither was the whiteness of the latter in the slightest degree impaired by the most powerful luminous focus that I could obtain from an olefiant gas flame.

13. In some experiments connected with the subjects of this communication, I have availed myself of a photometric thermometer, acting upon the principle of that described by Mr. LESLIE, but infinitely more sensible; it is constructed nearly in the same way as the differential thermometer, but instead of containing air, the balls are filled with the vapour of ether, and the stem contains a column of that liquid; it thus forms a very delicate differential thermometer. To convert it into a photometer, the upper bulb is covered with a thin coating of Indian ink, and the lower one with silver or gold leaf; the whole instrument is then placed in a pellucid glass tube: when taken out of its case the influence of light

is perceived at the instant of exposure, by the falling of the liquid from the blackened to the metallic side; it is powerfully influenced by the flame of a candle at the distance of one foot, and proportionally by other luminous bodies.

Fig 1

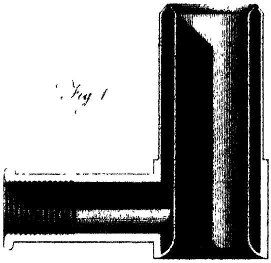


Fig 2

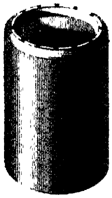
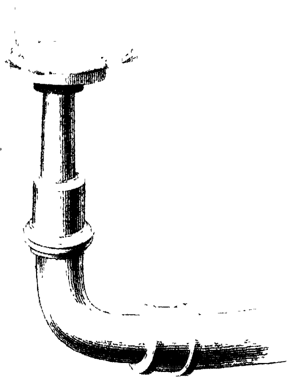


Fig 3



III. *On the Elasticity of the Lungs. By* JAMES CARSON, *M. D. Communicated by* THOMAS YOUNG, *M. D. For. Sec. R. S.*

Read Nov. 25, 1819.

IN a Treatise which I published a few years ago on the motion of the Blood and the mechanism of Respiration, it was contended, that a cause essential to the performance of these functions, had escaped the notice of physiologists. This cause was stated to be the elasticity or resilience of the lungs. The resilient property of the substance of the lungs had indeed been admitted by all anatomists and physiologists; and it is commonly demonstrated in the lecture room, that, if a piece of the substance of the lungs be cut out and stretched, it will recover its former dimensions when released from the extending power. But though the existence of this property had been universally admitted, no physiologist had attempted, so far as I know, to explain the means by which nature had contrived to render it subservient to the purposes of life. The statement and explication of this contrivance, with reference at least to certain purposes, constitute in a great measure the subject of the treatise to which I have alluded. Although it was proved in that Treatise, that, for the performance of those movements in which life is acknowledged chiefly to consist, a power of considerable extent is derived from the elasticity of the lungs, it was at the same time confessed, that no data had been discovered, from which the full extent of that power, as it is applied in the living system, could be calculated.

It was conceived that it would be a matter of no small importance to ascertain the extent of a power which, as I believe, discharges a part of the first importance in the scheme of life. With that view, a number of experiments have been performed, which I hope will be found to determine, in a considerable number of animals, the extent of the elasticity possessed by the lungs in their state of expansion in the living and sound body; or the extent of a power by which the heart and diaphragm, and perhaps various other organs, are as necessarily and as effectively influenced as the piston of the steam engine is by the expansive powers of steam.

To enable those Gentlemen who may not be familiar with the anatomy of the parts concerned, to perceive the tendency and import of these experiments, and to understand the arguments that may be advanced, it may be necessary to premise the narration with a short and general description of the structure, position, and connections of the lungs. It is hoped that no person, who has at all applied his attention to mechanical philosophy, will find any difficulty in comprehending this description of a simple but beautiful machinery.

The appearance of the lungs, or, in common language, the lights of oxen and of other animals slaughtered for the use of man, is familiar to every one. They are that irregular pale red spongy mass, which is daily seen suspended in the shambles by the windpipe. The windpipe, or in the language of anatomists, the trachea, which is a necessary appendage to the lungs, extends from the throat to the top of the chest, It is composed of a succession of cartilaginous rings, which nearly form complete circles. Each ring is connected to the next to it in succession by a strong elastic membrane. A small segment is cut off from each of those rings behind, where the place of cartilage is supplied by the same strong elastic membrane by which the rings are connected. By this contrivance the rings may be expanded, and the diameter of the windpipe is rendered susceptible of variation as well as its axis.

As soon as the windpipe has fairly entered into the confines of the chest, it divides into two branches, one of which proceeds to the right and the other to the left side. These branches possess the same structure with the windpipe itself, and each as it proceeds is divided into smaller branches, which again and again ramify into others still smaller. The branches for some time are formed like the trunk from which they proceed, but at length they drop the cartilaginous structure, and become simply membranous tubes. These tubes ultimately terminate in cells, between which there is a free passage for the air. The interstices between these tubes and cells are filled up by glands, the common cellular substance, and by blood vessels and nerves. The whole of this mass is enveloped in a fine membranous covering, termed the pleura pulmonalis, which however is impervious to the air. To each of the two original divisions of the windpipe a distinct lung is appended, called the right and left, between which there afterwards exists no communication.

The lungs occupy that part of the animal frame denominated the chest. This, in man, is the upper part of the trunk of the body. The shell of the chest is composed of bone, cartilage, muscular substance, and membrane. Behind, on each side, and in front, it is osseous. It is fenced by the spine behind, by the breast bone in front, and on each side by a tier of ribs, which are connected at one end to the spine by articulations, and at the other end to the breast bone by elastic cartilages The intercostal muscles secure the spaces between the ribs. The ribs on one side with the corresponding ribs on the other, and with the intervening parts of the spine and breast bone, form the circumferences of so many circles. The circle formed by the junction of one rib with its corresponding rib on the other side, is largest at the bottom, till at the top it is reduced to so small a diameter, as to serve for little more than a passage for the windpipe and gullet. Strong muscles and tendons traversing each other from the upper rib on one side to that on the other, and to the vertebræ of the neck and to the clavicles, and connected by a union of membranes with the exterior surface of the tubes, to which they afford a passage, form the secure boundary of the chest at the top. The form of the chest is evidently conical. The floor or base of this cone is formed by a thin circular plate, partly fleshy and partly membranous, but every where flexible and yielding, with its rim all around firmly concreted with the shell of the chest. It is called the diaphragm or midriff. It cuts the trunk of the body transversely, and separates the cavity of the chest just described from that of the belly. One condition of this flooring is worthy of particular notice. Its area is more extensive than that of the largest transverse section of the chest. In consequence of this, it admits of motion to a considerable extent; and, for reasons which will be afterwards explained, assumes in the living and sound state of the parts, the form of a cone, of which the apex looks towards the chest.

The form of the diaphragm, or base of the chest, may be illustrated by the comparison of a wine bottle; the bottom of which is convex inwardly and concave outwardly; the diaphragm bearing in form and position the same relation to the chest, which the bottom does to the rest of the bottle.

The cavity above the diaphragm is chiefly filled by the lungs, which therefore in the living system occupy a space nearly as large as the shell of the chest.

As soon, however, as the chest has been opened, and the external surface of the lungs exposed to the contact of the circumambient air, these organs shrink into dimensions far less extensive than those which they occupy in the living body.

The causes of the lungs being expanded in life into larger dimensions than those which are natural to them, or which they occupy when they have been extracted from the body, are curious and important, and may be thus explained.

The walls or boundaries of the chest are well secured. They are at several places indeed perforated by tubes, but these tubes, at the place of their entrance or exit, are, as has been described, always securely concreted by an union of membranes with the substance of the chest. The chief of these perforations are made by the windpipe, the gullet, and some large blood vessels at the top; and at the bottom, by the gullet again in its passage out of the chest, the great descending aorta, and the inferior cava. Conceive then, the lungs to be placed in this cavity in such a manner that the windpipe should pass out of it at the top, having its exterior surface well secured to the part of the chest at which it passes, but affording, in consequence of its cartilaginous and incompressible structure, an open passage into the interior substance of the lungs, and conceive that no substance should exist between the exterior surface of the lungs, and the internal surface of the chest. The pressure of the atmosphere, it is evident will, in these circumstances, necessarily bring the external surface of the lungs into contact with the whole of the interior surface of the chest; but as the walls of the chest are strong and incompressible, and as the lungs are pliant and dilatable, the lungs are necessarily expanded to the dimensions of the chest, which remain nearly the same. But the lungs, though dilatable, are powerfully elastic; and, when distended to the dimensions which they are thus forced to occupy, their substance is stretched far beyond its natural condition. As soon, however, as the full weight of the atmosphere shall be allowed to ponderate upon the external surface of the lungs, and an equal pressure shall be sustained by their exterior surface and the interior surface of the air vessels, of which their substance is composed, a condition which will arise as soon as the air shall have obtained a free passage through the walls of the chest to the external surface of the lungs; these organs will then shrink into the dimensions which are prescribed to them by the nature of their structure. To ascertain, then, the extent of the power required to distend the lungs to the dimensions which they occupy in the sound system, or, in other words, the extent of that force by which the walls of the chest are pressed, or, in popular language, sucked inwards, in consequence of the resilience of the lungs, is the object of the following experiments.

For the purpose of these experiments, an apparatus of glass, of the following simple construction, was used. An oblong glass globe, containing nearly two quarts, had tubular openings at each end, A, and B. (Plate IV.) A glass tube, nearly three feet in length, and bent at one end, was joined by the blowpipe to the opening at B, and is represented by B, C. To the other opening at A, a shorter tube was joined in the same manner, and in the form A, D. A free passage was established from D to C, where the tubes were both open. To D, a piece of the dried gut of some small animal was bound, of a few inches in length. The other end of the gut was fixed to a cylindrical tube of bone, metal, or wood, also of a few inches in length, and of a diameter corresponding with the diameter of the windpipe of the animal which was to be the subject of the experiment. The windpipe of an animal, which had been recently killed, was divided across near the throat, and separated by dissection from the rest of the neck, nearly to the top of the chest.

The first experiment was made on the 27th of August, 1817, on a cat, which had been hanged the day before. A small cylindrical tube of bone attached by gut to the end of the glass tube, A, D, was inserted into the windpipe, which had been prepared in the way described, and which was tied to the cylinder so tightly, that no air could pass between the external surface of the tube and the internal surface of the windpipe. An open and secure passage was thus established between the glass apparatus and the windpipe, and of course the lungs of the animal. Water was then poured into the apparatus at C, until it stood in the upright tube C B, at the height of eleven inches above the level of the water in the glass globe. An opening was then made through the chest of the animal on each side, and the air admitted into contact with the external surface of both lungs. The water instantly sunk about two inches in the upright tube, and the lungs were gently pressed out at the openings. Hence it was inferred, that the spring given to the air by the pressure of a column of water nine inches high, was stronger than the elasticity of this animal's lungs expanded to the dimensions of the chest. To my surprise and disappointment, the water began to sink still lower in the glass tube, and stood at last at the height of an inch above the level of the water in the globe, and the lungs at the same time gradually collapsed. Water was again poured into the upright tube, till it stood for a few seconds at the height of nine inches above its level in the globe, and the lungs again filled the chest. Upon applying the ear to the openings, the cause of the collapse of the lungs was discovered. The sound of air was distinctly heard pressed from the lungs at the openings. Hence it was concluded, that the pleura pulmonalis had been wounded in opening the chest.

On the 28th of August of the same year, a bullock recently slaughtered was made the subject of experiment. The same apparatus, using only a pipe of a larger diameter to be inserted into the larger windpipe of the animal, was applied as in the preceding case. Water was poured into the upright tube at C, until it stood at the height of one foot above the level of the water in the globe, and at that of four inches from the top of the tube. Openings were then made in the chest to admit the air. The water instantly rose in the tube two inches higher, and remained stationary at that point. The lungs, to appearance, were nearly collapsed to the usual degree. It was evident, from the ascent of the water in the upright tube upon the chest being perforated, that the spring given to the air by being compressed by a column of water twelve inches high, was not sufficient to balance the elasticity of the full dilated lungs of this animal. Water was poured into the apparatus till the tube remained filled. The lungs, with this additional pressure, still continued much shrunk. As the height of the tube was not great enough to ascertain the extent of the pressure necessary to balance the resilience of the lungs of animals of this size in the state of their usual expansion in the living body, an alteration in the apparatus became necessary.

On the 11th of September, 1817, a bullock was made the subject of experiment, with an apparatus of the same kind, but with a taller upright tube. Water was poured into the apparatus till it stood in the upright tube twelve inches above its level in the globe. The thorax was then opened. The water instantly ascended an inch and a half, and remained stationary. More water was then poured into the apparatus; but when it had risen an inch higher in the tube, the globe was found to be full. A further alteration therefore in the apparatus becomes necessary, before the resilience of the fully distended lungs of an ox can be ascertained; for the lungs continued shrunk to a considerable degree, in opposition to the spring of air compressed by a column of water of fourteen inches in height.

On the same day another bullock, with a less capacious chest, was made the subject of experiment with the same apparatus. Water was poured into the apparatus, as in the preceding instances, till it stood sixteen inches above the level of the water in the globe. An opening was then made into the abdomen. A fold of the fleshy part of the diaphragm was then drawn down on each side, and care being taken that no part of the lung was included in the fold, it was cut into. The sound of a current of air pressing through the openings into the chest was distinctly heard, and the water in the upright tube arose instantly to the height of eighteen inches above the level of the water in the globe. The diaphragm, before the openings, was still tense, and slightly concave towards the belly; but after that it became lax, wrinkled and flat. Some additional water was poured into the apparatus; but we were prevented in this case, as in the preceding, from ascertaining the amount of the force requisite to distend the lungs to the dimensions which they usually occupy in the living animal, by the deficient capacity of the glass globe.

By these experiments, I think it is clearly ascertained, that the spring of air compressed by a column of water of a foot and a half high, is not equal to the resilience of the lungs of an ox, at the usual stage of their dilatation.

In all these experiments, the oxen were placed upon the back with the shoulder raised a little above the rest of the body: some share of the collapsing effort of the lungs might be imputed to their specific gravity. But when the levity of the lungs is considered, and also that they were observed not to shrink more from the breast to the spine than from the diaphragm to the neck, in opposition in this case to gravity, and that their dimensions were not increased, nor their form varied by any change of position; little of the resistance which the lungs made to the spring of the contained air, is imputable, I think, to their specific gravity.

On the 16th of September, 1817, the apparatus was applied to a calf. When the water in the upright tube had reached the height of fourteen inches above its level in the globe, the lungs appeared to be distended to the full capacity of the chest. Upon cutting off the communication between the apparatus and the animal, the water instantly fell to its level, and the lungs shrunk into very small dimensions. The animal, in this experiment, was placed with the chest erect, so that the shrinking of the lungs upwards from the diaphragm to the neck, and which was observed to be as great as in any other direction, must have taken place in opposition to the specific gravity of the lungs.

In almost every experiment in which the chest was perforated by a sharp instrument, the lungs were found to be wounded, and the object in a great measure defeated. In the experiment about to be described, and the last which will be detailed at this time, great care was taken to prevent the accident now mentioned.

On the 31st of October of the same year, the apparatus was applied to the prepared trachea of a dog, which had been hanged on the preceding day. Water was poured into the apparatus until it stood in the upright tube at the height of six inches above its level in the globe. The abdomen of the animal was opened, and the diaphragm freely exposed. A part of the muscular substance of the diaphragm was drawn down in a fold, which was done without difficulty; and care being taken that no part of the lung was contained in the fold, an incision was made into it. This was done on both sides. The water ascended instantly about an inch in the upright tube, and the lungs were found to have receded from the openings. Water was poured into the apparatus, until its level in the tube stood above that in the globe at the height of ten inches. The openings in the diaphragm were carefully extended along the chest, and the lungs exposed freely to view. They were now apparently dilated to the boundaries of the chest. In this experiment, the water in the tube remained steadily at the same height, and the lungs continued fully dilated. In those experiments in which the water was observed to descend slowly, and the lungs to collapse gradually, it is very evident, that the lungs must have been wounded, so as to allow some air to escape through the pleura.

The appearance which the lungs exhibited in this situation was novel and interesting, and was no doubt the same which they would have exhibited in the living body, had it been possible to bring them into view. Their surface was smooth and polished, and their edges rounded, without any of those corrugations and sharp angles which they usually exhibit. Their colour was red, and life-like. They felt firm to the touch. The heart appeared like a bird nearly covered by its nest.

The state of the diaphragm in this experiment was worthy of remark. Before the incisions were made into it, it still exhibited a degree of concavity towards the belly, but not with the same degree of tenseness which it is generally observed to possess; for a fold of it was taken with ease, which in ordinary cases, before the chest has been opened, is done with difficulty. As soon as the openings were made into the diaphragm, it became lax, flat, and corrugated.

Frequent repetitions of these experiments, and much care and accuracy in conducting them, would be required, before the amount of the resilience of the lungs, in all the conditions in which it may be supposed to exist, could be estimated. But defective as these experiments in many respects are, the object, for which they were instituted, seems to have been fully attained by them. In the Inquiry into the causes of the motion of the blood, it was contended, that the elastic substance of the lungs, in consequence of the degree to which that substance was stretched in the living body, generated a permanent power of great extent, and that this power was employed by nature to circulate the blood, and to carry on the process of respiration. The existence of this power was inferred, from the elastic property of the substance of the lungs themselves; from the space which those organs must fill in the living body; from the phenomena exhibited upon opening the chest and admitting the external air; and from the ebullition on the surface of the water into which the inverted windpipe of an animal had been inserted, as soon as the lungs were allowed to collapse. In the various examinations which my opinions have undergone, the existence of this power has been admitted, and the claim to priority in the detection and application of it freely conceded to me; but it has been contended, that the amount of it, in some instances, is inconsiderable, and consequently that the effects ascribed to it have been greatly over-rated. By these experiments, the power has been proved to be greater, than my anticipations even made it, and fully adequate to all the important offices which I have ventured to assign to it. From a defect in the apparatus, the extent of the power in question could not be ascertained in the lungs of oxen and animals of their size; but it was proved to exceed, considerably, the force necessary to support a column of water of a foot and a half in height above its level. In calves, sheep, and in large dogs, the resiliency of the lungs was found to be balanced by a column of water varying in height from one foot to a foot and a half, and in rabbits and cats, by a column of water varying in height from six to ten inches.

The method by which nature has contrived to apply this powerful engine to the heart and diaphragm, I have attempted to explain at length in the treatise already mentioned. In a preceding part of this paper, however, I have stated, that during the life of the animal, and after death, until an opening shall have been made into the cavity of the chest, the diaphragm assumes the form of a cone, and that the causes of this phenomenon would be afterwards pointed out. The brief explanation which is now to be given of this appearance, will afford, at the same time, a perspicuous view of some of the important purposes to which, in my opinion, nature has turned the elasticity of the lungs.

While the chest is in a sound state, a balance of atmospherical pressure ponderates against the external surface of its walls; or these are pressed inwards more than they are pressed outwards by a given weight. The shell of the chest possesses sufficient stability to resist this pressure without changing in any considerable degree its form and capacity at all parts, except at the base, or diaphragm; which being muscular, pliant, and of a more extensive area than that of the transverse section of the chest, is in consequence of the greater weight sustained by its outward or inferior surface, necessarily pressed, or, in popular language, sucked upwards in the form of a cone. The extent of this cone will be necessarily regulated by the extent of the area of the diaphragm, compared with that of the area of the transverse section of the chest. But the contraction of the muscular fibres of the diaphragm diminishes its area, and reduces it to a nearer equality with the area of the transverse section of the chest, and thus diminishes the magnitude of the diaphragmatic cone, and in an inverse proportion enlarges the boundaries of the chest. But the diaphragm at the succeeding relaxation of its fibres, is restored to its former dimensions; becomes capable of being swelled into a larger cone; and, by this encroachment, reduces the boundaries of the chest to their former limits.

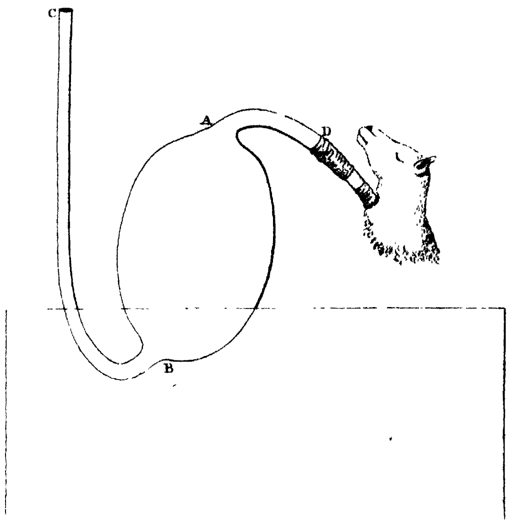
Two powers are therefore concerned in regulating the movements and in varying the dimensions and form of the diaphragm, the elasticity of the lungs, and the contractile power of the muscular fibres of the diaphragm. Of these powers the one is permanent and equable, the other variable and exerted at intervals. The contractile power of the diaphragm, when fully exerted, is evidently much stronger than its antagonist, the resilience of the lungs; but the latter not being subject to exhaustion, takes advantage of the necessary relaxations of the former, and rebounding, like the stone of Sisyphus, recovers its lost ground, and renews the toil of its more powerful opponent.

Breathing is in a great measure the effect of this interminable contest between the elasticity of the lungs and the irritability of the diaphragm.

The cause of the successive contractions of the diaphragm, in those cases at least in which the will is not concerned, seems to admit of the following explanation. A permanent and invariable load is sustained by its lower surface. By this load the relaxed muscular fibres become stretched to a degree which at length becomes painful and stimulating. To relieve itself from this irksome burden, the diaphragm is roused to contraction; but this contractile power, agreeably to the laws of muscularity, is soon exhausted, and falling into a quiescent state, allows the painful and stimulating distension of the relaxed fibres of the diaphragm to be again renewed. From the irksomeness of this condition it relieves itself by a fresh contraction. Thus, by the alternated superiority of two powers, on the balancing of which life itself depends, the chest is successively enlarged and diminished, and air alternately expelled and inhaled.

In a similar and equally effective manner, the elasticity of the lungs will be found to influence the movements of the heart and the motion of the blood. But as these movements are of a more complicated nature; as the consideration of them would lead into a long detail; and as my opinions respecting them are already fully recorded, I will not pursue the inquiry farther at this time.

In investigating the manner in which the elasticity of the lungs is rendered subservient to the great functions of respiration and the circulation, I shall be found, I trust, to have opened to the views of the physiologist, a new and a boundless field, which, I will venture to predict, will reward every skilful and assiduous cultivator with a rich harvest of discovery.



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IV. *On the action of crystallized bodies on homogeneous light, and on the causes of the deviation from* NEWTON'S *scale in the tints which many of them develope on exposure to a polarised ray. By* J. F. W. HERSCHEL, *Esq. F. R. S. Lond. and Edin.*

Read December 23, 1819

SINCE the period of the brilliant discovery of MALUS of the polarisation of light by reflection, the investigation of the general laws which regulate the action of crystallized bodies on light, has advanced with a rapidity truly astonishing, and the labours of an ARAGO, a BREWSTER, and a BIOT, have already gone far towards completing the edifice of which that distinguished philosopher laid the foundation. When MALUS wrote, the list of doubly refracting crystals was small, and the most remarkable among them possessing only one axis of double refraction, it seems to have been for some time, tacitly at least, presumed that the law discovered by HUYGENS, and since re-established in the most rigorous manner for that one,* might hold good in all. The discovery, by Dr. BREWSTER, of crystals possessing two axes of double refraction, or two directions in which a ray may penetrate their substance without separation into distinct pencils, has proved the fallacy of any such generalization, and rendered it necessary to enter on a far more extensive scale of investigation.

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* The author of the article on Polarisation, in the 63d Number of the Edinburgh Review, just published, is guilty of a most unpardonable mistake, in asserting, (p. 188), as deducible from Dr. BREWSTER's experiments, that the Huygenian law is *incorrect*, for carbonate of lime. Dr. BREWSTER's general formulæ for crystals with two axes resolve themselves into the Huygenian law when the axes coincide, of which case it is only an extension. That excellent philosopher, if I understand English, in the paragraph which gave rise to this strange assertion, only means to declare his opinion that it remains undemonstrated.

There are two methods which may be pursued in observations on double refraction and polarisation, the one direct, the other indirect. The former turns on immediate observations of the angular deviation of the extraordinary pencil, and is, of course, only applicable when the forces which act exclusively on the rays composing it are sufficiently intense to cause a sensible separation of the two pencils. There exist, however, a multitude of crystals in which the force of double refraction is so feeble as to produce scarcely any, or at most a very inconsiderable deviation of the extraordinary ray, and in which, consequently, the laws of double refraction could neither be investigated nor verified, without having recourse to some artificial means of magnifying the quantity to be observed; a thing easy enough in theory, but requiring, in practice, the greatest nicety on the part of the observer, and in many cases altogether impracticable, from the physical constitution of the crystals themselves. The indirect method depends on the discovery of Arago, scarcely inferior in intrinsic importance to that of Malus, of the separation of a polarised ray into complementary portions by the action of a crystallized lamina. It was reserved, however, for the genius of M. Biot, to trace this striking phenomenon to its ultimate causes, in the action of crystals on the differently coloured rays, and to develope, in a simple and elegant theory, the successive gradations by which the polarisation of a ray in its passage through a doubly refracting crystal is performed;

while, on the other hand, the splendid phenomena of the polarised rings, which we owe to Dr. BREWSTER, have established the connection of the tints so polarised with the force producing the deviation of the extraordinary pencil, and shown the legitimacy of conclusions respecting the intensity of the latter, drawn from observations on the former.

This indirect mode of observation, which consists in noticing the gradations of colour for different positions and thicknesses of the crystal, possesses three capital advantages. The first is its extreme sensibility, which enables us to detect the existence, and measure precisely the intensity of forces, far too feeble to produce any measurable deviation of the extraordinary pencil. It, in fact, affords the rare combination of an almost indefinite enlargement of our scale of measurement, with a possibility of applying it precisely to the object measured, arising from the distinctness of all its parts. Another, no less precious, is the leading us by mere ocular inspection to the *laws* of very complicated phenomena, and enabling us to form, and mould as it were our analytical formulæ, not on a laborious, and sometimes deceptive discussion of tabulated measures, but on the actual form of the curves themselves, which are loci of the functions under consideration. It is true, that a reference to tabulated measures is indispensible to give precision to such first approximations; but the power this mode of observation affords of copying our outline fresh from nature, and from the general impression of the phenomena, brought at once under our view, is an advantage not to be despised. Nor ought we, lastly, to omit, in our estimate of advantages, the means thus afforded us of subjecting the minutest fragments of a crystal to a scrutiny as severe as the most splendid specimen, and thus extending our researches to an infinitely greater variety of natural bodies, than we could otherwise hope to examine.

In order, however, to render observations on the tints developed by polarised light available, they must be comparable to each other; and it therefore becomes an object of the first importance, to ascertain the existence, and discover the laws of any causes which may operate to disturb their regularity. Ever since I first engaged in experimental enquiries on the polarisation of light, I was struck by the very considerable deviation from the succession of colours in thin laminæ, as observed by NEWTON, which many crystals exhibit when cut into plates perpendicular to one of their axes. I at first attributed this to a want of perfect regularity in their structure, or to inequalities in their thickness, arising from my own inexpertness in grinding and polishing their surfaces; and it was not till habit had rendered me familiar with all the usual causes of deception, that, finding the same phenomena uniformly repeated in different and perfect specimens, my curiosity became excited to enquire into their cause, the more so, as they now began to assume the form of a radical and unanswerable objection to the theory of M. BIOT, above alluded to, which affords so perfect an explanation of the tints in crystals with one axis.

These phenomena have not escaped the vigilance of Dr. BREWSTER. In his paper of 1818, he distinctly notices the fact of a deviation from NEWTON's scale, in crystals with two axes, and promises a more detailed account of it, which however has not yet appeared. But the object of the present communication is not thereby anticipated, as in the only passage in that paper in which he expresses himself otherwise than obscurely on its cause, he appears to regard the deviated tints as analogous to those developed along the axis of rock crystal and by certain liquids; an analogy which, in the present state of our knowledge on that perplexing subject, it seems not easy to admit. In a paper too, which has lately appeared, containing the interesting observations of the same excellent philosopher on the optical structure of the apophyllite, he remarks the very striking deviation of the colours of this crystal from NEWTON's scale " in the first orders" of its rings; and while he remarks that such deviations are common enough, and indeed universal in crystals " in which the rings are formed by the joint action of two axes," seems to think this analogy close enough to authorize the substitution of two rectangular axes of a negative character for the single positive axis actually observed, according to his own peculiar and ingenious views on this subject. I lost no time in endeavouring to procure a specimen of this mineral, and by the kindness of my friend, Sir SAMUEL YOUNG, (to whom I owe more than one obligation of this nature) was favoured with one sufficiently transparent for optical examination. From my observations on this body, I think I shall be able to demonstrate satisfactorily, that the phenomena of the apophyllite depend on a principle distinct from that which produces the chief part of the deviation of tints in most crystals with two axes.

The course I propose to pursue is, first, to describe the phenomena themselves. I shall then show how these phenomena, complicated as they are in appearance, are all reducible to one very simple and general fact; viz. that the axes of double refraction differ in their position in the same crystal for the differently coloured rays of the spectrum, being dispersed in one plane over an angle more or less considerable, according to the nature of the substance. In many bodies, the magnitude of this dispersion of the axes is comparatively trifling, while in some, not otherwise remarkable for a high ordinary or extraordinary dispersive power, it is enormous, and must render all computation of the tints in which it is not taken into consideration, completely erroneous; and indeed obliterating almost every trace of the Newtonian scale of colour. We have here, then, a new element, which for the future must enter into all formulæ of double refraction pretending to rigour, and at the same time are presented with another very striking instance of the inherent distinction between the differently coloured molecules of light, which, since the time of Newton, every new step in optical science has tended to place in a stronger point of view. At the same time, by the easy and complete explanation this principle affords of all the more perplexing anomalies in the tints, the theory of alternate polarisation to which they were hitherto so palpable and formidable an objection, stands relieved from every difficulty, and may now be received as fully adequate to the representation of all the phenomena of the polarised rings, and entitled to rank with the fits of easy transmission and reflection, as a general and simple physical law. In fact, if we investigate by this theory a general analytical expression of the tint developed for any position and thickness of the plate, taking this element into consideration, it will be found to include all the phenomena, as far as they can be computed, while the *law* of dispersion remains unknown. But we may go yet farther. The nature of the formula furnishes an equation by which the actual quantity of the separation of the extreme red and violet axes may be deduced from observations of the tints of a very simple and accurate nature, being perfectly analogous in principle to the "method of coincidences," which has of late been applied with such success to the most delicate investigations in every department of physical science. The comparison of the results afforded by that equation, with those deduced by direct observation on homogeneous light, while it leaves nothing to desire in point of accuracy, leads to another important result, viz. that the proportionality of the minimum lengths of the periods performed by differently coloured molecules, in a doubly refracting crystal to the lengths of their fits of easy reflection and transmission, supported as it is by an induction of no ordinary extent and accuracy, is yet not universal, admitting a deviation to a very large amount. Hence must of course arise a kind of secondary deviation in the scale of tints. In crystals with two axes, however, this is masked by the much more powerful effect of the separation of the coloured axes; yet even there, is not altogether insensible in an extreme case. In the apophyllite, however, the agency of this secondary cause is placed in the fullest evidence. The application of our general formula to the anomalous tints of that body, while it proves incontestably the exact coincidence of the axes for all the coloured rays, points out at the same time a peculiarity in its action on the more refrangible extremity of the spectrum, of a nature so singular, so entirely without example in all the multitude of natural and artificial bodies hitherto examined, as to render me extremely desirous of prosecuting the research, with the aid of more perfect specimens, and improved methods of observation.

Having arrived at the general result of a dispersion of the axes by the sole consideration of the gradation of tints in plates of various thicknesses, it becomes interesting to verify it by direct and independent observation. This I have accordingly done; and the fortunate discovery of a substance in which it is of enormous magnitude, puts it in our power to render the fact sensible to the eye of the most unpractised observer, by an exceedingly simple experiment, to be described in its place.

II. *Of the general phenomena of crystals which develope tints deviating from* NEWTON'S *scale, by exposure to polarised light.*

In describing the phenomena, I shall at present confine myself to the tints developed along the principal section of the crystal, which is supposed placed in an azimuth 45° with the plane of primitive polarisation. The observations of the tints in this position are most easily made, and least liable to error, and we shall see presently that it would be superfluous as well as embarrassing to examine other situations, the law of the phenomena being completely deducible from this. In this series of observations, then, we traverse the polarised rings (Pl. V. fig. 1.) in the direction of their axis of symmetry AA′, passing through their poles P,P′ and centre O. Now if we subject to this examination any one of the following substances:

Sulphate of soda?	Arragonite,
Sulphate of baryta,	Sugar,
Nitrate of potash,	Hyposulphite of strontia,

it will be seen that the tints between the poles PP′ correspond to lower orders of colour, than would result from assuming P, P′, as the origin of the scale, and agree much better with the assumption of certain points *p*, *p*′ without the poles, as their zero, or commencement of the scale. The poles themselves, too, instead of being absolutely black, are tinged with colour; and the tints beyond them, instead of descending in the scale from the poles outwards, continue to rise till they reach their maximum (which is a white, more or less brilliant, or an absolute black) at the points *p*, *p*′; after which they descend again to infinity. Not that in any case they coincide precisely with the scale of Newton, even with this correction, but, except in extreme cases, approximate to it within some moderate limit of error.

If, on the other hand, we examine in the same manner one of the following bodies:

Tartrate of soda and potash,	Sulphate of magnesia,
Borax,	Topaz,
Mica,	

it will be found that the imaginary points *p*,*p*′ (which we shall call the *virtual poles*) from which the tints must be reckoned inwards and outwards, to produce the nearest possible agreement with Newton's scale, lie between the poles P, P′.

In all these crystals, as the thickness of the plate examined increases, the virtual poles *pp*′ recede from the actual ones PP′, at least in respect of the number of alternations of colour which intervene between them: in other words, the tint developed in the poles, or along the apparent axes of the crystal, descends in the scale of colour, as the thickness of the plate increases, and *vice versa*. In very small thicknesses, the tints approximate pretty closely to NEWTON's scale, or wholly coincide with it, while in very great ones, the tint developed in the poles is the composite white of the extremity of the scale. The angular distance, however, of the virtual poles from each other and from the axes, remains absolutely unchanged for all thicknesses; and this striking fact, which I have proved by numerous and satisfactory experiments, was first suggested for examination as a result of theory, and would equally hold good, as will presently be proved, for every conceivable law of double refraction.

The substances which I have examined most attentively, are sulphate of baryta, nitre, mica, and Rochelle salt, and the subjoined tables of tints developed for different inclinations in plates of the first and last of these, may serve as examples of the mode of action of the respective classes to which they belong on light, and will afford data for some calculations to follow. The first two columns contain the inclinations corresponding to similar tints of the incident ray on the moveable plate which carries the crystal, in the general apparatus imagined by M. BIOT, for observations of this kind. Were the plate cut in a direction precisely perpendicular to the optic axis, (or line bisecting the angle between those of double refraction) and adjusted with perfect accuracy on the instrument, the excesses or defects of these angles above or below 90° would represent the angles of incidence. Neither of these conditions were, of course, exactly fulfilled. But it is obvious that the small errors in these particulars (which were ascertained not to exceed 1° or 2°), must affect the computed angles of refraction on both sides of the perpendicular with equal and opposite errors. The same may be said of any error arising from a slight prismaticity of the plate, which, however, must have been extremely small, the plate having always been rendered parallel by the delicate test of the sphærometer, within a very few divisions.* Consequently, in calculating on these data, the mean angle of refraction determined by the simultaneous use of both observations, (their semi-difference being taken for the angle of incidence) may be expected to differ from the truth by an extremely minute quantity. The third column contains the tint developed in the ordinary pencil, and the fourth in the extraordinary. The last notices the remarkable points in the system of rings to which the tints and angles in the other columns correspond. The positions of the poles were determined by interposing a red glass between the crystal examined and the reflector used to polarise the incident light. The glass used for this purpose, was of that kind occasionally found in old church windows, and whose manufacture seems to be numbered among arts now forgotten. It transmits almost the whole of the red rays, and part of the orange, while it completely stops all the more refrangible colours. I have endeavoured in vain to procure a specimen, whose limits of transmission are more confined. Such are said to exist, though very rare, and in the absence of such, the indications of that employed may be taken to correspond to the mean red rays.

* Each equal to the 23809th part of an inch.

TABLE I. *Sulphate of baryta. Thickness of plate* = 0.11964 in.

Corresponding inclinations.				Ordinary pencil.	Extraordinary.	Remarks, &c.
°	′	°	′			
43	55	134	37	Pink	Very pale blue green	Tints beyond the poles.
45	0	133	42	Bluish green	Pink	
46	0	132	45	Rich pink	Beautiful green	
		132	0	Whitish	Dull purple	
46	58	131	37	Splendid green	Rich crimson	
		131	0	Blue	Yellow	
47	53	130	45	Crimson	Fine green	
				Yellow inclining to orange	Blue	
				Yellow	Purple	
48	51	129	30	Blue green	Rich crimson	
				Blue	Orange	
		128	58	Purple	Pale yellow	
				Rich crimson red	Pale green or greenish white	
				Fine orange	Light blue	
				Pale orange yellow	Dark blue	
50	58	127	45	White	Sombre & very narrow purple	
				Bluish white	Scarlet or fiery red	
				Light blue	Orange	
				Sombre greenish blue	Ruddy white, or very pale [orange	
52	0	126	50	Dirty and very sombre purple	White	
				Sombre and narrow pinkish red	White	
				Very pale violet or pinkish white	Greenish white	
				White	Dirty bluish green	
53	1	125	47	White	Narrow and very sombre violet [purple	Virtual poles pp′, or points of coincidence.
		125	30	White slightly yellowish	Violet	
		125	0	Pale and dirty olive green	Violet white	
				Very narrow violet	White	
54	8	124	30	Blue very sombre and narrow	White slightly yellowish	
				Light blue	Pale yellow	
				Bluish or greenish white	Indifferent purplish pink	
55	25	123	30	Yellowish white	Sombre and narrow purple	
				Light yellow	Dark blue	
				Dull orange pink	Pale greenish blue	
56	53	122	15	Sombre purple	Pale yellow green	
				Blue	Yellow	
57	30			Pale green	Fine pink verging to crimson	
57	50	121	0	Pale yellow	Purple	
58	15	120	15	Light yellowish pink	Greenish blue	The poles P, P′
				Rich pink	Bluish green	Tints between
59	12	119	35	Pale purple	Greenish white	the axes.
				Blue green	Crimson	
60	45	118	20	White	Very pale purple	
		117	40	Pink	Blue green	
62	0	117	5	Pale purple	Whitish	
62	58	116	25	Greenish blue	Pink	
63	25	115	35	White	Dull purple	
64	28	114	47	Pink	Bluish green	
65	40	113	20	Greenish blue	Pink	
67	30	111	30	Pink	Pale greenish blue	
69	15	110	0	Pale greenish blue	Pale pink	

In this plate the virtual poles correspond very nearly to the second minimum of the extraordinary pencil beyond the poles PP′. The same plate was now reduced by grinding to the thickness 0.08816 inches. In this operation, care was taken to grind away the side of the plate most distant from the eye only, leaving the other perfectly untouched and unimpaired in its polish. The plate being reduced to exact parallelism by the sphærometer, was again examined, the same side still remaining next the eye. By this arrangement the same angles of emergence from the posterior surface correspond rigorously to the same directions of the ray in the interior of the crystal, with respect to the axes of its molecules; and thus we avoid completely any errors which might arise from using plates cut at different angles, it being almost impossible to cut two plates precisely alike in this respect.

TABLE II. *Sulphate of baryta.* *Thickness* = in. 0.08816.

Corresponding inclinations.				Ordinary pencil.	Extraordinary.	Remarks, &c.
°	′	°	′			
58	0	120	0	Pink, verging to orange yellow	Blue somewhat greenish	Poles for the mean red rays.
				Yellow	Dark blue	
57	30	120	47	Pale yellow	Purple	
				Greenish white	Pink	Tints beyond the poles.
				Light blue	Yellow verging to orange	
				Dark blue	Bright yellow	
55	57	122	15	Sombre purple	Yellow white	
				Very indifferent sombre pink	Bluish white	
				Pale yellow	Dark indigo blue	
54	5	124	0	White	Sombre violet	
				Very pale violet white	Dusky greenish yellow	
52	57	125	33	Very sombre violet, almost black	Pure brilliant white	Virtual poles.
				Sombre and dirty olive green	White rather ruddy	
				Very pale blue	Orange white	
				White	Sombre orange or brick red	
51	30	127	0	White	Narrow purple	
				Orange white	Blue	
				Bright orange	Pale blue	
				Bright scarlet	Bluish white	
				Narrow crimson	White	
50	0	128	30	Purple	White	
				Blue	Yellow	
				Bluish white	Rich crimson	
48	40	130	0	Yellowish or greenish white	Purple	
				Orange yellow	Bright blue	
47	35	131	10	Rich crimson	Green	
47	10	131	28	Purple	Good yellow	
				Bright blue	Pink yellow	
46	7	132	33	Good green	Rich pink or crimson	
45	3	133	45	Rich pink	Splendid green	
44	33	134	30	Greenish purple	Pinkish white	
43	35	135	15	Good green, but pale	Fine pink	
42	23	136	30	Pink red	Greenish blue	
40	55	138	0	Pale bluish green	Pale pink	
39	30	139	50	Pale pink	Very pale greenish blue	

In this plate the virtual poles correspond to the second maximum of the extraordinary pencil. It is needless to detail the tints between the poles. The same plate once more reduced, with the same precaution to leave the posterior surface untouched, developed the following series of colours beyond the poles.

TABLE III. *Sulphate of baryta.* *Thickness* = 0.05758 in.

Corresponding inclinations.				Ordinary pencil.	Extraordinary.	Remarks, &c.
°	′	°	′			
60	57	122	50	Fine yellow	Indigo	Poles, for the mean red rays. Tints beyond the poles.
60	20	123	27	Pale yellow	Purple	
				White inclining to yellow	Dull crimson red	
				Bluish white	Dull orange	
				Indigo	Yellow	
57	55	126	3	Sombre purple	White	
57	50	126	7	Sombre reddish violet	White	
57	20	126	33	Dirty violet yellow	White	
		127	30	Pale yellow	Violet white	
55	40	128	2	White	Sombre violet	
55	33	128	20	Pure brilliant white	Black	Virtual poles.
		128	30	White	Sombre dirty green	
				Pale orange	Pale dirty bluish green	
53	40	130	10	Sombre orange or brick red	White	
53	27	130	30	Sombre and narrow purple	White	
				Blue	Ruddy white	
				Pale blue	Orange	
				Bluish white	Orange red	
		132	20	White	Narrow crimson	
51	3	132	40	Pale yellow, &c. &c.	Purple, &c. &c.	

Here the virtual poles p, p' correspond precisely to the first minimum of the extraordinary pencil.

In a plate of Rochelle salt, cut nearly, but not quite perpendicular to the optic axis, and whose thickness was 0.194425 inches, the rings beyond the poles were almost entirely obliterated, while those between them exhibited the following singular succession of colours, which will show to what an extent the deviation from NEWTON's scale is carried in this substance.

TABLE IV. *Rochelle salt, perpendicular to the optic axis. Thickness*=0.194425 in.

Corresponding inclinations.				Ordinary pencil.	Extraordinary.	Remarks, &c.
°	′	°	′			
201	30	330	10	White	White	Poles PP′ for
207	10			Exceeding pale blue	Exceeding pale pink	red rays.
209	0			Exceeding pale pink	Exceeding pale blue	
210	30			Very pale blue	Very pale pink	
212	30			Very pale pink	Very pale blue	
213	20			Very pale blue	Very pale pink	
215	25			Pale pink	Pale blue green	
216	40			Pale blue	Pale yellow pink	
218	0			Pink	Blue green	
219	29			Pale blue	Pale yellow pink	
220	58			Pink	Greenish blue	
				White	Very pale purple	
222	30			Blue	Yellow pink	
223	45	309	30	Yellowish pink	Greenish blue	
224	50	308	15	Pale greenish blue	Yellowish pink	
				Blue	Pale pinkish yellow	
226	10	307	0	Pale pink	Pale greenish blue	
		306	10	Pale yellow	Dark blue	
				White	Pale purple	
227	25			Bluish or greenish white	Very pale violet pink	
228	30			Blue	Very pale yellow	
				Violet almost imperceptible	White	
229	3	304	10	Pure white	Pure white	Virtual poles
				Exceeding pale yellow	A little violet	*p*, *p*′.
229	50			Pale yellow	Very narrow dark blue	
				Very pale tawny orange	Blue, sombre and pale	
231	20	302	20	Fine purplish crimson	Pale yellow green	
				Very pale purple	White	
232	25			Very pale green	Fine crimson	
				White	Pale purple	
234	0	299	20	Splendid crimson	Very pale green	
				Pale purple	White	
235	40			Very pale green	Rich crimson	
				White	Pale purple	
236	23	296	15	Rich crimson	Very pale green	
				Pale purple	White	
238	50	294	35	Pale blue green	Good pink, almost crimson	
				White	Very pale purple	
240	12	292	55	Pink rather pale	Pale greenish blue	
				White	White	
242	10	291	30	Pale blue	Pale yellowish pink	
243	40	289	50	Pale pink yellow	Pale blue	
245	10	287	45	Very pale blue	Very pale yellow	
246	50	286	45	Very pale yellow green	Very pale lilac blue	
248	25	284	55	Pale lilac	Yellow green	
				White	White	
250	40	282	30	Fine yellow green	Fine lilac	
				White	White	
252	57	280	12	Lilac	Fine yellow green	
				White	White	
256	5	277	25	Fine yellow green	Fine lilac	
				White	White	
259	20	274	40	Pale lilac	Fine yellow green	
261	40	271	40	White	White	
266	40	266	40	Green yellow	Pale lilac	The middle tint

In order however to avoid the effect of the dispersive power, which at such considerable obliquities would render the observations liable to some uncertainty, I cut another plate, in such a manner that the perpendicular to its surface, instead of coinciding nearly with the optic axis, was directed very nearly to one of the virtual poles. Its thickness was then gradually reduced in the manner above described for sulphate of baryta, though, owing to the nature of the body, it was found impossible to avoid the necessity of re-polishing the posterior surface at each operation; but as this was done with all possible care, only a very slight error can have arisen from this cause.

TABLE V. *Rochelle salt.* *Thickness* = 0.11518 in.

Inclinations.	Ordinary pencil.	Extraordinary.	Remarks, &c.
° ′			
277 8	Very pale pink . . .	Very pale bluish green . .	Pole P for mean red rays.
261 8	. . . . . .	. . . . .	Perpendicular incidence.
261 0	White tinged with orange .	Very fine intense indigo	
260 25	Yellowish or greenish white .	Purple rather sombre	
259 45	Very pale greenish blue .	Indifferent purplish pink	
	Dull blue	Yellowish pink white	
258 50	Fine deep indigo . . .	White inclining to orange	
258 30	Violet purple . . .	Yellowish white	
257 35	White, a little tinged with violet .	White, not very brilliant .	The virtual pole *p*.
	Yellowish white . . .	Pale violet blue	
256 30	Pale yellow . . .	Sombre indigo inclining to violet, narrow & well defined.	
	Pale pink yellow . . .	Sombre violet white	
255 10	Pinkish purple . .	Very pale greenish yellow	
254 30	Rich sombre purple somewhat fiery .	White tinged with greenish yellow	
	Pale green . .	Fine crimson	
252 30	Extremely pale green .	The richest deep damask crimson	
	White . . .	Livid imperfect purple	
250 27	The richest damask crimson .	Fine pale green	
	Livid imperfect purple .	White	
248 5	Pale bluish green . .	Fine rich crimson	
245 45	Pink approaching to red .	Pale blue green	
243 30	Sky blue . . .	Light pink strongly inclining to orange red	
240 25	Pink orange . .	Pale greenish blue	
	Sky blue inclining to lilac .	Fine yellow	
237 30	Pale bluish green . .	Fine pink a little purple	
237 0	Splendid yellow green . .	Lilac	
234 20	Rich lilac . .	Splendid green	
230 10	Splendid green . .	Rich lilac	
	White . . .	White	
225 45	Lilac blue . . .	Splendid yellow green	
	White . . .	White	
220 40	Splendid green yellow . .	Pale lilac blue	
	White . .	White	
211 40	Pale lilac . .	Pale greenish yellow	
197 30	Fine yellow green . . .	Fine lilac	

Here the virtual pole was coincident with the 5th maximum (or thereabouts) of the extraordinary ray from the pole P; the succession of tints, however, unless close to the virtual pole, is omitted, in order to shorten the table.

TABLE VI. *Rochelle salt.* *Thickness* = 0.08557 in.

Inclinations.		Ordinary pencil.	Extraordinary.	Remarks, &c.
262°	0'	Good light pink	Light blue green	Pole P for mean red.
278	0	. . . . . .	. . . . .	Perpendicular incidence.
		Very pale yellow green	Bluish purple	
278	20	Very pale yellow green	Violet	
278	55	Very pale bluish green	Very light pink, or pinkish white	
280	0	Indigo	Very pale yellow, almost white	
		Pale violet	Very pale yellow	
281	30	White	White, perfectly equal and alike	Virtual pole.
		Very pale greenish yellow	Pale lilac	
282	35	Pale greenish yellow	Sombre lilac purple	
		Pale pinkish yellow	Dull and impure blue	
		Pale pink	Pale yellow green	
285	15	Crimson	Yellow green	
285	45	Rich fiery damask crimson	Very pale yellow green	
		Livid imperfect purple	Very pale pink yellow	
287	50	Fine light green	Fine pink	
288	45	Very pale green	Splendid crimson	
		Good crimson	Blue green	
291	40	Crimson, almost scarlet, &c.	Pale blue green, &c.	

In this plate the virtual pole fell about half way between the 4th maximum and the 5th minimum of the extraordinary ray from the apparent pole P for the mean red rays. When once more ground down, it gave as follows:

TABLE VII. *Rochelle salt. Thickness* = 0.05487.

Inclinations.	Ordinary pencil.	Extraordinary.	Remarks, &c.
262° 25′	Fine pink	Fine light blue green	Pole P for mean red rays.
. .	. . . . . .	. . . . .	
	Indifferent purple	Yellowish white	
277 20	Indifferent lilac pink	Very pale greenish yellow	
278 25	Pale yellow inclining to orange	Blue rather pale	Perpendicular incidence.
	Fine pale yellow	Beautiful sombre indigo	
280 50	Yellowish white or pale yellow	Violet	
282 0	White	White with an almost imperceptible tinge between yellow [and violet	Virtual pole
	Very pale blue	Yellow white	
284 50	Sombre indigo	Pale yellow white	
	Very pale blue	Extremely pale pink white	
	Yellow green	Lilac pink	
	Pale yellow green	Deep lilac pink	
288 25	Greenish white	Rich but sombre purplish crim-	
	White	Dull purple [son	
	Very pale pink	Good blue green	
293 33	Deep fiery crimson	White	
	Very dull purple (greenish)	White	
	Blue	Pink yellow [&c.	
294 50	Very pale blue, &c.	Rich orange, bordering on red,	

III. *On the causes of these phenomena.*

The developement of colour along the axis of double refraction, is at first sight analogous to the production of the secondary tints along the axis of rock crystal, discovered by M. ARAGO, and recently explained by M. BIOT, in a masterly memoir communicated to the Academy of Sciences, on the hypothesis of a force inherent in its molecules independent of their state of aggregation, by which they communicate a rotation in an invariable direction to the axes of polarisation of the luminous rays. And this analogy is partially supported by the fact, that the tint developed along the axis, descends in the scale of colour as the thickness increases.

A more scrupulous examination however will show, that its origin must not be sought in any cause of this nature, for (not to mention the impossibility of explaining the phenomena of the virtual poles by this hypothesis) if we place the principal section of the crystal in the azimuth zero, the extraordinary image will be found to vanish completely for every angle of incidence, and whatever be the thickness of the plate. I may add too, that I have in my possession a crystal of quartz, which exhibits with tolerable distinctness in some parts the phenomena of two axes, and the appearances produced by the interference of the secondary tints in this specimen, while they agree completely with M. Biot's explanation, differ entirely from those which form the subject of this Paper.

Neither are the phenomena above described explicable on any supposition of a peculiar action of the crystal on the differently coloured rays, analogous to its ordinary or extraordinary dispersive power, by which the periods of alternate polarisation of the molecules of some colours, should be lengthened, and of others contracted, so as to disturb that exact proportionality to their periods of easy reflection and transmission, which M. Biot has proved to be a necessary condition for the production of the tints of Newton's scale. It is true, such laws of action may be imagined, and I shall presently show must really exist; in all crystals probably to a small extent, but in two instances at least, to a surprising degree. But this alone will avail us nothing. To show this, and at the same time obtain a general analytical expression for the tint developed at any inclination, and for every hypothesis of the action of the crystal on the differently coloured molecules, let us denote by c, the length of a complete period of easy transmission and reflection, or the extent of one pulse, on the undulatory hypothesis, in vacuo, and at a perpendicular incidence for any homogeneous ray, and let C denote its colour and proportional intensity or illuminating power in the prismatic spectrum. Then will the formula representing a beam of white light intromitted into the crystal, be

$$C + C' + C'' + \&c.$$

from one end of the spectrum to the other.

Let n be the number of periods (each consisting of a double alternation) and parts of a period performed by the elementary pencil C, in its passage through the medium: then, according to the theory of M. BIOT, when n is 0, 1, 2, 3, &c. ad inf. the pencil will wholly pass into the ordinary image; but when the values of n are $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, &c. it will wholly[*] be thrown into the extraordinary one, and in the intermediate states of n, partly into one and partly into the other. These conditions are satisfied if we represent by $\sin.^2 (n\pi)$ the intensity of the ray in the ordinary image, taking unity for its original intensity; and it will I believe be found, that the gradation of intensity given by this formula for the intermediate values of n, will agree sufficiently with the judgment of the eye to warrant its adoption.* The part of the elemen-[illegible] will be $C.\ \sin^2(n\pi)$. Let us denote by $S\left\{C.\ \sin^2(n\pi)\right\}$ the aggregate of all such elements from one extremity of the spectrum to the other, or take

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[*] The amplitude or total extent of each oscillation of the plane of polari here supposed 90°, in which case the contrast of colour in the two pencils is at its maximum. This is the case in the situation we are considering, but in general the intensity of the extraordinary ray, instead of being represented by $\sin.^2 n\pi$, will have an additional factor, a function of the azimuth A of the principal section of the crystallized plate and the position of the refracted ray, and which becomes unity when $A=45°$, and the plane of incidence is that of the principal section. It is on this factor that the gradation of *brightness* in the isochromatic lines, and the black cross or hyperbolic branches which intersect them, depend. But it is not my intention at present to enter on this part of the subject for reasons to be explained farther on. See note in page 84.

$$S\left\{C.\ \sin^2(n\pi)\right\} = C.\ \sin^2(n\pi) + C'.\ \sin^2(n'\pi) + \&c.$$

Then will this expression represent the tint developed in the extraordinary image, and consequently, $S\left\{C.\ \cos^2(n\pi)\right\}$ that in the ordinary one.

Now, n, the number of periods performed depends, 1st, on the nature of the ray, or on c; 2dly, on the intrinsic energy of the action of the medium on that ray; and 3dly, on the direction of its course, the thickness of the plate, and whatever other cause or limit of periodicity may happen to prevail. Hence we may take $n = M \times k$, k being a function of c, dependent only on the nature of the body through which the ray C passes, and M being a certain multiplier whose form we shall consider presently. This substitution made, the expression for the tint becomes $S\left\{C.\ \sin^2(Mk.\pi)\right\}$.

In the theory of the Newtonian colours of thin plates and the polarised rings in crystals with one axis, the multiplier M is independent on c, varying only with the direction of the ray and the thickness of the plate. It is therefore the same for all the coloured rays, and the tint, for any value of M, will be

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[*] No part of our subsequent reasoning depends on the *form* of this function. It is sufficient to know that it must be a periodical, and even function of n. It is only in the computation of numerical values that it is necessary to make any more precise assumption.

$$\ldots\ldots C.\sin^2(M\,k\,\pi)+C'.\sin^2(M\,k'\,\pi)+\&c.\ldots\ldots(a)$$

riation either in the direction of the ray or thickness of the medium, or both, through all gradations of value, to infinity, or to its maximum, if not susceptible of infinite increase: then we see that for every value of M a certain peculiar tint will arise, and that, provided M commence at zero and continue increasing, the same succession of tints will invariably be developed in the same order. Consequently, if we fix upon any two tints in this scale of colour, or any two values of M, the same succession and the same number of alternations of colour must invariably intervene between them, however we pass from one to the other.

In a crystal with two or more axes, the value of M for any ray C must of course be zero in the direction of the axis, and therefore if the same supposition of the independence of M on *c* be made, the same conclusions should follow; namely, 1st. that the extraordinary ray must always vanish in the pole, whatever be the thickness of the plate; and 2dly, that the same succession and number of alternations of colour should intervene between the pole and any assigned unequivocal tint, such as black, or the pure brilliant green of the 3d order of Newton's scale. Both these conclusions are totally at variance with the facts above detailed, as to the developement of colour in the poles, and the situation in the order of the rings of what we have called the virtual poles. Hence we are necessitated either to give up the theory of alternate polarisation altogether, or to admit the dependence of the multiplier M on *c*, or on the nature of the ray. Let us see to what this will lead us.

According to the theory of the polarised rings if extended to crystals with two axes, the number of periods performed in a given space ($= 1$) by a molecule of a given colour, transmitted in a direction making angles θ, θ' with the axes, can only be a function of the form $k . \psi(\theta, \theta')$, k depending on the intensity of the polarising force; or as before, being a function of c, the nature of the ray, and of the intrinsic energy of the molecules of the crystal. Now if we call t the thickness of the plate, and φ the angle of refraction, $\frac{t}{\cos\varphi}$ is the length of the path described, and therefore we must have for the number of periods

$$n = \frac{kt}{\cos\varphi} . \psi(\theta, \theta');$$

so that the value of M must be $\frac{t.\psi(\theta, \theta')}{\cos\varphi}$, which must be a function of c. Now t is obviously independent of it; and if we neglect at present the very trifling effect at moderate incidences of the ordinary dispersive powers of the media examined,* φ is so also. It is therefore in the form of the function $\psi(\theta, \theta')$ that we must look for the cause of the phenomena; and since, we have $\theta' = \theta + 2a$, $2a$ being the angle between the axes (because the observations are made in the principal section) we see that $\psi(\theta, \theta + 2a)$ must involve c, and consequently, θ being arbitrary and independent, a must be a function of c. In order then to render the theory of alternations applicable, we must admit the angle between the axes of double refraction to differ in the same crystal for the differently coloured rays. We must now show that this supposition is sufficient to represent the phenomena correctly.

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* It is easy to see that in the two classes of crystals above described, the effects of the dispersive powers will be opposite to each other, in one opposing, and in the other conspiring with the causes which produce the deviation of tints. In the tables, Nos. V, VI, VII, where the virtual poles were observed almost at a perpendicular incidence, the influence of the dispersive power is quite insensible.

The symmetry of the rings and total evanescence of colour in the principal section at an azimuth zero, requires that the axes of all the different colours shall be symmetrically arranged, on either side of a fixed line (which may be called the optic axis) in this plane, or in one perpendicular to it. At present we need only consider the former case. Let a represent the angular distance of the axis for any one standard species of ray C (the extreme red, for instance) from this line, $a + \delta a$, the same distance for any other ray. Then the distance of the transmitted ray C, from the axes of rays of that colour being θ, θ', the corresponding distances from *their* respective axes for rays of any other colour C′ emerging in the same direction will be $\theta - \delta a + \delta\varphi$ and $\theta' + \delta a + \delta\varphi$, $\delta\varphi$ being the difference $(= \varphi' - \varphi)$ of the angles of refraction, corresponding to the same incidence, for the colours C, C′. The positive values of θ here reckon outwards from the pole; δa is negative for crystals of the second class, and $\delta\varphi$ is negative or positive according as C or C′ is the less refrangible colour.

Let us for a moment consider rays of only these two colours. The portion of the extraordinary pencil due to them will be

$$\mathrm{C}.\ \sin^2\left\{\frac{kt}{\cos\varphi}\,\psi\,(\theta, \theta').\,\pi\right\} + \mathrm{C}'.\ \sin^2\left\{\frac{k't}{\cos\varphi'}\,\psi(\theta - \delta a + \delta\varphi, \theta' + \delta a + \delta\varphi)\right.$$

The rays of these colours of the same order in their respective series of rings will therefore coincide, and that in the proper degree of proportional intensity for the production of a white image, provided we suppose

$$\frac{k}{\cos\varphi}\cdot\psi(\theta,\theta')=\frac{k'}{\cos\varphi'}\cdot\psi(\theta-\delta a+\delta\varphi,\theta'+\delta a+\delta\varphi)\ldots(b)$$

which, since $k, k', a, \delta a$, are constant elements, φ, φ' determinate functions of θ; and $\theta'=\theta+\delta a$, suffices to determine θ.

If we suppose C and C' to represent the extreme red and violet rays, it is evident that the coincidence of the extraordinary pencils of the same order for these two extremes, will ensure that of the intermediate ones, at least very nearly. It would do so precisely, were the value of δa for any intermediate ray, such a function of k as would result from making θ constant in the preceding equation, because the two laws, that of the dispersion of the axes, and that of the magnitude of the rings of different colours, would then act in exact opposition to each other throughout their whole extent. It is, in fact, a case precisely analogous to that of the compound achromatic prism, where if the law of dispersion in the one medium were identical with that in the other, a perfectly colourless pencil would emerge, and when these laws differ, the coincidence of the red and violet rays ensures an approximate coincidence of all the rest. Should these laws however differ very considerably, an uncorrected colour will appear at the point so determined, and a nearer approximation will be obtained by uniting two of the more powerful intermediate rays, such as for instance the mean red and the blue, or limit of the green and blue.

This then is the origin of the virtual poles or points beyond or between the axes where the tint rises to a white of the first order, more or less feeble, or even to an absolute black; and we may now see the reason why the tints in reckoning from these points approximate in a general way to the Newtonian scale. In fact, the periods of the more refrangible rays being performed more rapidly than those of the less, if we suppose the coincidence above spoken of to take place at any point (the minimum for instance), of the n^{th} ring, the intervals between the n^{th} and $(n \pm 1)^{th}$ minimum will be greatest for the red and least for the violet, &c. Consequently, when the violet next disappears totally from the extraordinary pencil, there will remain yet a little of the red, less of the orange, and so on, and this difference increasing at every succeeding minimum on either side, will produce a succession of colours approximating in a general way to NEWTON'S scale. This approximation will however be much less close on the side of the virtual pole towards the nearest axis, because the disturbing influence of the separation of the axes on the figure of the rings and the law of their successive intervals, is much more sensible than at a distance from the pole. This will be evident if we consider that in the interval between the extreme coloured axes, the tints will be regulated entirely by the law of their distribution. Now this is perfectly corroborated by the succession of tints in the foregoing tables, as well as by numerous experiments made on other bodies.

Our equation (b) gives room for a remark of some consequence, as it affords a striking verification of the theory here delivered. It will be observed that this equation does not involve t, and in consequence, the angle θ determined from it, at which the coincidence takes place, is the same for all values of t, or for all thicknesses of the plate. The observations of the tints in the tables given above, afford us ample means of putting this remarkable consequence to the test of experiment. In fact, in the three series of tints observed in sulphate of baryta, the apparent angles between the axes for the mean red rays are respectively 62° 0′, 62° 2′, and 61° 53′, the mean of which is 61° 58′, while the apparent angles between the virtual poles in the same series are 72° 46′, 72° 36′, and 72° 47′. The semi-excesses of the latter angles over the mean value of the former, are the apparent angular distances of the virtual poles from the axes of mean red rays, and are respectively 5° 24′, 5° 19′, 5° 25′, neither of which differs more than 4′ from the mean.

To ascertain the real value of θ, we have only to compute the angles of refraction. In the specimen employed, I found 1·6475 for the index of ordinary refraction, and the angles of incidence (the halves of the above angles) being 30° 59′ and 36° 23′, 36° 18′, 36° 24′; the corresponding angles of refraction are 18° 12′ 30″ ($= a$), 21° 6′ 10″, 21° 3′ 40″, 21° 6′ 40″; and since $\theta = \phi - a$ in this case, we find for the values of θ, 2° 53′ 40″, 2° 51′ 10″, 2° 54′ 10″, the mean of which gives 2° 53′ 0″ for the real angular distance of the virtual pole from the axis of mean red rays in sulphate of baryta.

Again, in the series of tints, tables V, VI, VII, for Rochelle salt, the apparent angular distances of the mean red axis from the virtual pole were

277° 8′ − 257° 35′ = 19° 33′; 281° 30′ − 262° 0′ = 19° 30′,
and 282° 0′ − 262° 25′ = 19° 35′

of which neither differs more than 3′ from the mean 19° 33′.

Dr. BREWSTER (Phil. Trans. 1814, p. 216) has stated the refractive index of this salt at 1·515; but this is certainly a little too large. In four experiments made at distant intervals of time, and by different modes of observation, I have found

1·49640 1·50220
1·49670 1·49858

for the index for the mean yellow-green rays, of which determinations the last is to be preferred, having been made with great care. The same experiment gave 1·49298 for the index for mean red rays. The apparent angular distance of the axis for red rays from the perpendicular was 16°, which leaves 3° 33′ for the angular distance of the virtual pole from the perpendicular. These angles of incidence correspond to the respective angles of refraction 10° 38′ 20″ and 2° 22′ 40″, of which the sum 13° 1′ is the real angle between the virtual pole and mean red axis in Rochelle salt. The series in table IV, gives 13° 2′ 25″ for the value of θ, which agrees completely with the foregoing determination.

I took seven plates of nitre of various thicknesses, and cut from different crystals, and by a mode of observation to be described hereafter, found as follows:

TABLE VIII.

Designation of the plate.	Mean distance of the virtual poles from the axis of mean yellow rays (apparent.)	Excess above the mean.	Order of the coincidence.
1	1° 50′	− 4′	Between the 4th minimum and 5th maximum.
2	1 51	− 3′	A little beyond the 3d maximum.
3	1 57	+ 3′	Different at the two extremities.
4	1 53	0′	2d maximum.
5	1 55	+ 2′	2d minimum.
6	1 59	+ 5′	9th minimum.
7	1 53	0′	A little beyond the 8th minimum.
	Mean 1 53		

Although the constancy of the position of the virtual pole for different thicknesses is sufficiently made out here, the small differences which exist are certainly not attributable to errors of observation, which, in the method I employed, are usually confined within much narrower limits. They are due to minute irregularities in the crystals themselves, consisting, probably, in a state of imperfect equilibrium of the molecular forces of aggregation, to which this salt is so subject, that it is rather rare to find a specimen in which the rings beyond both poles have exactly the same breadth or tints.

ART. IV. *Of the tints developed by crystals with two axes out of the principal section.*

If we place a crystallized plate at an azimuth zero in a tourmaline apparatus, having the axes of the tourmalines at right angles, we shall observe, if its thickness be at all considerable, that the two oval spots on either side the axis of symmetry (which is now perfectly black) instead of being exactly regular in their figure, as in Fig. 2. Pl. V, and tinged with colours symmetrically disposed on either side of a line *m n* perpendicular to the principal section, are invariably coloured at one extremity *r* with a strong prismatic red hue, and drawn out at the other *v* into more or less elongated and tapering spectra or tails of blue and violet light. The extremities *r*, *r* of the rings too have a large excess of the red rays, and the opposite *v*, *v* of the violet rays. In crystals of the first class above described, the red extremity is turned towards the other pole, while in those of the second it is directed from it. If we subject a plate of Rochelle salt to this examination, the ovals *a*, *a*, are drawn out to a surprising length, and the whole prismatic spectrum is displayed in them with great vividness of colour, while the violet portions of the rings are greatly elongated also, and appear to run into one another. If the plate be turned round in azimuth, the phenomena assume the most singular appearances of distortion; and as the rotation approaches to 45°, the rings in the vicinity of the pole are gradually obliterated by their mutual overlapping, which is the greater the thicker the plate. In all situations, however, the interposition of a red or dark green glass immediately restores the perfect symmetry and regularity of the rings, which are then seen in much greater number, and completely well defined.

All this is the necessary consequence of the want of coincidence of the axes for different colours. The lateral spots, for example, are formed for each homogeneous colour with perfect regularity close to their corresponding pole, and regularly decreasing in size from the red to the violet. Their arrangement will therefore be as represented in Fig. 3, Pl.V, R, O, Y, &c. being the poles corresponding to the several colours red, orange, &c. The oval spots composed of red rays being represented by *r*, *r*, those of the other colours will be super-imposed on them in their order, overlapping, as represented by the dotted ovals *oo*, *yy*, &c. like the circular coloured images of the sun in the spectrum of an ordinary prism, giving rise to the long prismatic tails above described. Similar considerations will apply to the anomalous appearances presented by the rings of all the other orders and in every situation.

This suggests a very simple and pleasing experiment, which affords an ocular demonstration of the adequacy of the explanation I have advanced. Let a plate of Rochelle salt be placed in a tourmaline apparatus in any azimuth (45° is the most convenient) and firmly fixed on a proper stand in a dark room. The eye being now applied, let an assistant illuminate the emeried glass or lens of short focus* which disperses the light previous to its incidence on the first tourmaline, with the several colours of the prismatic spectrum in succession, beginning with the red. The rings will then be seen formed successively of each of the colours, perfectly regular in their figure, but contracting rapidly in dimension as they become illuminated with the more refrangible rays.† At the same time the pole about which they form will be seen to move regularly in the direction of the other axis of the crystal, and if we pass alternately from a red to a violet illumination, will shift its position accordingly, backwards and forwards through a very considerable angle. If rays of two colours be thrown at once on the apparatus, their two corresponding sets of rings will be seen at once, crossing, but not obliterating one another, and the distance between their respective centres will be observed to increase with the difference of their colours.

By measuring the interval occupied by the projections of the centres of the last visible red and violet rings, as well as those of the intermediate colours, on a screen at a known distance seen with the other eye, I found the following values of the apparent and real separation of the several coloured axes:

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* See the description of an apparatus of this kind, subjoined.

† See Lectiones Opticæ, lib. ii. Pars. i. Obs. xiii. from which the idea of this experiment is taken. "Magnâque voluptate perfusus" says NEWTON, with the enthusiasm of the true philosopher who loves the field he labours in, "videbam eos dilatare aut contrahere se, gradatim pro eo ac color luminis immutabatur." It is impossible to witness the very beautiful phenomenon described in the text without entering into the same train of feeling.

Between	Apparent interval.			Real Interval.	
	°	′		°	′
Red and orange	0	37	very uncertain	0	25
Yellow	1	50	rather uncertain	1	13
Green	3	43	Do.	2	29
Blue	6	5		4	3
Indigo	8	19		5	33
Violet	9	46		6	31

As a mode of measurement this method is very inaccurate, especially in the extreme red and violet rays, both of which would be copiously, and indeed almost entirely absorbed in their passage through two plates of tourmaline of a yellowish-green colour. Much more exact and unexceptionable measures will be presently given, but these are quite sufficient to establish the *reality* of the phenomenon described.

V. *Of a secondary cause of the deviation of tints, subsisting in certain crystals, and of the anomalous tints of the apophyllite.*

To determine the dispersive power of any medium, and obtain some rough knowledge of its law, we make a prism of it act in opposition to one of a standard substance. To ascertain the total dispersion of the axes of a crystal, or the angle by which the extreme red and violet axes differ, we may make it act against itself. Since the violet rings are more elevated by refraction than the red, from the situation in which they would appear to an eye immersed in the medium, a plate may be conceived cut in such a direction as to make their apparent centres coincide, in which case the tints immediately about the poles will coincide with NEWTON's scale; and the extraordinary image will totally disappear in the pole at an azimuth 45°. This condition gives $\theta = 0$, $\theta - \delta a + \delta \varphi = 0$, whence (supposing R, R′ the indices of refraction for extreme red and violet rays and $\delta R = R' - R$) we find

$$\delta a = \delta \varphi = -\frac{\delta R}{R} \cdot \tan \varphi$$

The angle φ however becomes imaginary, and this method, in consequence, inapplicable when the separation of the extreme axes (δa) is greater than the maximum dispersion of the colours of an intromitted white ray, that is, when

$$\delta a > \frac{\delta R}{R . \sqrt{R^2 - 1}}$$

Let us resume our equation (*b*), and supposing the form of the function ψ, and the constants a, k, k', R and δR ascertained, let the angle θ, at which the coincidence takes place be observed, and the value of δa will then become known. If we suppose it small, which it is in the generality of crystals, we get

$$\delta a = \frac{\frac{k-k'}{k'} \cdot \psi - \delta \varphi . \left\{ \frac{k}{k'} \cdot \sin \varphi . \psi + \frac{d \psi}{d \theta'} + \frac{d \psi}{d \theta} \right\}}{\frac{d \psi}{d \theta'} - \frac{d \psi}{d \theta}} ; \qquad (c)$$

(ψ being put for $\psi(\theta, \theta')$ for the sake of brevity). At incidences nearly perpendicular, $\delta \varphi$ may be neglected, and the expression reduces itself to

$$\delta a = \frac{k-k'}{k'} \cdot \frac{\psi}{\frac{d \psi}{d \theta'} - \frac{d \psi}{d \theta}} \qquad (d)$$

The comparison of these formulæ with observation, which will lead to some very remarkable consequences, requires us to know the form of the function ψ and the values of k, k'. We will begin with the former, and in this investigation the first step is to determine the general equation of the isochromatic lines. In order to this, we must separate in all cases the law of the tint from that of its intensity. The latter depends entirely on the greater or less facility which the emergent ray finds in penetrating the prism of Iceland-spar employed for its analysis, and will not enter into the present investigation. When we examine a crystallized plate in a convenient graduated apparatus, between tourmaline plates crossed at right angles, turning it slowly round between them in its own plane, the form of the coloured bands, if illuminated with homogeneous light, will remain perfectly unchanged during the rotation, but the two black hyperbolic branches passing through the poles, will obliterate in succession every part of their periphery; and the space over which the darkness extends, as well as the degree of illumination of what remains visible, varying at every instant, give rise to so great a variety of appearances, that some little attention is required to recognize this perfect identity of figure. When the tourmaline next the eye is made to revolve, the crystallized plate remaining fixed, the complicated changes which take place, are perfectly reconcilable with the superposition of the primary on its complementary set of rings, the relative intensities of the two sets at any point being regulated by laws we have no occasion to consider at present, but the figure of the isochromatic lines, where visible, remains absolutely unchanged by any rotation in this part of the apparatus.

To form a first hypothesis on the nature of the function which determines the equation of any one of these curves, we must select a crystal, where the proximity of the axes and intensity of the polarising forces are such, as to bring the whole system of rings within a very small angular compass; as by this means we avoid almost entirely the disturbing effect of the variation in thickness, arising from obliquity of incidence. Dr. Brewster, in his Paper of 1818, has chosen nitre, as affording the best general view of the phenomena, and it is admirably adapted for this purpose; the whole system of rings being comprised at a very moderate thickness within a space of 10°, allowing us to regard their projection on a plane perpendicular to the optic axis as a true representation of their figure, undistorted by refraction at the surface, &c. If we examine the rings in this crystal (illuminated with homogeneous light, or by the intervention of a red glass in common day-light) it will be evident that the general form of any one of them is a re-entering symmetrical oval, which no straight line can cut in more than four points, and which, by a variation of some constant parameter, is in one state wholly concave, as 1 (Fig. 4. Plate V.) then becomes flattened, as 2; then acquires a minimum ordinate and points of contrary flexure, as 3; then a node, as 4; after which it separates into two conjugate ovals, as 5; which ultimately contract themselves into the poles P, P′ as conjugate points. The general idea bears a striking resemblance to the variation in form of the curve of the fourth order, so well known to geometers under the name of the lemniscate, whose equation is

$$(x^2+y^2+a^2)^2 = a^2(b^2+4x^2)$$

when the parameter b gradually diminishes from infinity to zero, $2a$ being the constant distance between the poles. In order, however, to put this to a satisfactory examination, accurate measures must be taken, which, in the case of nitre, from the minuteness of the system of rings, presented at first some difficulties. These I obviated, after many fruitless trials, by a mode of observation which I have found extremely convenient and accurate, and which applies particularly well to the present purpose. It consists in projecting the rings by solar light on a screen in a darkened room, by which means they may be magnified to any required extent, examined at perfect leisure, and in all their phases, and measured or traced with a pencil with the utmost exactness and facility. They may be thus exhibited too to a number of spectators at once; a thing which may prove serviceable to the Lecturer, for which reason I have subjoined to this paper a brief description of the apparatus I employ.

Having cut a very perfect crystal of nitre at right angles to its axis of crystallization, and adjusted it properly on this apparatus, the rings were projected on a large sheet of paper, stretched, while moist, on a drawing board, by which means it assumes a truly plane surface by the contraction it undergoes while drying. The poles were then marked, and the loci of the successive maxima for red rays carefully outlined. The screen being then removed, a series of lemniscates were laid down by points, having the same poles and one common point in each, chosen where the tint was most decided. It is unnecessary to give any comparative statement of measures in the observed and constructed curves, as the points, graphically laid down, uniformly fell on the pencilled outlines, or, in the few instances to the contrary, within limits less than the very trifling irregularities of the outlines themselves.

The graphical construction of these curves is rendered extremely easy by the elegant and well-known property of the lemniscate, in which the rectangle under two lines drawn from the foci (or poles) to any point in the periphery, is invariable throughout the whole extent of the curve. This is easily shown from its equation, and the value of this constant rectangle in any one curve is expressed by a b.

We must next enquire how the constant parameter b varies in passing from ring to ring. To this end I projected the rings, illuminated by red light only, on a screen as before, and having outlined the successive loci of the minima of illumination, and laid down the poles, found the values of $a\,b$ in the several lemniscates, as in the following table:

Order of the minimum.	Observed values of $a\,b$ in square inches.	Differences.	Values of $a\,b$ computed from formula $a\,b = 1{\cdot}59 \times n$	Excess of computed above observed values of $a\,b$
$n = 0$	0·00		0·00	0·00
1	1·62	1·62	1·59	− 0·03
2	3·165	1·545	3·18	+ 0·02
3	4·69	1·525	4·77	+ 0·08
4	6·27	1·58	6·36	+ 0·09
5	7·87	1·60	7·95	+ 0·08
6	9·56	1·69	9·54	− 0·02
7	11·09	1·53	11·13	+ 0·04
8	12·77	1·68	12·72	− 0·05
9	14·33	1·56	14·31	− 0·02
10	15·93	1·60	15·90	− 0·03
		Mean 1·59		

The nature of the illumination not allowing the delineation to be performed with the same freedom and precision as in a fuller light, the values of a b in the second column are the means of a great number of measures, taken in every part of their respective curves. The numbers in the 5th column exhibit the excesses of the terms of the arithmetical progression in the 4th (whose common difference is 1·59, the mean of all the differences in the third column) above the observed values of ab, and are so small as fully to authorize the conclusion, that these values, and of course those of the parameter b, increase in arithmetical progression with the order of the rings; or in other words, that the number of periods performed in a given space (=1) by a luminous molecule going to form any point M in the projection of any ring, is proportional to the rectangle of the distances PM, $P'M$ of that point from the two poles.

Now, if we extend our views to crystals in which the distance between the axes is considerable, we may reasonably expect that the usual transition which takes place in analytical formulæ from the arc to its sine, when we pass from a plane to a spherical surface, will hold good. If this be the case, we shall have at once, and in all cases

$$\psi(\theta, \theta') = \sin\theta . \sin\theta'$$

and the nature of the isochromatic curve for the n^{th} complete period will be expressed by the equation

$$\sin\theta . \sin\theta' = \frac{n}{kt} \cdot \cos\varphi = nh . \cos\varphi \qquad (e)$$

putting h for $\frac{1}{kt}$. If the plate be cut at right angles to the optic axis

$$\cos\varphi = \frac{\cos\theta + \cos\theta'}{2 . \cos a}$$

and consequently

$$\sin\theta . \sin\theta' = \frac{n}{2kt . \cos a}(\cos\theta + \cos\theta'); \qquad (f)$$

To put this to the trial, I took a plate of mica, whose thickness measured 0·023078 inch, and having adjusted it accurately on a divided apparatus, placed it in an azimuth 45°, and, by the intervention of the red glass above mentioned, observed the maxima and minima of the extraordinary pencil between the poles. As these observations, when repeated, seldom agreed unless within a few minutes, ten were taken of each maximum and minimum. The angles of incidence, deduced from a mean of similar observations on each side of the perpendicular, are set down in the 2d column of the following table, each number in which is therefore a mean result of 20 observations. The 1st column contains the values of *n*, or the order of the ring observed; the 3d, the angles of refraction, to obtain which I used the index 1·500, employed by M. Biot.* The 4th and 5th columns contain the values of θ, θ' thence computed, and the 6th, values of the coefficient *h*, deduced from the formula $h = \frac{\sin\theta . \sin\theta'}{n . \cos\varphi}$

TABLE X. *Mica. Thickness*=0·023078 in.

Values of *n*	Angles of Incidence.	Angles of Refraction.	Values of θ.	Values of θ′	Values of *h*.	Excesses above mean.
	° ′ ″	° ′ ″	° ′ ″	° ′ ″		
0	35 3 30	22 31 0	0 0 0	45 2 0		
0·5	32 55 20	21 14 40	1 16 20	43 45 40	0·032952	—0·000195
1	30 34 40	19 49 30	2 41 30	42 20 30	0·033622	+0·000475
1·5	28 15 40	18 24 0	4 7 0	40 55 0	0·033035	—0·000112
2	25 34 20	16 43 30	5 47 30	39 14 30	0·033327	+0·000180
2·5	22 46 20	14 57 15	7 33 45	37 28 15	0·033148	+0·000001
3	19 35 40	12 55 10	9 35 50	35 26 10	0·033058	—0·000089
3·5	15 48 40	10 27 50	12 3 10	32 58 50	0·033026	—0·000121
4	10 48 50	7 11 10	15 19 50	29 42 10	0·033010	—0·000137
					0·033147	

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* Recherches sur les Mouvemens des Molecules de la lumiere, &c. p. 482. He takes it equal to that of glass—" ce qui ne doit pas etre fort éloigné de la verité." I have attempted, without success, to measure its value. What has satisfied M. Biot and Dr. Brewster (for the latter has evidently used this index, or one very near it, Phil. Trans. 1818, p. 230) ought to satisfy every one: yet it is fortunate that in the present instance, a slight variation in the refractive index will produce but a very trifling change in the *relative* values of *h*.

The last column of this table exhibits the deviations in excess or defect of the values of the quantity h, so computed from the mean of all of them. Their smallness, in comparison with the quantity itself, and their alternations of sign, are evident proofs of the constancy of this coefficient, and we are therefore authorized to take $\sin\theta \times \sin\theta'$ as the general value of $\psi(\theta, \theta')$. The observations on Rochelle salt, presently to be noticed, confirm this law.* If we denote by l the minimum length of a double oscillation, or the space passed over during one complete period by a ray transmitted at right angles to both axes, we have $k = \frac{t}{l}$; and consequently $h = \frac{l}{t}$, $l = h\,t$. If we substitute for h and t their values above found, we obtain

$$l = 0{\cdot}00076497 \text{ inch}$$

for the minimum length of a period performed by a mean red ray in mica.

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* When $\theta = \theta'$, as in crystals with one axis, we have $\psi(\theta, \theta') = \sin\theta^2$, a result long since confirmed by the accurate experiments of Brewster and Biot. The velocity of the extraordinary ray in such crystals is given by the formula $v^2 = V^2 + \alpha \,.\, \sin\theta^2$. Following this analogy, we may conclude that in crystals with two axes we should have $v^2 = V^2 + \alpha \,.\, \sin\theta \,.\, \sin\theta'$. Now this is precisely the expression at which M. Biot has recently arrived. This very simple and elegant result was communicated to me by that eminent philosopher in the spring of this year, and subsequently in a letter of the 2d May. His Memoir on the subject, which appears (by the Ann. de Chim.) to have been read to the Institute in April, I have not seen, nor do I know by what precise steps he was led to it, but presume it must have been by some considerations of the nature above described. In the foregoing investigation of the law of periodicity, I beg leave therefore to disclaim all intention of arrogating to myself any share in this beautiful discovery, but have thought it necessary to state the steps in the text, in order to demonstrate a truth essential to the investigations to follow, which could not have been taken for granted, or deduced by any legitimate reasoning, independent of experiment, from the equation $v^2 = V^2 + \alpha \,.\, \sin\theta \,.\, \sin\theta'$, by reason of our ignorance of the nature and mode of action of the polarising forces; and, have purposely abstained from entering any farther into the general laws of double refraction and polarisation than I could possibly avoid.

Resuming our general equations *(b)* and *(d)* if we substitute the value now determined for ψ, and write $\frac{l}{l'}$ for $\frac{k'}{k}$, we have

$$l' . \cos\varphi' . \sin\theta . \sin\theta' = l . \cos\varphi . \sin(\theta - \delta a + \delta\varphi) .$$
$$\sin(\theta' + \delta a + \delta\varphi) ; \qquad (g)$$

whence it is easy to derive (independent of any approximation)

$$\cos 2(a + \delta a) = \cos 2\varphi' + 2\frac{l'}{l} . \frac{\cos\varphi'}{\cos\varphi} . \sin\theta . \sin\theta' ; \qquad (h)$$

while our approximate equation *(d)* furnishes the following very convenient formula for incidences nearly perpendicular

$$\sin\delta a = \frac{l - l'}{l} . \frac{\sin\theta . \sin\theta'}{\sin 2a} . \qquad (i)$$

The simplest supposition we can frame relative to the values of the constant elements l, l' is their proportionality to those of c, c', or the lengths of the fits of easy reflection and transmission. This cannot certainly be far from the truth in crystals with one axis, in which the coincidence of the tints, with those of NEWTON's scale, is for the most part exact. In sulphate of lime too, and mica, the only crystals with two axes which have been examined with sufficient exactness, and under the proper circumstances for ascertaining this important point, the law of proportionality seems to be sustained with great precision. This may seem to authorize the general conclusion, that in all cases, $\frac{c}{c'} = \frac{l}{l'}$. Let us see how this agrees with the measures given in the former part of this paper.

In sulphate of baryta, if we take Dr. BREWSTER's measure of the dispersive power,* we have $\delta R = 0{\cdot}019$, and consequently, calculating on the data determined in page 71, we must have, at the virtual pole,

$$\varphi = 21^\circ\, 5'\, 30'' \qquad \varphi' = 20^\circ\, 50'\, 30'' \qquad \delta\varphi = -15'$$

Now, if we suppose $l = 6{\cdot}3463$ $l' = 3{\cdot}9982$, the values of c

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* $\delta R = 0{\cdot}019$. Treatise on new Philosophical Instruments.

and c' respectively for the extreme red and violet rays,* we shall find by substitution in our formula (h)

$$\delta a = 51' \, 10''$$

But a red ray penetrating the surface from within the crystal at an angle $a = 18°12'30''$, and a violet one at an angle $a + \delta a = 19° \, 8' \, 40''$, would emerge at the respective angles $30°59'$ and $32°58' \, 20''$, and would include between them an angle of $1°59' \, 20''$, which should be the *apparent* separation of the red and violet axes in the plate employed. Now, previous to the computation of this result, I had carefully measured this angle, by observing the incidences at which the extreme red and violet rays of the prismatic spectrum, received on the reflector of a graduated apparatus, respectively disappeared from the extraordinary image at the poles P, P'. I thus found

Interval of the poles P, P' for red rays - 62° 2'
Do. for violet - - 66° 5'

Semi-difference, or apparent separation of the axes 2° 1' 30'' which differs from its computed value only by 2' 10''. We may therefore fairly conclude, that in the case of sulphate of baryta, the hypothesis $\frac{l}{l'} = \frac{c}{c'}$ does not deviate sensibly from the truth.

If we apply our formula (i) to the measures above given for Rochelle salt, the result will be widely different. The same supposition as to the values of l, l' being made, we get

$$\delta a = 4° \, 2' \, 50''$$

The incidence being nearly perpendicular, and the angle small, we need only increase it in the proportion 1·499 : 1, to have the apparent angle, which thus comes out 6° 4'. We have already found 9° 46' for the same angle, by a method which must necessarily give a result much below the truth. This difference is by far too great to arise from any errors of observation; but to obtain more exact measures, I took several times the apparent angular separation of the axis of each colour from that of the extreme red by the direct homogeneous light of a sunbeam, separated by the prism, and received on the reflector of a divided apparatus, when, after the proper reductions for refraction and dispersion, the results were as follow:

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* BIOT, Traitè de Physique. Vol. IV.

Colour.	Apparent separation of the axes.	Real separation = δa =	Values of 2 *a*.	Number of observations.
	° ′	° ′	° ′	
Extreme Red.	0 0		75 42	13
Mean Red.	1 33	1 2	73 38	45
Do. Orange.	2 37	1 44	72 14	18
Do. Yellow.	4 0	2 40	70 23	20
Green.	5 49	3 52	67 57	16
Blue.	8 2	5 21	65 0	15
Indigo.	10 21	6 54	61 54	33
Indigo Violet.	11 17	7 31	60 40	2
Mean Violet.	13 58	9 17	57 8	2
Extreme Violet.	15 23	10 14	55 14	8

Though the total separation of the red and violet axes in this table so far exceeds what we had before estimated it at, I am fully satisfied that it is no way exaggerated, but rather falls short of the truth. It is very practicable, by combinations of coloured glasses, liquids, &c. to insulate either extremity of the spectrum in a state of the most absolute purity. In this climate, the dispersed light of the sky in the neighbourhood of the sun, which always mixes with the prismatic beam, is so considerable as to obliterate the feeble rays which compose the two extremities of the spectrum, and it is only by interposing such combinations between the eye and the Iceland crystal used to analyze the polarised ray, that they can be examined with any certainty. The combination I employed for the extreme red was such, that when the whole spectrum thrown on a white screen was viewed through it, it was seen reduced to a perfectly circular, well defined, deep red image, whose *centre* fell on the very farthest *termination* of the red as seen by the naked eye, and whose circumference attained, or perhaps surpassed the point where the maximum of the calorific rays has been supposed to be situated. In like manner, when the same spectrum was examined with the violet combination, a very slightly elongated violet image became perceptible, but every trace of the indigo, and the brighter portion of the violet rays was extinguished. For observations on the indigo, and all the more refrangible portion, I employed similar artifices, without which I found it perfectly impracticable to obtain any regular and comparable results.

The coefficient $\frac{l-l'}{l}$ in our formula being the only part not immediately deduced from observation, it is evident that the assumption $\frac{l}{l'}=\frac{c}{c'}$ must be widely erroneous in the present instance, and it therefore becomes necessary to ascertain the values of l by direct measures. This is rendered easy by the equation (e) which gives

$$l=t.\frac{\sin\theta.\sin\theta'}{n.\cos\varphi}.$$

We have only therefore to observe the inclinations of a plate of known thickness, properly cut and adjusted to 45° azimuth, which correspond to the alternate disappearances of the ordinary and extraordinary images, at which points the values of n are $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}$, &c; computing then the values of θ, θ', and φ, and substituting, we get the values of l, without detailing particular experiments. The following table expresses the final result of a great number of such measures.

Colour.	Values of l in inches.	Number of Observations.
Extreme Red.	0·0056158	64
Mean Red.	0·0050032	14
Mean Orange.	0·0045852	24
Mean Yellow.	0·0040583	52
Mean Green.	0·0036549	62
Mean Blue.	0·0032863	22
Mean Indigo.	0·0029868	52
Extreme Violet.	0·0025093	49

The observations from which this table was calculated, were made indiscriminately on the maxima and minima of all orders. Those of different orders were of course computed separately, and found to agree without exception in giving the same values of l within limits of error less than those to which the observations are liable; thus affording another proof of the exactness of the law of periodicity above employed. Now, if we compare these, one with another, and with those of c as deduced by M. Biot from Newton's observations, we shall have as follows:

Colour.	Values of $\frac{l'}{l}$	Values of $\frac{c'}{c}$
Extreme Red.	1·00000	1·00000
Mean Red.	0·89093	0·96215
Mean Orange.	0·81659	0 90490
Mean Yellow.	0·72266	0·85550
Mean Green.	0·65082	0·79433
Mean Blue.	0·58520	0·73725
Mean Indigo.	0·53166	0·69641
Extreme Violet.	0·44684	0 63000

It appears from this comparative statement, that the forces of polarisation and double refraction in the body now examined, act with much greater proportional energy on the more refrangible rays than in mica, sulphate of lime, and other similar bodies, and consequently that, even were its axes coincident, its tints, though perfectly regular, would still differ very sensibly from the colours of thin plates. This secondary cause of deviation ought to become sensible in plates cut so as to contain both axes, if examined at a perpendicular incidence; but I have not yet had an opportunity of making the trial.

If we calculate on the numbers above given, it will soon appear that a perfect coincidence of all the colours in a single virtual pole is impossible. For this purpose we may employ our equation (i) which easily affords the following

$$\cos 2(a+\theta) = \cos 2a \left\{ 1 + \frac{2l}{l-l'} \cdot \tan 2a \,.\, \sin(-\delta a) \right\}$$
$$= \frac{\cos 2a}{(\cos M)^2}$$

taking M an auxiliary angle such that

$$\tan M = \sqrt{2 \,.\, \tan 2a \,.\, \frac{l}{l-l'} \,.\, \sin(-\delta a)}$$

whence the value of θ or the position of the coincidence of any two coloured rays becomes known, the values of l, l', and $-\delta a$ being given from the foregoing tables. If we unite the mean red with the mean green, these formulæ give $\theta = -11^\circ\, 29'$, and if with the mean blue, $\theta = -14^\circ\, 8'$, of which the one falls short of, and the other exceeds the angle $-13^\circ\, 1'$ given by observation. If we determine by interpolation the values of l' and $-\delta a$, which give $\theta = -13^\circ 1'$, we shall find very nearly

$l' = 34581 \qquad -\delta a = 3^\circ\, 37' \qquad -\delta a + 1^\circ\, 2' = 4^\circ\, 39'$

which correspond to a blue ray strongly inclining to green, and in the brightest part of the colour. Now it is evident that when a rigorous union of all the rays in the proportion in which they exist in white light, is impossible, that of the strongest and brightest colours in opposition to each other will at least ensure the nearest approach to a virtual pole on the principles above demonstrated, and a white will thus be produced, not indeed mathematically perfect, but containing no marked excess of any of the more powerful colours.

The apophyllite is the only crystal with one axis whose tints exhibit a sensible deviation from the scale of NEWTON. Its phenomena, however, are entirely independent of the first and principal cause which produces the deviation in crystals with two axes, viz. the separation of the axes of differently coloured rays, and are referable solely to the secondary and subordinate cause, of which Rochelle salt has just afforded an example, viz. a peculiarity in the law which regulates the lengths of the minimum oscillations of the differently coloured rays within the medium.

1. The tints of the apophyllite commence at the centre of the rings and increase in regular progression outwards, following the same order, whatever be the thickness of the plate.

It follows from this, that the multiplier M in our general formula, (*a*) is the same for all the coloured rays, being zero at the commencement of the scale; and hence it follows, as a necessary consequence, that the axes of all the colours are united in one, and the virtual and actual poles coincide with each other and with the centre. Did any sensible separation of the axes exist, it must become perceptible by the ellipticity of the rings when examined with homogeneous light of that colour from which they are farthest asunder; but with the greatest attention, in plates of considerable thickness, I have not been able to observe the slightest shifting of the axis, or deviation from the circular figure, in passing from a red to a violet illumination. Moreover, it is evident from the preceding theory, that any difference which may exist in their position, if too small to be sensible to the eye, can produce only an imperceptible deviation of tints. In fact, if we suppose $a = o$ for any colour, we get, for the position of the virtual pole,

$$\sin \theta = \sqrt{\frac{l - l'}{l}} \cdot \sin \delta a$$

θ being the angular distance of the point of coincidence from the single axis of that colour. It is, consequently, insensible when δa is so. Now, the polarising force of the apophyllite being very feeble, the diameters of the rings in any plate of moderate thickness must so far exceed this very minute quantity, that the virtual poles, did any exist, must fall within the limit of the central blackness; the Newtonian scale would still appear to commence from the centre, nor could any sensible deviation from it arise from this cause.

2. When the prismatic spectrum is passed over an apparatus containing a plate of this mineral, no perceptible change in the *magnitudes* of the rings for different colours takes place. Hence it appears that the value of the function l for all the coloured rays is nearly alike. By measures taken on a divided apparatus, a slight difference is observed. Taking the mean refractive index R at 1·5431 (by a very careful measure) and the dispersion δR at 0·017, the formula

$$l = t \cdot \frac{\sin \theta^2}{n \,.\, \cos \varphi} = \frac{t}{n} \cdot \sin \theta \,.\, \tan \theta$$

gave as follows:

Extreme Red.	$l =$ 0·0093066
Mean Red.	0·0092810
Mean Orange.	0·0092337
Mean Yellow.	0·0091503
Mean Green.	0·0090643
Mean Blue.	0·0092059
Mean Indigo.	0·0093964
Extreme Violet.	0·0100660 ?

This table, though not given as exact, owing to imperfections in the specimen examined, agrees with the succession of tints which, as far as the fourth order, were as follows:

Apophyllite. Thickness = 0·0829 in.

Incidence. ° ′	Ordinary pencil.	Extraordinary.	
0 0	Bright white . .	Black . . .	The axis.
13 50	White with a trace of purple .	White slightly greenish	1st. maximum of illumination.
21 50	Exceeding sombre violet .	Pure bright white .	
25 12	Pale greenish yellow . .	Purplish white	
29 45	White . . .	Sombre violet blue	
30 50	White . . .	Extremely sombre violet .	1st. minimum.
33 3	White with a strong tinge of violet	Pale yellow green	
35 50	Blue strongly inclining to purple	Greenish white	
37 20	Sombre indigo inclining to violet	White	
38 5	Sombre violet . .	White . . .	2d. maximum.
40 10	Tolerably good yellow green .	Purplish white	
43 55	White with a trace of yellow .	Obscure indigo inclining to purple.	
44 27	White . . .	Sombre violet . .	2d. minimum.
46 45	Pale purple . .	Tolerable yellow green	
49 57	Sombre purple blue . .	Yellowish white	
50 30	Sombre violet . .	White . . .	3d. maximum.
53 40	Green yellow . .	Pale purple	
56 0	Yellowish white . .	Sombre indigo blue . .	3d. minimum.
56 40	Yellowish white . .	Sombre violet	
	White . . .	Livid grey	
59 35	Pale purple . .	Yellow green	
61 45	Sombre indigo . .	Pale yellowish white . .	4th. maximum.
62 10	Sombre violet . . .	Yellowish white	
63 0	Faint violet white . .	White	
	Livid grey . .	White	
66 0	Tolerable green yellow .	Purple	
69 30	Yellow white . .	Very sombre indigo . .	4th minimum.

In the colours of thin plates and others of the like composition, the difference in the lengths of the periods of the different rays is so considerable, that after seven or eight alternations of colour the rings confound one another, and are blended into a uniform whiteness. Were the periods more nearly equal, a greater number of rings should be visible, and were they strictly so, the succession of alternate whiteness and blackness should be continued to infinity. As the values of

l, l', &c. in the apophyllite approximate pretty closely to this limit, we should expect to see a much greater number of rings, and this I find to be really the case. By enclosing a thick plate in balsam of copaiba in a proper apparatus to increase the range of incidence, I have counted as far as the 35th order, when I desisted; not from any want of alternate colours, but owing to their extreme closeness, which rendered it impossible to number them distinctly. Indeed I have no doubt, that could a very thick and limpid specimen be procured, hundreds might be seen without artificial aid.

In two instances then, at least, and probably in many more, or perhaps to a certain small degree in all cases, the minimum lengths of the periods deviate in their respective proportions from those of the fits of easy transmission and reflection; a circumstance which of itself is sufficient to prove the independence of the causes of these laws of periodicity. If we take $Rr = RA$, Fig. 5, Pl. V. and construct a curve whose abscissas A P are the values of c, c', &c. and ordinates those of l, l', &c. the straight line $roygbiv$ inclined at 45° to A R will represent the locus for crystals, such as carbonate of lime, in which the periods follow the Newtonian law, $r'o'y'g'b'i'v'$ will represent the same locus for tartrate of soda and potash, while $r''o''y''g''b''i''v''$ is the curve similarly traced for apophyllite.*

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* Having communicated to Dr. Brewster my observations on the deviation of tints, and the conclusion I had thence deduced as to the separation of the axes of the differently coloured rays, I received in answer a letter, from which, in justice to that indefatigable observer, I subjoin the following extracts.

"My Dear Sir, *Esk Hill, by Roslin Laswade, Sept.* 18, 1819.

"In consequence of having been some time from home, I have only now received your letter, and hasten to reply to that part of it in which you request me to state what results I had obtained respecting the deviation of the tints from NEWTON's scale. The following general points will enable you to judge of the progress which I had made in the enquiry.

"1. In almost all crystals with two axes there is a deviation from the tints of NEWTON's scale.

"2. This deviation is greater in some crystals than in others, being a maximum in acetate of lead and tartrate of potash and soda.

"3. That all these crystals may be divided into two classes, viz. those which have the *red* ends of the rings inwards and the blue ends outwards, and those which have the red ends outwards and the blue ends inwards.

"4. That in all crystals with two axes, the doubly refracting force of *one axis* in general acts differently upon the differently coloured rays from the doubly refracting force of the *second* axis.

"5. That as the polarising force is always proportional to the force of double refraction, the polarising force of one axis will act differently on the differently coloured rays from the polarising force of the other axis.

"6. . . . .

"7. The consequence of this is, that there will be different resultant axes, or different points of compensation for the differently coloured rays.

"8. All these effects may be calculated with the utmost accuracy, if the ratio of the dispersive powers of the two extraordinary refractive forces is given, or *vice versâ*, the dispersive powers may be obtained from the angles of the resultant axes for the red and violet rays of the spectrum.

"9. I have found crystals in which these phenomena *are decidedly connected* with the rotatory phenomena; and from this highly important fact I am led to conclude, that both have the same origin, and that all the rotatory phenomena are, as I have stated in my paper, the result of the uncompensated tints of two axes, equal *for the mean ray*, but unequal for all the rest. (*Here follows an illustration by a diaphragm.*)

"10. The division into two classes in § iii. as founded merely on observation, is converted into another division into two classes, viz. 1. That in which the doubly refracting force of the principal axis acts more powerfully on the blue rays than the other axis does; and 2. That in which it acts less powerfully. The first class comprehends those crystals in which the blue ends are inwards, and the second those in which the red ends are inwards, or nearer the principal axis."

In a subsequent letter (Oct. 4), he adds,

"The *virtual poles*, which you mention, I discovered in the year 1815, and I have two accounts of them in my Journal, the one signed on the 24th January, 1816, and the other 6th January, 1817, by Sir G. MACKENZIE, President of the Physical Class of the Royal Society."

No comments on the above extracts are necessary. They establish at once the priority of Dr. BREWSTER's observations, and the independence of mine. With regard to the division of crystals into two classes, which observation has alike suggested to both of us, it is unnecessary, if we regard either of the two classes as having the angle between the resultant axes greater than a right angle. In Dr. BREWSTER's table, Phil. Trans. 1818, p. 230, succinic acid and sulphate of iron are stated as having this angle 90°. If this determination corresponds, as in all probability it does, to the yellow rays, they belong at once to both classes, and are, in fact, instances of the limit where one class passes into the other. Bi-carbonate of ammonia, in which I can perceive no separation of the axes of different colours, nor of course, any virtual poles, belongs in like manner to both classes, or to neither.

JOHN F. W. HERSCHEL.

APPENDIX.

Description of an instrument employed in the foregoing experiments on the polarised rings.

The singular property possessed by the tourmaline, by which a plate of it of any moderate thickness cut in a direction parallel to its axis of double refraction, is enabled to absorb the whole, or nearly the whole, of an incident pencil polarised in a plane parallel to that axis,* was pointed out by

* The same property is observable in the epidote, the axinite, and all other natural and artificial crystals which exhibit any degree of dichroism when examined by *unpolarised* light. Muriate of palladium and potash possesses it in the highest perfection. This remarkable effect is easily explained by a reference to the general principles laid down by Dr. BREWSTER in his paper on absorption, Phil. Trans. 1819, p. 11. The incident pencil is separated by the doubly refractive force into two, oppositely polarised, one of which is partly absorbed, the other emerges (polarised in its proper plane) of nearly its original intensity.

M. Biot, in the fourth vol. of his Traitè de Physique, and he has availed himself of it with his accustomed ingenuity, as affording an extremely ready and convenient mode of viewing the phenomena of polarisation, much more so than by the use of plates of agate, prisms of Iceland spar, or a second reflection. It follows, from the above mentioned property, that if a beam of ordinary light be made to traverse such a plate, the whole of the emergent pencil, or nearly so, will be polarised in a plane at right angles to the axis; for the incident ray being divided by the doubly refracting force into two pencils, polarised in planes, the one parallel, the other perpendicular to the axis, the former is extinguished in its passage, while the latter emerges with nearly its full intensity.

Hence, if two such plates are crossed at right angles, though separately very transparent, their combination will be opaque. There is a great difference, however, in the degree in which tourmalines of different colours possess this power. Those of a light green, pink, or bluish colour, are quite improper, allowing a considerable portion of light to pass when so crossed, while, on the other hand, those whose colour verges strongly to the honey yellow, or to the hair brown, or purplish brown, effect nearly a complete absorption, and afford, when crossed, a combination almost impervious to light. In ignorance of this distinction, I sacrificed several fine and valuable specimens before I could obtain proper plates.

When a crystallized lamina, cut in a proper direction, is interposed between such a combination of plates, it disturbs the polarisation which the light has received in traversing the first plate, and renders a certain portion of it capable of traversing the second: the colour and intensity of this portion varying with the direction of the ray, give rise to the phenomena of the polarised rings, which may accordingly be seen by applying the eye, and receiving on it the dispersed light of the clouds, &c.

In order, however, to equalize as well as disperse the light, which is of great importance to obtaining a perfect view of the phenomena, an emeried glass may be cemented on the anterior plate, or the first surface of the plate itself roughened; but it will be found more convenient in practice to employ a double convex lens of short focus for this purpose, by which, if necessary, a very strong illumination may be obtained, and an extremely minute portion of a crystal subjected to examination. I have thus, occasionally, examined the rings in a portion not exceeding the hundredth of an inch in diameter, and thus detected irregularities of crystallization of a very singular nature, in many bodies, which would have eluded any other mode of observation. For this purpose, the crystal must be cemented over a small aperture in a thin sheet of brass, on which the focus of the lens must be exactly adjusted to fall.*

If, instead of applying the eye to receive the light so dispersed, we place a screen at some distance in a darkened room, the apparatus is converted into a solar microscope, and the rings will be seen projected on the screen. The construction of the apparatus I employed is as follows: AB is a brass-tube, within which are fitted, 1st, a fixed diaphragm, *aabb*, carrying the first plate of tourmaline in its centre; 2dly, a diaphragm, *ccdd*, moveable freely in its own plane by means of the pin *g* passing through a slit in the side of the cylinder A B, which occupies an arc of about 120° of its circumference. This is destined to receive the crystallized plate *dd*, while a cylinder, *hheeff*, made to slide and turn smoothly within A B, carries the second tourmaline, *ff*. It is essential that the tourmalines employed for this purpose, and especially the posterior one, should be perfectly free from all flaws and blemishes; but large plates not being required, this condition is easily satisfied. The plates so arranged, and brought as near together as possible, the extremity A, of the cylinder A B, is fitted to slide somewhat stiffly on the brass tube P Q, furnished with a lens L, of about two inches focus, and a screw P P, by which it can be adapted to the apparatus usually employed for reflecting a sunbeam into a darkened chamber. The sliding motion of the cylinder A B allows the focus of the lens to be adjusted so as to fall exactly on the first surface of the posterior tourmaline *f*, while its rotation suffers the axis of the anterior one to be placed perpendicular to the plane of reflection. By this arrangement two advantages are gained. The reflector employed (though metallic) always polarises a more or less considerable portion of the reflected beam, which in any other position is partially, or totally extinguished by the first tourmaline, and a great loss of light ensues, which it is of the utmost consequence to avoid: moreover, by this disposition, the action of the reflector is brought to conspire with that of the tourmaline, and the polarisation of the light which traverses it (which is never rigorously exact) is thereby rendered more complete.

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* I have now an apparatus preparing, in which the first plate of tourmaline itself is formed into a double convex lens, by which the loss of light at two surfaces will be suppressed. It is easy to adapt such a lens to a double microscope, for the purpose of detecting microscopic irregularities; and I have reason to suppose a variety of curious results will be brought to light by these means.

It is convenient to have sliding tubes containing lenses of different focal lengths according to the crystal examined, for the intensity of illumination of any point in the screen being, *cæteris paribus*, as the square of the focal length, consequently, when the rings lie within a very small angular compass, a greater illumination of every part of them may be obtained by using a lens of a longer focus.

The dimensions of the figure, Fig. 6. Pl. V. are nearly of the actual size.

Fig: 1.

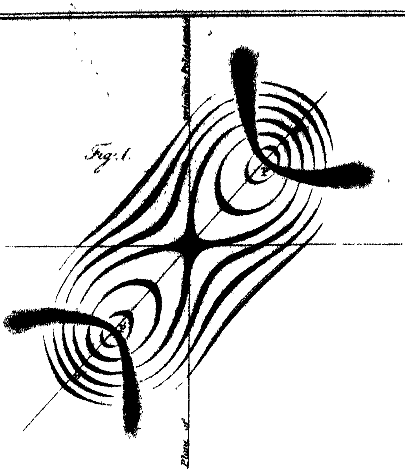


Fig 2

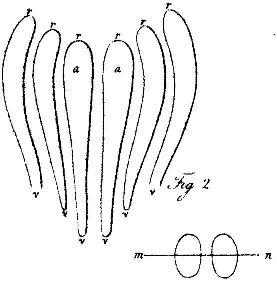


Fig 3

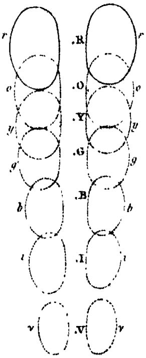


Fig 4

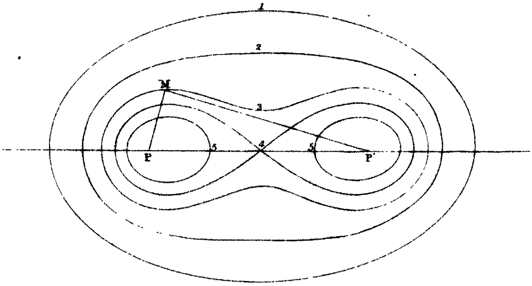


Fig 5

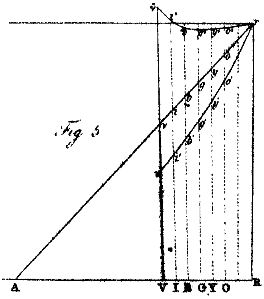
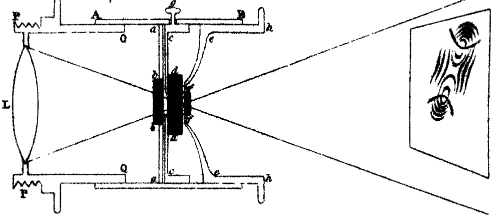


Fig 6



J. Basire sc.

V. *A case of the human fœtus found in the ovarium, of the size it usually acquires at the end of the fourth month.* By A. B. GRANVILLE, *M. D. F. R. S. In a letter addressed to Sir* EVERARD HOME, *Bart., V. P. R. S.*

Read January 13, 1820.

MY DEAR SIR,

HAVING met with a well defined case of a fœtus in the human ovarium, which illustrates in a very satisfactory manner the explanation contained in your paper on the *corpora lutea*, respecting the mode in which the ovum is sometimes retained, and the embryo formed, within the cavity of the *corpus luteum*, I beg leave, through you, to communicate it to the Royal Society, that it may have a place amongst your observations on that subject.

To render it still more deserving of that honor, Mr. BAUER, to whom you and the public are so much indebted, has made drawings of the parts in a manner which, I may say, no one but himself could have executed.

A lady, aged 39 years, died on the 9th of last June, after having experienced various and severe sufferings, which, with the exception of a short period of apparent convalescence, had lasted from December the 12th, 1818, to the moment of her dissolution.

After death, the body was examined at the request of her friends. It presented nothing which, on a general inspection, might be said to indicate disease, except a considerable tumour, soft and moveable, yet perfectly distinct, placed immediately above the region of the pubis, and a little to the left of the linea alba. My attention was, therefore, directed to the investigation of this point.

The integuments and muscular coverings of the abdomen being successively and carefully removed, a considerable quantity of fluid resembling blood, to the amount of several pounds, was discovered to fill every space which the viscera did not occupy. Many large coagula of pure blood lay here and there on the surface, and amongst the convolutions of the intestines; under the stomach; in the right hypochondrium principally; and in the cavity of the pelvis, where both the coagula and the fluid were in so large a quantity as completely to obscure from sight the internal parts of generation. Another obstacle to the view of these organs, even after the intestinal mass had been laid aside, was a tumour of the size of four times that of a hen's egg, partaking of the general black-reddish hue of all the surrounding parts.

A blood-vessel of the size of a large crow-quill, which penetrated the dense portion of the tumour, was traced upwards to the descending aorta, and was ascertained to be a branch of the left spermatic artery. A smaller and much shorter vessel, arising from the tumour, was also found to communicate with the spermatic vein; thus establishing a complete circulation to and from the parts. The inferior and left half portion of the tumour presented a surface consisting, in two or three places, of diaphanous membranes, through which a *fœtus* of about four months growth was readily discovered.

The fact of an extra-uterine conception being thus made out, I proceeded to detach the parts from the pelvis, and brought them to that state of demonstration, which Mr. BAUER has so beautifully represented in the drawing (see Plate VI.)

The left ovarium (O) was the seat of the tumour, which, as it gradually enlarged, distended the coverings of that organ in the same progressive manner, in proportion to its own size, so as to remain, for a time, wholly enveloped by it.

As the fœtus however increased further, the ovarium burst in three different places (1. 2. 3) and thus the membranous sac forming the tumour, protruded partially into the cavity of the abdomen. During this destructive process, that part of the covering of the ovarium was also lacerated, over the inner surface of which the placenta (P) was engrafted, so as to tear the adhesions of the latter, thereby producing that sudden and fatal hemorrhage, which destroyed the life of the mother and the child, and filled the cavity of the abdomen with blood.

EXPLANATION OF PLATE VI.

The womb (A) had acquired considerable developement during the increase of the fœtus, so as nearly to have reached the size which it is known to attain when bearing the produce of conception. Its parietes (*a a*) were thickened in proportion; the orifice was closed, and transversely oblong, as I have generally found it to be in women who have borne children.* Several small prominent vesicles (*bb*) surrounded the internal orifice, and part of the neck of the uterus, within whose cavity (B) neither fluid, membrane, nor production of any description was found.

The right ovarium (C) was of a full size, and healthy, with a great number of the usual cells, containing fluid, distinctly visible; it was terminated by a long slender thread-like appendix (*c c*) of a tendinous nature, which connected it with the neighbouring viscera.

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* The lady in question had been the mother of seven children. She had regularly menstruated up to December, when conception took place; and from that time there were occasional irregular discharges of a coloured fluid from the vagina.

The Fallopian tube of the same side (D) was likewise healthy in its structure, but adhering to the lateral part of the *fundus uteri* by means of a short and tough band (*d*), (no doubt the result of inflammatory action), which seemed a prolongation of one of the *fimbriæ* of its floating extremity, so as to give to the tube that peculiar turn, which is so well expressed in the drawing.

The ligamenta rotunda (E E) seemed larger than usual, although healthy; and the peritoneal foldings, forming what are commonly called the large ligaments, offered not the slightest trace of disease. The latter were removed in the progress of the dissection, in order the better to exhibit the more important parts just described.

Both the Fallopian tubes were pervious only from their loose extremity inwards, to about half their length, and no farther; nor did any artifice carefully employed, enable me to discover the opening of the tubes into the womb.

The left Fallopian tube (F) was perfectly sound and loose, particularly at its fimbriated extremity, which had no connection whatever with the neighbouring tumour.

All the above anatomical and pathological features are accurately pourtrayed, and to their natural size, in the drawing, to which I beg to refer you and the Society, in order the better to comprehend my description.

It ought to be stated here, that one of my principal reasons for dissecting away the broad ligaments, was to ascertain whether the peritoneal foldings, within which the ovarium is placed, rather than the membranes of the ovarium itself, formed the covering of the tumour. The dissection, and the drawing, prove that the latter was the case.

In order to supply as much information as possible, respecting a fact so curious and important in itself, the only one I believe on record, which offers such complete and authentic evidence of a perfect ovario-gestation, I thought it expedient to know the internal anatomy of the tumour, and readily acceded to Mr. BAUER's suggestion, of giving a second view of the case, which should represent as many of its important details as could be rightly ascertained. The result of this ulterior investigation, for which the preparation, after having been drawn in its original state, as seen by yourself, Mr. CLIFT, and others, was necessarily altered and cut into, for better demonstration, is given in a second drawing of Mr. BAUER, of the most accurate and finished execution.

I shall only give you an explanation of the references marked in that drawing, which speaks for itself.

EXPLANATION OF PLATE VII.

A. Amnion.

B. Chorion.

C C C. Placenta.

D. D. Fragments of the *corpus luteum* which surrounded the ovum, and was broken to pieces by the enlargement of the fœtus.

E E E E. Ovarium.

F. Fallopian tube.

G. A portion of the omentum adhering to the ovarium.

From the above explanation, and, on a mere reference to the plate, it appears evident,

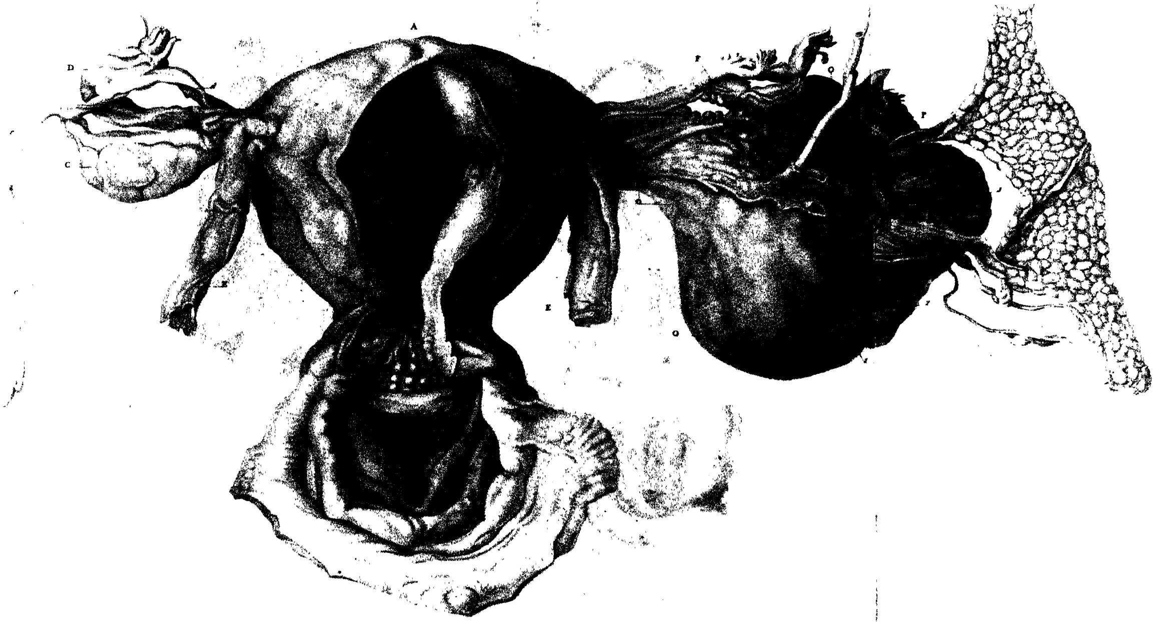
1st. That conception took place in the ovarium.

2dly. That the formation of the placenta and the developement of the fœtus, does not seem to have been different in this case, from what occurs within the womb under similar circumstances.

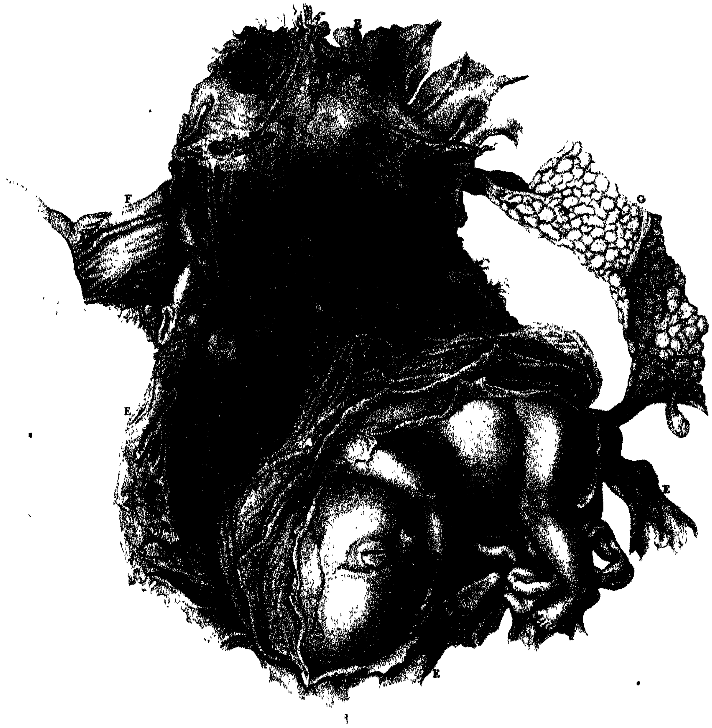
3dly. That the ovum was perfect, being formed of the placenta, and the two well known membranes quite distinct, without, however, the *decidua* of HUNTER.

4thly. That the *corpus luteum* does not follow, but precedes conception, as you have shown in the paper alluded to, since, in this case, considerable portions of the corpus luteum were still found adhering in part to the placenta. To all of which I may add, that the fœtus, in every respect as perfect as a fœtus of four months is known to be when borne in *utero*, was, in the present instance, contained in one of the outer and inferior cavities of the ovarium, within which it must have been completely inclosed, up to the period of the bursting of its covering; and that the fragments of the corpus luteum, together with the placenta, (which in this case was only half the size of what it should have been, though more vascular and spongy) adhered firmly, by means of short filaments, to the inner parietes of the cavity within which conception had taken place.

In examining the right ovarium, as well as that within which the fœtus had been developed, several cells were found, as usual, containing fluid, which hardened into small white masses by the action of alcohol, and appeared to be what anatomists have called *ova*. But having summoned the



Franz Bauer del 1819



Franz Bauer del 1819

Js Basire sc.

microscopical power of Mr. Bauer to my assistance, he has enabled me to give the following explanation, consistent with your doctrine, contained in the last volume of the Philosophical Transactions.

The small round, white, and opaque bodies found in the cells of the human ovarium, as well as in that of other animals in a healthy state, after they have been immersed for some time in alcohol, consist entirely of coagulable lymph, which is easily reduced, in water, to a mass of globules like those observed in *serum;* but in the present instance, the substance of which the coagula consisted, seemed very different, it being a very strong and tenacious mucus, not soluble in water, and which, when pressed, yielded a vast quantity of oil, without the slightest appearance of any globules; a phenomenon supposed to be the effect of previous disease. In the right ovarium there was a *corpus luteum* perfectly well preserved, not far from the cells, which did not contain any such opaque white bodies; while within two other cells equally near, and containing the coagula or white bodies above described, no traces of *corpora lutea* could be discovered. I abstain from all professional observations, this not being the place to enter into them.

I am, dear Sir,

yours most truly,

A. B. GRANVILLE.

Saville Row,
June, 1819.

VI. *On some combinations of Platinum. By* Edmund Davy, *Esq. Professor of Chemistry, and Secretary to the Cork Institution. Communicated by* F. Babington, *M. D. F. R. S.*

Read February 17, 1820.

In my communication to Sir H. Davy, Bart, "On a new fulminating platinum," which has been honoured with a place in the Transactions of the Royal Society,[*] I stated, that I had obtained some other new compounds of this metal: these have since occupied no inconsiderable portion of my leisure hours, and I now beg leave to lay the results of my inquiry before the Royal Society. A constant attention to other necessary duties, has not allowed me sufficient time to render this investigation so complete as I could have wished; but as I presume the facts are novel, I shall venture to bring them forward in a form, which, though imperfect, may not be wholly destitute of interest.

I. *On a peculiar compound of platinum, obtained from sulphate of platinum, by the agency of alcohol.*

Sulphate of platinum, unlike the other metallic sulphates in general, is, to a considerable extent, soluble in alcohol and in ether; as these fluids are capable, in certain circumstances, of partially or wholly reviving some metallic oxides from their solutions in acids, I wished to try their effects on the sulphate of platinum. Accordingly, I put into a small phial about equal volumes of a strong aqueous solution of the sulphate, and alcohol; and after agitating the mixed fluids, the phial was put aside. Some weeks afterwards, I found the dark colour of the sulphate had entirely disappeared, a dense black substance had subsided, and the supernatant fluid remained colourless and transparent. On opening the phial, an odour similar to that of ether was perceived, the fluid had a strong acid taste, and afforded a copious precipitate with nitrate of barytes. After the black substance had been well washed and dried, a few preliminary experiments, served to show that it was a peculiar compound which had not been noticed. To confirm these results, and procure more of the substance, I repeated the experiment with the sulphate and alcohol. In about two days the fluid assumed a darker tint, the black substance began to precipitate in a finely divided state, and in about a week it had all subsided, leaving the fluid colourless and transparent. I afterwards found that the substance in question may be readily obtained by boiling the sulphate and alcohol* together for a few minutes; it separates in small particles, leaving the supernatant fluid colourless, or with only a slight tinge of yellow. In cases when it is thus procured, a little volatile inflammable fluid, having a peculiar ethereal smell, is also obtained. The substance, after being washed till the water is tasteless and does not affect litmus paper, and dried at a temperature of about 250° Fahrenheit, exhibits the following properties.

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[*] Phil. Trans. 1817.

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* The alcohol used in this experiment, may vary considerably in its strength and quantity, without materially affecting the results. Ether may also be employed as a substitute for alcohol.

2. *Properties of the peculiar compound.*

The substance is of a black colour, and in small lumps, which are soft to the touch, and easily reduced to an impalpable powder. It readily soils the fingers, or paper. It is destitute of lustre. It is tasteless, and apparently unaffected either by cold or hot water. It has a peculiar ethereal smell that is not easily removed, and probably arises from the presence of a little inflammable matter occasioned by the action of the alcohol. It seems to undergo no change by exposure to the air for some time. When it is gently heated, on a slip of platinum or paper, a hissing noise or a feeble explosion is produced, and this effect is accompanied by a flash of red light, and the platinum is reduced. It is insoluble in nitrous, sulphuric, and phosphoric acids, but it dissolves slowly in muriatic acid. It is scarcely affected by chlorine, except moisture be present, when a little muriate is gradually formed. When the powder is put into liquid ammonia, minute globules of air are evolved from it, and after some time it acquires fulminating properties. The quantity of air I have hitherto obtained in this way, has been too small to allow me to ascertain its nature with precision. When the powder is brought in contact with ammoniacal gas, a crackling noise is produced, and it becomes red hot and scintillates; but by this treatment, its external appearance is scarcely altered, though it undergoes a partial decomposition. The powder is immediately decomposed by the agency of alcohol. This fact is shown in an interesting manner by moistening different substances, such as paper, sand, cork, &c. with alcohol, and placing the smallest particle of the powder on them; it hisses, a sufficient degree of heat is produced to reduce and ignite the platinum, and it remains in a state of ignition until the alcohol is consumed. During the agency of alcohol on the powder, acetic acid is produced. This is shown by putting a little of the powder on a paper filter and moistening it with alcohol; a moderate action takes place, and in a few minutes, the odour of acetic acid is very perceptible. In some experiments of this kind, the action, though comparatively feeble at first, has presently increased, the powder has become red hot, and the bottom of the filter completely charred. If two or three grains of the powder are placed in a glass, and a few drops of alcohol added, in about half an hour acetic acid will be produced; and as it evaporates and disappears, it may be successively renewed, at longer or shorter intervals, for some weeks, by occasionally adding a little alcohol.

When the powder is boiled in alcohol, it is partially decomposed, and assumes a lighter colour; if it be then thrown on a filter, the odour of acetic acid is soon perceived, and in a few hours the platinum is found reduced and the paper charred. When the powder is mixed with flowers of sulphur, and heated, a sulphuret of platinum is formed of a blue colour. When the powder is heated with phosphorus, there is a brilliant combustion, and a dark grey phosphoret is formed. Oxygen gas does not affect the powder at the common temperature of the air, but by a moderate heat there is a slight combustion, which seems to indicate the presence of a little inflammable matter.

3. *Composition of the peculiar compound.*

In my first attempts to ascertain the nature of the black powder, I was limited to very minute quantities of it; and I made several trials, before I gained any satisfactory evidences of its constitution. I decomposed the powder in long green glass tubes filled with mercury; in such cases, by a gentle heat, the powder became ignited, the reduced platinum amalgamated with the mercury, a little fluid appeared, and some gas was evolved. The fluid reddened litmus, and had an acid taste. The gas rendered lime water turbid, and was in part absorbed by water and by ammonia; and the unabsorbed portion exhibited properties similar to those of nitrogen. These results seemed to prove, that the powder contained acid and inflammable matter; but they were not sufficiently uniform to enable me to place much reliance on them. I then used very small glass retorts, varying in capacity, from $\frac{4}{10}$ to $\frac{7}{10}$ of a cubic inch, and decomposed the powder over pure water and over mercury; but the results were most satisfactory when I operated over mercury. From two experiments of this kind, which I beg briefly to detail, as they very nearly agree, I think I may venture to state the composition of the powder under examination.

Experiment 1. Ten grains of the powder were decomposed in a little retort, over dry mercury, by the heat of a spirit lamp. On the first impression of the heat, gas was disengaged, and shortly after, the interior of the retort assumed a reddish yellow colour (like that exhibited by the vapour of fuming nitrous acid), and small drops of a colourless fluid condensed in the neck of the retort. After the utmost heat of the lamp had been given to the retort, it was suffered to cool, and the results were immediately examined.

(*a*) *Examination of the gas.*

The gas remaining in the retort made an ignited piece of wood glow brighter; that which came over (deducting the common air) was $\frac{34}{100}$ of a cubic inch, which diminished to $\frac{25}{100}$ on being transferred to water and agitated. $\frac{20}{100}$ of the unabsorbed gas, on being mixed with an equal volume of pure hydrogen and fired by an electric spark, diminished to $\frac{26}{100}$. Hence, the unabsorbed portion of gas contained more oxygen than could have been furnished from the common air of the retort.

From other experiments, the gas absorbed by water was found to be carbonic acid; it rendered lime water turbid, was absorbed by ammonia, and again disengaged by muriatic acid.

(*b.*) *Examination of the fluid.*

The fluid which rose in the neck of the retort reddened litmus paper, and resembled the nitrous acid in odour, colour, and taste. It acted upon the mercury in contact with the retort, and when washed out by pure water, the solution did not affect the nitrate of barytes, or silver.

(*c.*) The platinum was perfectly reduced, and its particles formed a loosely coherent mass, which could not be removed until the bulb of the retort was broken. It weighed $9\frac{5}{8}$ grains, and suffered no diminution on being again heated to redness in a platinum cup.

Experiment 2. Ten grains of the same powder as that used in the first experiment, afforded by its decomposition $9\frac{5}{8}$ grains of platinum, a little fluid agreeing in its properties with that noticed in the former experiment, and $\frac{34}{100}$ of gas, which was examined in a different manner from that of Experiment 1. The gas remaining in the retort, was treated with pure nitrous gas; red fumes were produced, and the absorption was so great that the mercury presently rose near the bulb of the retort, and was still rising, when its neck was intentionally broken to secure the platinum. Hence, it seems the gas in the retort was oxygen.

The gas that came over was first treated with lime-water; an immediate turbidness was produced, and increased by agitation, and $\frac{9}{100}$ of the gas were absorbed. To the residual gas, nitrous gas was added, which occasioned a considerable absorption; and the remaining gas, which exhibited the properties of nitrogen, was principally derived from the common air of the retort. By adding a little diluted muriatic acid to the turbid fluid, it immediately became transparent, and the absorbed carbonic acid was slowly disengaged, and the mercury was studded with innumerable little globules of it.

From these experiments, 100 grains of the black powder appear to contain

96·25	platinum.
3·75	nitrous acid, a little oxygen, and a minute portion of carbon.
10·000	

Though the powder was dried at a heat considerably above 212°, it may contain water; and if this is the case, its composition may be differently stated, as deduced from the foregoing experiments:

96·2500	platinum.
0·1200	oxygen.
0·0106	carbon.
3·6194	nitrous acid and water.
100·0000	

4. *Observations, &c. on the peculiar compound.*

From the preceding experiments, the black powder obtained by the agency of alcohol on the sulphate of platinum, appears to consist almost solely of platinum, with a little oxygen, and the elements of the nitrous acid. The very minute portion of carbonaceous matter it contains, is probably accidental. If the constitution of the powder is such as I have stated, a doubt may arise whether it can be considered as a definite compound; but its solubility in the muriatic acid, the facility with which it combines with sulphur, and resists the action of a strong solution of potash at a boiling heat, and its acquiring fulminating properties in liquid ammonia, are all circumstances which favour the notion of its being a true chemical compound. It seems rather doubtful, whether the powder can be regarded as a sub-nitrate of platinum, or a combination of platinum with oxygen and nitrogen, in a different state from that in which they co-exist in the nitrous acid. On the idea that the powder is a compound of the metal with a little oxygen and nitrous acid, something may be said on the mode of its formation, and on the more remarkable properties it exhibits.

From the manner in which the sulphate of platinum is formed, (namely, by the agency of nitrous acid on the hydrosulphuret of platinum) there can be no difficulty in accounting for the presence of a small portion of nitrous acid in it; and my experiments incline me to the opinion, that it is scarcely possible to separate the last portions of nitrous acid from the sulphate, without entirely decomposing it. That the quantity of nitrous acid in the sulphate must, however, be very limited, appears from this circumstance, that the addition of a little nitrous acid to the sulphate, entirely prevents the formation of the black powder, though successive portions of alcohol be added, and the whole boiled for a considerable time.

When sulphate of platinum, containing a little nitrous acid, is treated with alcohol, a mutual action takes place; slowly at the common temperature of the air; but rapidly by the assistance of heat: the sulphuric acid being united to the oxide of platinum by a weak affinity, seems to form a new combination with the alcohol, whilst the oxide combines with the portion of nitrous acid present, to form the black powder. In certain cases, as is well known, alcohol separates salts from their aqueous solutions, in consequence of a stronger affinity for the water in which they are dissolved; but in this instance, the agencies of alcohol and of nitrous acid, are probably concerned in separating the sulphuric acid from the sulphate.

The vivid action of ammoniacal gas on the powder, may be referred to the mutual energy with which the alkaline gas, and loosely combined nitrous acid in the powder, act upon each other. I found by experiment, that ammoniacal gas is absorbed in this instance; thus, 3 grains of the powder were placed in a graduated glass receiver, and filled with dry mercury. 2·3 cubic inches of ammoniacal gas, containing only $\frac{1}{100}$ impurity, were let up into the receiver: an immediate action took place, the powder became ignited, and after two hours, $\frac{3}{10}$ of a cubic inch of the gas were absorbed; recently boiled pure water, whilst yet hot, was let up into the receiver, and the residual gas was all absorbed, except a small globule, which did not exceed the original impurity in the ammonia.

The action of alcohol on the powder is curious, and is connected with the decomposition of both substances. When the powder is brought in contact with the vapour of alcohol, at the common temperature of the air, there is an immediate chemical action; the heat generated is sufficient to reduce and ignite the metal, and to continue it in a state of ignition, until the alcohol is consumed. In this case, the acid first noticed by Sir H. DAVY, (in his beautiful experiment of the ignited platinum wire, and since, more fully examined by Mr. DANIELL), is produced. In other instances, the acetic acid, as has been mentioned, is formed. It would be premature to speculate on the uses to which this powder may be applied, but from its peculiar properties, there is reason to think it will admit of some useful applications. I have already employed it as an easy means of affording heat and light. To produce heat, it is only necessary to moisten any porous substance, such as sponge, cork, cotton, asbestos, sand, &c. with alcohol or whiskey, and to let a particle of the powder fall on the substance so moistened; it instantly becomes red hot, and remains so until the spirit is consumed; nor is the ignited metal extinguished by exposure to the atmosphere, or by blowing the breath on it; on the contrary, partial currents of air only make it glow brighter. The heat produced in this way, may be accumulated to a considerable extent, by increasing the quantity of the materials employed. I have also constructed a tinder box, to procure immediate light by means of the powder. It consists of two small phials placed in a japanned box, and some sulphur matches tipped with phosphorus. One of the phials contains the powder; the other, alcohol. The stopper of the phial containing the alcohol, has a bit of sponge inserted in a small aperture at the bottom of it. When a light is wanted, it is only necessary to shake the bottle so as to moisten the sponge with the alcohol, take out the stopper, and put the smallest particle of the powder on the moistened sponge; it instantly becomes red hot, and will readily kindle one of the matches. This mode of igniting a metal seems to be quite a new fact in the history of chemistry; but the means of keeping it in a state of ignition, is only another illustration of the facts previously pointed out by Sir H. DAVY, in his late valuable researches, which have thrown so much light on the philosophy of flame, and led to such very interesting, important, and unexpected results.

5. *On the effects of sulphate of platinum upon gelatine.*

When an aqueous solution of sulphate of platinum is added to any solution of gelatine, such as isinglass, size, or glue, a precipitate occurs, and all the sulphate is separated in union with the gelatine; or if a minute portion remain, it is precipitated on boiling the fluid. This precipitate, whilst in a moist state, is of a brown colour, and has some degree of tenacity; but when well washed and dried at a temperature a little above the boiling point of water, its colour changes to a jet black; it becomes hard and brittle, and has a resinous lustre. It is not decomposed by being boiled in water or in weak alkaline solutions. When it is gently heated by a spirit lamp on a slip of platinum, a violent action is produced, and a dense white vapour is exhaled, in which the odour of sulphureous acid is perceptible, the substance becomes ignited, and is presently decomposed, leaving the reduced platinum in small grains.

When this compound is decomposed by heat in close vessels over water or mercury, it yields a grey sulphuret of platinum,* nitrogen, sulphureous, carburetted hydrogen and carbonic acid gases, carbonate of ammonia, and an oily-like fluid. This compound of sulphate of platinum and gelatine, when dried at a heat just above that of boiling water, afforded, by its decomposition in two experiments, half its weight of platinum ; and if my former statement of the composition of sulphate of platinum is correct, 100 grains of the above compound will consist of about

56·11	oxide of platinum,
20·02	sulphuric acid,
23·87	gelatine and water.
100·00	

6. *On the sulphate of platinum, as a test for gelatine.*

As I found that minute quantities of gelatine in solution, were readily detected by the sulphate of platinum, I made some experiments to ascertain the efficacy of this substance as a test for gelatine, and I am inclined to think it merits a decided preference over the re-agents at present used by chemists for this purpose. The best known substances for detecting the presence of gelatine are, I presume, those which contain the tanning principle, as the infusions of oak-bark, nutgalls, catechu, &c. And a variety of gelatine, isinglass, (as is well known), is employed to ascertain the quantity of tanning principle in different astringent substances; but for this purpose, as Sir H. DAVY has shown,* many precautions are necessary; and from his experiments it appears, that tannin may exist in a state of combination, in which its presence cannot be made evident by means of a solution of gelatine. I have made several comparative experiments on the efficacy of those astringent infusions, and of the sulphate of platinum, as tests for gelatine; and I think I may venture to conclude, that the sulphate is a test of superior delicacy, and more certain in its operation. Thus, in cases where the gelatine was in very minute quantity, or in a very diluted state, when no effect was produced by strong infusions of oak-bark, nutgalls, or catechu, there was an immediate precipitate on adding sulphate of platinum. In instances also, when the quantity of gelatine was too minute to be readily detected by simply adding the sulphate, the effect was immediately produced on boiling the fluid.

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* In the "Annales de Chimie," &c. Tome V., M. VAUQUELIN treats of the sulphuret of platinum as a new compound which he had formed ; but I published an account of it in the Philosophical Magazine, in the year 1812.

The effects of sulphate of platinum on solutions of the different varieties of gelatine, as isinglass, glue, and size, appear to be precisely similar, and the precipitates obtained in such cases, seem to be uniform in their properties and composition; nor are they affected by the presence of any of the mineral acids in excess. The operation of astringent infusions, as oak-bark, nutgalls, and catechu, on solutions of the different varieties of gelatine, is not uniform. According to Sir H. DAVY, catechu contains a much larger quantity of the tanning principle than oak-bark; yet I found that an infusion of it produced no precipitate in solutions of size, of different degrees of concentration. The size I employed was such as paper-hangers use; it had been recently prepared, and was, previous to its being dissolved in water, in the form of a tremulous jelly. The sulphate of platinum occasions, after a short time, a brown precipitate in astringent infusions; but this substance I have not examined.

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* Phil. Trans. 1803.

7. *On a grey oxide of platinum.*

In the course of my experiments to ascertain the composition of fulminating platinum, I treated it with nitrous acid, and thus procured, as I have elsewhere stated, a grey oxide of platinum, which has not yet been described. It may be obtained by adding strong nitrous acid to fulminating platinum, boiling it to dryness, and exposing the dry mass to a heat just below redness, so as to expel all the nitrous acid. The oxide of platinum remains. It is to be finely pulverized and boiled, first in pure water, and then in a weak solution of caustic alkali to separate the last portions of acid, which adhere with great tenacity to it. It is now to be well washed and dried at a heat not exceeding that of boiling mercury. I have usually made the experiment in a platinum crucible on a hot sand bath. The oxide thus prepared exhibits the following properties.

8. *Properties and composition of the grey oxide of platinum.*

Its colour is dark iron grey. It has the metallic lustre. It is sufficiently hard to cut brass, which it polishes, and when the polished surface is rubbed a little with the oxide, a delicate coating of platinum remains. It does not touch steel. It is not affected by cold or hot water, nor by the nitrous, sulphuric, or phosphoric acid at a boiling heat. It is insoluble in nitro-muriatic acid, and in cold muriatic acid, but it slowly dissolves in this last acid by the assistance of heat. It is not acted upon by a strong solution of the fixed alkalies. When the oxide is put into liquid ammonia, minute globules of air are evolved from it, but the quantity has been too small to admit of being examined; probably it is common air, as the oxide appears to undergo no change by being kept for some weeks in ammonia. When heated with sulphur, the oxide yields sulphureous acid gas and a grey sulphuret of platinum. When mixed with zinc filings and heated, the oxide is decomposed with vivid ignition, and white oxide of zinc is formed.

When the oxide is mixed with borax, and exposed to a strong red heat before the blowpipe, it forms a black glass, which becomes of a lighter colour on urging the heat to whiteness, and the oxide appears to be reduced. If the oxide is mixed with powdered glass and fused, a glass is obtained of a dull brown colour. The oxide is readily reduced by moistening it with oil of turpentine, and heating it moderately; or by exposing it to a dull red heat in the atmosphere; but it requires a strong red heat to reduce it in close vessels. Some of the oxide which had been well dried, first on a hot sand bath, and then exposed to a heat just below redness, on a slip of platinum, was decomposed in very small green glass retorts, over mercury. In two experiments in which I used 7 grains of the oxide, I obtained in each instance 6 grains of platinum, and 2·1 cubic inches of oxygen, the thermometer being at 60° and barometer 30°. I found also in the necks of the retorts, a slight trace of a fluid that reddened litmus paper, and had an odour similar to that of nitrous acid. Now, if

6 grains of platinum combine with 2·1 cubic inches of oxygen, 100 grains will take 34 cubic inches; and calculating from Sir H. DAVY's statement, that 100 cubic inches of oxygen weigh 34 grains, the grey oxide of platinum will be found to consist of

100 platinum	or per cent, of	89·366 platinum,
11·9 oxygen		10·634 oxygen.
		100·000

It will be readily seen, that I have here deduced the composition of the grey oxide from the actual quantity of oxygen and metal obtained in the experiments; and this mode of analysis seems liable to little objection, and can very rarely be resorted to, in ascertaining the composition of metallic oxides. On comparing my previous experiments upon the grey oxide, with the above results, I am most inclined to place confidence in the latter. There is, indeed, a near coincidence between them, and the difference, which is only about one per cent, may be referred to the presence of a little more acid in my first experiments. The grey oxide is insoluble in aqua regia, a fact which seems to add additional support to Sir H. DAVY's opinion respecting the action of aqua regia on platinum.* This menstruum, according to Sir H. DAVY, does not oxidate platinum, but merely causes its combination with chlorine. Now, if the metal were oxidated previous to its solution, the oxygen, there is reason to think, would be derived from the nitrous acid, and the grey oxide formed by this acid, be produced, which can scarcely be the case, as it is insoluble in aqua regia. Add to this, the fact, that by evaporating a common solution of platinum to dryness, no nitrate can be obtained, but only a muriate, or a compound of the metal and chlorine.

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* Journal of Science and the Arts, Vol. I.

If, according to the statements of Professors VAUQUELIN and BERZELIUS, the black oxide of platinum contains about 15 per cent. of oxygen, the grey oxide may be considered as the protoxide, containing 1 proportion, and the black oxide $1\frac{1}{2}$ proportion of oxygen; and the number representing the element or proportion in which platinum combines with bodies will be 126, taking Sir H. DAVY's number 15, to represent the proportion in which oxygen unites with bodies.

Mr. COOPER states the black oxide of platinum to consist of 100 platinum, with only 4·317 of oxygen;* but he has, I think, considerably under-rated the oxygen in it. On repeating his experiments on a small scale, I obtained results different from those he has stated. Thus, he says the powder obtained from the muriate of platinum by a neutral solution of mercury, is a compound of calomel and the protoxide of platinum; but by decomposing this powder in a little retort over mercury, I found the neck of the retort partially lined with metallic mercury; and this fact alone, I think, is sufficient to awaken suspicion as to the accuracy of his results. Mr. COOPER, I presume, used a nitrate of mercury to decompose the muriate of platinum, but he seems to have overlooked the nitrous acid in stating his results.

The chemical history of platinum, is far from being complete. The great want of uniformity in the statements of chemists respecting the composition of the known compounds of this valuable metal, and the circumstance of their not harmonizing with the doctrine of definite proportions, prove the necessity of submitting them to a more rigid examination; and this could not be done without rendering our information on the subject more accurate and extensive.

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* Journal of Science and the Arts, Vol. III.

Cork Institution,
Sept. 1, 1819.

VII. *On the methods of cutting rock crystal for micrometers. By* WILLIAM HYDE WOLLASTON, *M. D. F. R. S.*

Read February 24, 1820.

FOR the mere purpose of examining the phenomena of double refraction, it is extremely easy for any skilful workman to combine a wedge of rock crystal, or any other doubly refracting substance, with another wedge of crown glass opposed to it, in such a manner that a luminous object seen through them shall appear in its true place by ordinary refraction, accompanied by a second image at a small distance, produced by the extraordinary refraction of the crystal.

In consequence of the dispersion of colours which occurs in employing different substances, the above combination is not suited for the purpose of the micrometer invented by the Abbé ROCHON; but it is not difficult to obtain such a section of rock crystal as may be substituted for the wedge of glass, so that the pencil of light shall be restored to its original direction void of colour, without diminishing the separation of the images occasioned by the first wedge.

But since the degree to which the double refraction of rock crystal separates the two portions of a beam of light transmitted through it, is not so great as may frequently be wished, it becomes desirable to increase this effect beyond what can be produced by the most obvious method of employing that substance; and it does appear from M. ROCHON's own account of his contrivance,* that he fully succeeded in accomplishing this end. But although he informs us that the means employed, as best suited to his views, had exactly the effect of doubling the amount of deviation produced by ordinary means, he has not chosen to explain the mode of construction he adopted, and has merely referred to a certain artist living at that time in Paris, who was in possession of his secret, and skilful in applying it to the construction of micrometers.

As I have reason to think that the method to which he alludes in his memoir has never yet been described, I design, in the present communication, to explain a combination which I have found advantageous, and which I think must be the same as that of M. Rochon.

I shall hope to render the principles of this construction intelligible to every one acquainted with the original observation of Huygens on the properties of polarised light, and to enable any competent artist to cut wedges from hexagonal prisms of rock crystal, in the positions requisite to produce, by their combination, the double effect to which I allude.

There are three principal directions in which a crystal may be cut specifically different from each other, which require to be distinctly understood.

In the first place, let us suppose a prismatic crystal to be placed with its axis in a vertical position, and a portion to be cut off from the base by a plane surface at right angles to the axis, and sufficient to form a wedge of 20 degrees, by giving it a second surface duly inclined to the former. For distinction, this may be called the *horizontal* wedge.

Next, let the crystal be bisected vertically by a plane passing through two opposite edges of the prism, in order to make two other wedges which are to be cut in different directions from the two portions, and to have each the same angle of 20 degrees.

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* Journal de Physique, An. 9.

Let one of the halves thus obtained be slit in a plane which meets the surface of bisection in one of the edges of the original prism, and consequently, in a line parallel to the axis. The wedge thus formed may be called a *lateral* wedge.

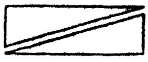
Let the remaining half be cut by another plane not vertical, but inclined to the vertical plane at an angle of 20°, and meeting it in a line parallel to the base, or at right angles to the axis. This may be called a *vertical* wedge.

We have thus three wedges cut in different directions at right angles to each other, and, accordingly, having their axes of crystallization differently placed in each.

In the first, or horizontal wedge, the axis is at right angles to the first surface. In the second, or lateral wedge, the axis is parallel in the first surface, and parallel to its acute edge. In the third, or vertical wedge, the axis is also in the first surface, but it is at right angles to the acute edge.

An object seen through the first wedge in the direction of the axis, does not appear double; but, since rays transmitted through the second or third, pass at right angles to the axis, both of these wedges give two images of any object seen through them.

There are obviously three modes in which these wedges may be combined in pairs, by placing two of them together with their acute edges in opposite directions. The first pair may be represented by L H; the second by V H; the third by V L. In the two first cases



the separation of the images will be the same, since the angles of all the wedges are supposed to be made equal, the compound medium will be comprised under parallel surfaces, so that a ray ordinarily refracted by both, emerges in its original direction; but since the extraordinary ray is made to deviate about 17 minutes from the ordinary course by the wedge which refracts doubly, this difference is not corrected by the horizontal wedge, so that an object seen through either of the combinations L H or V H, appears doubled to the amount of 17′.

The third combination, consisting of the vertical and lateral wedges combined, as in the former cases, with their acute edges in opposite directions, produces an effect perfectly distinct from either of the former combinations; for by reason of the transverse position of their axes of crystallization, the separation of the two images becomes exactly doubled. The consequence of that position is, that the pencil ordinarily refracted by the first wedge, is refracted extraordinarily by the second, and that which has been refracted extraordinarily by the first, suffers a similar interchange, and is now ordinarily refracted, so that neither of the divided pencils returns to its true place; and since one falls as much short of the mean as the other exceeds the truth, they emerge ultimately separated twice the usual difference between the ordinary and extraordinary refractions, and thus present two images separated 34 minutes, just double of that which is effected by either of the preceding combinations.

Though it could scarcely be doubted that this is essentially the construction which was employed by M. Rochon, there is an additional circumstance concerning the effect of such a pair of wedges when otherwise combined, which fully establishes the identity of the method here proposed with his. If the two wedges be placed with their edges together, so as to form by their union a wedge of 40°, the consequence is, that though a pencil of light is in fact divided into two parts by the first wedge, both parts in the end emerge together; the refraction of one being $o+e$, and of the other $e+o$: they both deviate from their original direction by exactly the same quantity, and present only a single image of the luminous object; but it is coloured, as usual, in proportion to the amount of deviation occasioned by the sum of the wedges. This, without doubt, is the first of two opposite directions mentioned by M. Rochon, in which he says the double refraction was not perceptible.

" Pour cet effet," says M. Rochon, " j'employai deux " prismes égaux taillés dans le sens le plus favorable a mes " vues, et en les présentant dans les deux sens opposés je " trouvai, que dans la première disposition la double réfrac- " tion n'étoit pas perceptible, mais, en faisant prendre à mes " prismes un sens inverse, la double réfraction de chaque " prisme étoit presque doublée."

The correspondence in the effect which I have described, renders this passage from M. Rochon perfectly intelligible; and I hope the directions above given will be sufficient to enable any one to cut a crystal to the greatest advantage for making this sort of micrometer. But it must be observed, that in attempting such a construction, great nicety is requisite, not only in cutting the wedges so that the refraction in each shall take place at right angles to the axis, but also in cementing them together, so that the axes of the two wedges shall be at right angles to each other. And it may farther be remarked, that even then, unless the pencil of light pass truly in the common plane of refraction of the wedges, four images will be formed, so as to destroy the effect of the combination.

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PRESIDENT AND COUNCIL.

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METEOROLOGICAL JOURNAL

for January, 1819.

1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Jan. 1	8	0	32	44	30,48	2⅓	34	Rain this Month 0,789 Inches.	W	1	Dark and hazy.
	2	0	37	51	30,49	3	38		SSW	1	Dark and cloudy.
2	8	0	33	44	30,48	5	34		S	1	Thick fog.
	2	0	36	49	30,46	3	39		S	1	Thick fog.
3	8	0	37	44	30,35	7	39		S	1	Fine.
	2	0	38	46	30,31	10	42		S	1	Fine.
4	8	0	35	43	30,21	7	34		SE	1	Fine.
	2	0	40	52	30,18	11	40		SE	1	Fair.
5	8	0	33	59	30,17	7	32		W	1	Thick fog.
	2	0	41	51	30,20	6	39		SE	1	Cloudy.
6	8	0	37	47	30,20	6	38		SW	1	Cloudy, and hazy.
	2	0	42	56	30,16	12	45		SE	1	Fine.
7	8	0	41	48	29,91	8	39		SE	1	Cloudy.
	2	0	45	53	29,78	15	46		SSE	1	Cloudy.
8	8	0	37	51	29,88	7	39		W	1	Fine.
	2	0	42	57	29,92	17	42		NW	1.2	Fine.
9	8	0	48	51	29,58	10	41		S	1.2	Cloudy.
	2	0	45	55	29,56	10	51		S	1	Cloudy.
10	8	0	45	50	29 91	9	42		SW	2	Cloudy.
	2	0	48	50	29,75	8	49		S	3	Cloudy.
11	8	0	43	50	29,70	16	44		S	1.2	Cloudy.
	2	0	46	56	29,84	28	46		WSW	1	Cloudy.
12	8	0	39	49	30,13	17	39		SW	1	Fine.
	2	0	48	55	30,15	17	49		SW	1	Fine.
13	8	0	40	50	30,01	7	41		S	1	Fine.
	2	0	47	57	29,93	12	47		S	2	Cloudy.
14	8	0	48	51	29,95	4	40		S	1.2	Rain.
	2	0	52	57	29,88	10	52		W	1.2	Rain.
15	8	0	43	52	30,02	8	44		W	1	Cloudy.
	2	0	49	59	29,92	7	52		NW	1	Cloudy.
16	8	0	36	49	30,20	13	38		NW	1.2	Fine.
	2	0	43	57	30,29	20	44		S	1	Fine.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Six's Therm.		Winds.		Weather.
	H.	M.	o	o	Inches.				Points.	Str.	
Sep. 17	8	0	47	51	29,50	6	42	Rain this Month 0,789 Inches.	WSW	2	Rain.
	2	0	48	51	29,35	25	52		WSW	1.2	Fine.
18	8	0	40	49	29,37	12	39		W	2	Cloudy.
	2	0	41	55	29,57	18	45		W	1	Fine.
19	8	0	37	49	29,78	14	37		W	1	Cloudy.
	2	0	43	55	29,82	17	44		W	1	Cloudy.
20	8	0	37	51	29,58	11	38		SW	1	Cloudy.
	2	0	43	59	29,60	22	45		W by N	1	Fine.
21	8	0	36	41	29,49	13	37		W	1	Fine.
	2	0	43	55	29,52	25	44		WSW	1	Fine.
22	8	0	38	51	29,53	11	39		SW	1	Cloudy.
	2	0	47	55	29,42	15	49		SE	1	Cloudy.
23	8	0	38	50	29,58	13	39		SW	1	Fine.
	2	0	45	57	29,56	26	48		SW	1	Fine.
24	8	0	42	50	29,61	3	41		SE	1	Fine.
	2	0	47	50	29,42	15	60		SE	1	Fine.
25	8	0	40	49	29,43	8	40		S	2	Cloudy.
	2	0	43	53	29,22	7	44		SSE	3	Rain.
26	8	0	41	51	29,40	7	42		S	1	Cloudy.
	2	0	45	55	29,44	7	47		SW	1	Cloudy.
27	8	0	43	52	29 46	3	41		SE	1	Cloudy and thick fog.
	2	0	46	53	29,47	2	50		SE	1	Cloudy.
28	8	0	46	50	29,35	9	44		SSE	1	Cloudy.
	2	0	46	56	29,42	10	46		S by E	1	Cloudy.
29	8	0	38	53	29,53	3	39		SE	1	Cloudy and hazy.
	2	0	45	60	29,50	28	47		SSE	1.2	Fine.
30	8	0	40	52	29,36	8	40		E	1.2	Rain.
	2	0	42	56	29,35	4	44		E	1	Cloudy and hazy.
31	8	0	39	51	29,47	5	38		N	1	Cloudy.
	2	0	43	51	29,51	12	44		SE	1	Cloudy.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Feb. 1	8	0	31	47	29,60	5	31	Rain this Month 1,638 Inches.	W	1	Fine.
	2	0	40	56	29,61	30	40		W	1	Fine.
2	8	0	36	47	29,54	4	35½		E	1	Dark with snow.
	2	0	37	49	29,66	12	38			1	Cloudy.
3	8	0	30	47	29,72	8	30		NW	1	Hazy.
	2	0	39	51	29,61	11	40		S by E	1	Cloudy.
4	8	0	37	48	29,56	11	36		W	1	Fine.
	2	0	44	52	29,70	14	44		W	1	Cloudy.
5	8	0	39	50	29,73	5	39		E	1	Rain.
	2	0	45	56	29,66	4	46		S by E	1	Rain.
6	8	0	43	51	29,61	6	45		W	1	Fine.
	2	0	48	55	29,64	15	52		W	1	Cloudy.
7	8	0	39	50	29,44	5	40		WSW	1	Fine.
	2	0	46	51	29,41	18	54		NW	1	Cloudy.
8	8	0	36	48	29,80	8	37		W	1	Fine.
	2	0	45	55	29,96	18	47		N	1	Fine.
9	8	0	44	50	29,94	3	41		SW	1	Cloudy.
	2	0	48	56	29,88	5	58		NW	1	Cloudy.
10	8	0	45	51	29,74	14	46		NW	1	Fine.
	2	0	49	51	29,99	28	49		W	1	Fine.
11	8	0	45	50	30,04	12	42		W	1.2	Cloudy.
	2	0	51	57	30,03	19	54		W	1	Fine.
12	8	0	45	52	29,73	13	46		W	2	Cloudy.
	2	0	44	57	29,60	13	49		W	1.2	Rain.
13	8	0	37	51	29,75	8	39		W	1	Fine..
	2	0	46	56	29,78	13	49		W	1	Hail.
14	8	0	35	50	29,30	10	35		W	1	Fine, rather hazy.
	2	0	40	53	30,04	34	42		S	1	Fine.

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1818	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Feb. 15	8	0	36	48	30,04	11	35	Rain this Month 1,638 Inches.	W	1	Cloudy and hazy.
	2	0	46	56	30,93	14	46		SW	1	Cloudy.
16	8	0	43	50	29,68	13	40		S	2	Cloudy.
	2	0	43	55	29,57	6	46		S	2	Rain.
17	8	0	48	52	29,42	17	44		SW	1.2	Cloudy.
	2	0	52	58	29,56	30	52		W	1	Fine.
18	8	0	43	53	29,51	10	43		N	1	Hazy.
	2	0	48	58	29,64	18	50		W	1	Cloudy. [night.
19	8	0	47	54	29,24	10	44		S	3	Rain, a very stormy
	2	0	50	59	29,39	23	52		W	2	Cloudy.
20	8	0	37	53	29,83	7	37		WSW	1	Fine.
	2	0	46	58	29,88	21	52		SW	1	Fine.
21	8	0	43	52	29,18	20	41		E	1.2	Rain.
	2	0	46	52	29,02	7	48		W	1	Rain.
22	8	0	40	50	29,75	11	40		N	1	Cloudy.
	2	0	45	55	29,89	20	46		N	1	Fine.
23	8	0	38	53	29,59	7	38		S	1	Rains hard.
	2	0	43	55	29,51	19	45		SW	1	Cloudy.
24	8	0	34	49	29,65	14	33		SW	1	Hazy.
	2	0	41	54	29,47	27	44		SW	1	Cloudy. [yesterday even.
25	8	0	34	47	29,70	10	28		E by S	1	Cloudy, much snow
	2	0	41	54	29,78	10	43		SE	1	Fine.
26	8	0	32	49	29,70	11	31		N	1	Cloudy.
	2	0	36	53	29,55	8	39		E	1	Rain.
27	8	0	37	48	29,89	12	35		E	1	Cloudy.
	2	0	43	55	29,38	10	45		SW	1	Cloudy.
28	8	0	40	48	29,30	9	39		SW	1	Cloudy.
	2	0	40	48	29,31	7	41		SE	1	Cloudy.

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1818	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	o	o	Inches.				Points.	Str.	
Mar. 1	7	0	36	46	29,21	6	36	Rain this Month 1,010 Inches.	SSE	1,2	Rain and sleet.
	2	0	40	51	29,22	6	41		N	1	Rain.
2	7	0	41	48	29,30	6	38		E	1	Rain.
	2	0	44	54	29,41	5	45		SE	1	Cloudy.
3	7	0	39	47	29,53	15	37		E	1	Cloudy.
	2	0	40	50	29,65	17	41		E	1	Cloudy.
4	7	0	37	46	29,89	16	36		NE	1	Cloudy.
	2	0	45	56	29,88	14	46		NNE	1	Cloudy.
5	7	0	41	50	29,98	4	38		N	1	Cloudy.
	2	0	45	56	29,97	17	46		NE	1	Cloudy.
6	7	0	43	51	29,87	6	41		E	1	Cloudy.
	2	0	47	56	29,91	27	57		SSE	1	Fine.
7	7	0	40	50	30,07	10	39		E	1	Cloudy.
	2	0	45	51	30,09	5	47		SE	1	Cloudy.
8	7	0	40	49	30,10	10	39⅓		E	1	Cloudy.
	2	0	46	53	30,09	22	46		NE	1	Cloudy.
9	7	0	39	48	30,09	15	38½		ESE	1	Cloudy.
	2	0	47	53	30,11	5	48		SW	1	Fine.
10	7	0	40	48	30,10	10	37		NW	1	Cloudy and hazy.
	2	0	46	52	30,05	15	47		NW	1	Cloudy.
11	7	0	43	50	30,05	15	40		NW	1	Cloudy.
	2	0	49	57	30,08	10	55		SW	1	Cloudy.
12	7	0	45	53	30,14	18	43		NW	1	Cloudy.
	2	0	49	53	30,16	26	51		W	1	Cloudy.
13	7	0	46	53	30,27	7	44		NW	1	Cloudy.
	2	0	48	57	30,29		50		NW	1	Cloudy.
14	7	0	43	52	30,30	27	43		W by S	1	Cloudy.
	2	0	46	54	30,19	30	50		SW	1	Fine.
15	7	0	39	51	30,18	15	37				Thick fog.
	2	0	51	59	30,12	35	52		W	1	Fine.
16	7	0	48	54	30,02	10	44		WSW	1,2	Cloudy.
	2	0	57	59	30,03	30	57		W	1.2	Cloudy.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Mar. 17	7	0	45	53	30,10	15	43	Rain this Month 1,010 Inches.	W	1.2	Fine.
	2	0	53	61	30,14	36	54		W	1.2	Fine.
18	7	0	39	51	30,21	19	36		N	1	Fine.
	2	0	53	60	30,17	37	55		SW	1	Fine.
19	7	0	44	54	29,51	4	39		S	1.2	Rain.
	2	0	48	60	29,51	23	50		NW	1	Fine.
20	7	0	43	54	29,42½	14	41		SW	2	Cloudy.
	2	0	47	62	29,56	14	47		NW	2	Rain.
21	7	0	41	54	29,77	22	40		NNE	1	Fine.
	2	0	46	54	29,79	20	56		NW	1	Fine.
22	7	0	40	51	29,81	14	39½		SW	1	Cloudy.
	2	0	51	59	29,81	39	49		W	1	Cloudy.
23	7	0	42	53	29,75	14	40		S	1	Cloudy and hazy.
	2	0	49	57	29,73	7	52		S	1	Cloudy.
24	7	0	47	52	29,56	5	43		SW	1	Cloudy.
	2	0	54	57	29,62	40	57		W	1	Fine.
25	7	0	47	51	29,5	11	46		SW	1	Cloudy.
	2	0	51	58	29,83	32	55		W	1.2	Fine.
26	7	0	45	54	29,88	16	39		W	1	Fine.
	2	0	55	58	29,95	42	53		W	1	Cloudy.
27	7	0	47	53	29,86	15	47		W	1.2	Cloudy.
	2	0	48	56	29,77	14	55		W	1.2	Rain.
28	7	0	46	55	29,78	5	46		W	1	Rain.
	2	0	48	55	29,74	7	54		W	1	Cloudy.
29	7	0	50	54	29,72	50	47		W	1	Fine.
	2	0	54	61	29,97	24	55		W	1	Showery and squally.
30	7	0	46	56	29,62	13	49		WSW	2	Cloudy.
	2	0	57	61	29,99	16	60		WSW	1	Cloudy.
31	7	0	52	57	29,09	13	50½		W	1	Cloudy.
	2	0	57	63	30,11	9	60		WbyN	1.2	Cloudy.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Apr. 1	7	0	51	56	30,16		48	Rain this Month 1,010 Inches.	SW	1	Fine.
	2	0	61	64	30,17	26	63		SW	1	Fine.
2	7	0	52	61	30,16	13	49		W	1	Fine.
	2	0	61	66	30,14	48	63		NW	1	Fine.
3	7	0	50	60	30,05	22	52		S	1	Fine.
	2	0	65	66	29,99	37	67		W	1	Fine.
4	7	0	45	59	30,16	17	48		E	1	Cloudy.
	2	0	55	60	30,14	25	62		SE	1	Fine.
5	7	0	50	61	30,07	12	47		NE	1.2	Cloudy.
	2	0	54	66	30,06	41	65		N	1	Fine.
6	7	0	44	58	29,95	22	41½		E	1	Fine.
	2	0	52	64	29,82	22	55		SE	1	Fine.
7	7	0	47	58	29,58	11	54		SE	1	Cloudy.
	2	0	57	66	29,71	26	62		SE	1	Fair.
8	7	0	54	61	29,85	17	54		SSE	1	Cloudy.
	2	0	55	62	29,91	7	56		N	1	Rain.
9	7	0	48	59	29,99	14	46		N	1	Fine.
	2	0	57	61	30,02	25	51		SE	1	Cloudy.
10	7	0	45	56	30,01	14	43		W	1	Cloudy.
	2	0	58	64	29,84	33	62		SW	1	Fine.
11	7	0	48	59	29,46	12	45		W	1	Cloudy.
	2	0	56	60	29,35	29	58		W	1	Fine.
12	7	0	45	56	29,33	5	45½		SW	1	Rain.
	2	0	48	57	29,31	7	54		SE	1	Rain.
13	7	0	52	58	29,21	6	49		S	1	Cloudy.
	2	0	52	58	29,29	16	55		WSW	1	Fine.
14	7	0	47	57	29,49	11	44		S	1.2	Cloudy.
	2	0	53	62	29,49	35	58		S	1	Fine.
15	7	0	48	56	29,41	9	46		NW	1	Fine.
	2	0	57	62	29.45	22	58		NE	1	Fine.
16	7	0	49	57	29,11	10	46		NE	1.2	Cloudy.
	2	0	55	65	29,22	24	57		NE	1	Fine.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Apr. 17	7	0	47	57	29,36	13	43	Rain this Month 1,694 Inches.	S	2	Rain.
	2	0	49	63	29,45	34	55		NE	1	Fine.
18	7	0	48	56	29,63	24	45		W	1.2	Fine.
	2	0	52	57	29,61	20	59		WNW	1	Fine.
19	7	0	46	55	29,82	21	41		W	1	Fine.
	2	0	52	61	29,81	8	57		SSE	2	Cloudy.
20	7	0	52	57	29,68	3	50		W	1	Dark and Cloudy.
	2	0	57	61	29,70	11	58		S	1	Cloudy.
21	7	0	50	58	29,64	14	48		W	1.2	Cloudy.
	2	0	56	62	29,57	12	59		SW	1	Fine.
22	7	0	48	56	29,81	52	45		SE	1	Fine.
	2	0	50	60	29,90	20	50		NNE	1	Cloudy.
23	7	0	46	56	29,81	5	45		SE	1	Cloudy.
	2	0	50	57	29,73	16	51		SE	1	Fine.
24	7	0	48½	56	29,58	3	47		ENE	1	Rain.
	2	0	46	57	29,58	5	53		ESE	1	Cloudy.
25	7	0	48	55	29,71	4	46		NNW	1	Cloudy.
	2	0	48	55	29,85	10	49		ESE	1	Cloudy.
26	7	0	44	52	30,11	18	38½		N	1	Fine.
	2	0	50	55	30,13	20	52		ESE	1	Fine.
27	7	0	45	52	30,14	15	41		NE	1	Fine.
	2	0	47	60	30,18	30	59		SE	1	Fine.
28	7	0	39	53	30,22	15	41		ESE	1	Fine.
	2	0	52	61	30,19	50	56		E	1	Fine.
29	7	0	48	55	30,07	20	42½		ESE	1	Fine.
	2	0	52	61	30,02	46	55		ESE	1	Fine.
30	7	0	46	54	29,94	15	42		ESE	1	Fair.
	2	0	56	69	29,59	44	58		ESE	1	Fine.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds. *		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
May 1	7	0	49	55	29,84	17	43	Rain this Month 1,400 Inches.	SE	1	Fine.
	2	0	56	62	29,84	56	62		SW	1	Fine.
2	7	0	52	57	29,75	25	43		SE	1	Fine.
	2	0	49	60	29,68	34	65		NW	1	Fine.
3	7	0	49	56	29,56	20	41		N	1	Fine.
	2	0	60	62	29,56	40	69		SE	1	Fine.
4	7	0	50	61	29,47	17	40		S	1	Fine.
	2	0	61	61	29,52	47	65		SSE	1	Fine.
5	7	0	50	61	29,56	19	40		SE	1	Fine.
	2	0	61	63	29,55	22	63		SSW	1	Fine.
6	7	0	49	57	29,49	18	40		S	1	Fine.
	2	0	60	65	29,97	25	65		SE	1	Fine.
7	7	0	52	56	29,48	16	42		SSE	1	Fine.
	2	0	61	63	29,54	27	66		SE	1	Fine.
8	7	0	48	54	29,51	18	44		S	1	Fine.
	2	0	62	64	29,53	30	69		SE	1	Fine.
9	7	0	55	56	29,53	10	45		SSE	1	Fine.
	2	0	68	65½	29,58	34	71		SW	1	Fine.
10	7	0	56	62	30,17	19	50		SSW	1	Fine.
	2	0	65	65	30,17	30	70		SW	1	Fine.
11	7	0	68	64	30,06	19	64½		W	1	Fine.
	2	0	62	62½	30,07	34	66½		NW	1	Fine.
12	7	0	61	60	30,08	16	56		W	1	Fine.
	2	0	62½	63	30,04	28	67		W	1	Fine.
13	7	0	59½	61	30,04	21	66		SW	1	Fine.
	2	0	61	62	30,06	36	65		NW	1	Fine.
14	7	0	53	59	30,03	24	52		SWbN	1	Fine.
	2	0	61	63	30,01	43	64		W	1	Fine.
15	7	0	57	57½	30,13	23	54		SW	1	Fine.
	2	0	60	52	30,15	40	65		S	1	Fine.
16	7	0	55	58½	30,17	17	52		SSW	1	Fine.
	2	0	59	61	30,21	36	64		SE	1	Fine.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
May 17	7	0	55	58½	30,17	17	52	Rain this Month 1,400 Inches.	SSE	1	Fine.
	2	0	64	65	29,91	46	67		NE	1	Fine.
18	7	0	62	60	29,68	10	51		SW	1	Fine.
	2	0	63	63	29,10	30	68		W	1	Fine.
19	7	0	55	59	29,12	5	54		E	1	Rain.
	2	0	60	61	29,16	15	62		NE	1	Rain.
20	7	0	58	59	29,16	9	66		W	1	Rain.
	2	0	63	64	29,19	19	64		SW	1	Rain.
21	7	0	54	58	29,14	18	56		S	1	Fine.
	2	0	59	60	29,17	34	67		E	1	Fine.
22	7	0	66½	58	29,55	10	51		S	1	Fair.
	2	0	61	61½	29,52	17	64		S	1	Showery.
23	7	0	58	59½	29,91	16	50		ESE	1	Fine.
	2	0	60	63	29,91	24	58		SE	1	Fine.
24	7	0	57	61	29,84	12	51		SE	1	Rain.
	2	0	61	61	29,84	17	63		SE	1	Fine.
25	7	0	53	55½	29,63	4	50		E	1	Fair.
	2	0	60	62	29,68	7	63		E	1	Fine.
26	7	0	54	58	29,51	5	51		SE	1	Fine.
	2	0	58	59	29,81	11	61½		WSW	1	Fine.
27	7	0	49	56	29,73	6	51		E by N	1	Fine.
	2	0	56	59	29,64	14	62		E	1	Fine.
28	7	0	46½	55	29,91	8	51		SW	1	Fine.
	2	0	53	56	29,93	19	58		SSW	1	Fine.
29	7	0	51	52	29,62	16	52		NE	1	Fine.
	2	0	59	56	29,62	18	55		SWbN	1	Fine.
30	7	0	59	66	29,84	12	52½		SW	1	Fine.
	2	0	49	53	29,84	20	67		S	1	Fine.
31	7	0	58	55½	29,08	17	55		SW	1	Cloudy, rain in the night.
	2	0	57	57	29,08	21	61		NE	1	Fine.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
June 1	7	0	57	56	30,08	10	51	Rain this Month 1,017 Inches.	SW	1	Fine.
	2	0	61	58	29,95	19	67		SW	1	Fine.
2	7	0	58	57	29,95	10	54		SSW	1	Fine.
	2	0	69	61	30,01	25	67		SSW	1	Fine.
3	7	0	56	58	30,01	10	50		SW	1	Fine.
	2	0	64	61	29,99	24	67		SW	1	Fine.
4	7	0	59	59	29,93	10	56		SW	1	Fine.
	2	0	64	62	29,91	14	68		WSW	1	Fine.
5	7	0	54	59	30,01	9	48		WSW	1	Fine.
	2	0	64	63	30,06	17	68		WSW	1	Fine.
6	7	0	56	59	30,04	8	51		SW	1	Fine.
	2	0	63	64	29,93	27	68		WSW	1	Fine.
7	7	0	58	60	29,62	10	51		WSW	1	Fine.
	2	0	63	61½	29,63	32	66		SW	1	Fine.
8	7	0	58	60	29,60	16	51½		S	1	Fine.
	2	0	59	63	29,58	19	66		S	1	Fine.
9	7	0	57	61	29,58	10	54		NW	1	Fine.
	2	0	66	66	29,61	36	67½		S	1.2	Fine.
10	7	0	55	61½	29,67½	19	53		W	1	Fine.
	2	0	63	65	29,74	15	67		SW	1	Showery.
11	7	0	53	61	29·96	11	48		W	1	Fine.
	2	0	62	62	29,98	17	67		SSW	1	Fine.
12	7	0	54	60	30,00	13	49		SWbyS	1	Fine.
	2	0	62	63	30,01	25	64		SW	1	Fair.
13	7	0	55	59	30,07	16	49		SW	1	Hazy.
	2	0	65	62	30,03	39	69		SSW	1	Fine.
14	7	0	55	60	29,99	11	51		W	1	Fine.
	2	0	63	62	29,93	22	65		SW	1	Cloudy.
15	7	0	51	60	29,78	15	55		S by W	1 2	Rain.
	2	0	62	63	29,76	20	64		N	1	Cloudy.
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1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
June 16	7	0	54	59	29,92	12	46½	Rain this Month 1,017 Inches.	SSE	1	Cloudy.
	2	0	62	61	29,94	26	63		N	1	Cloudy.
17	7	0	57	59	30,06	11	51		W	1	Cloudy.
	2	0	63	63	30,07	28	66		S	1	Fine.
18	7	0	56	61	30,04	3	55½		N	1	Hazy.
	2	0	60	61½	30,02	15	62		ENE	1	Cloudy.
19	7	0	58	61	30,10	18	54		N	1	Fine.
	2	0	66	67	30,13	37	70		NE	1	Fine.
20	7	0	60	63	30,19	16	55		S	1	Fair.
	2	0	66	66	30,28	39	68		SSE	1	Fine.
21	7	0	51	62	30,18	23	58		N	1	Fine.
	2	0	70	70	30,12	29	72		SE	1	Fine.
22	7	0	56	64	30,07	19	56		W	1	Fine.
	2	0	65	65	30,05	26	69		NE	1	Fine.
23	7	0	60	64	30,03	16	58		NW	1	Cloudy, rather hazy.
	2	0	66	67	30,01	19	70		N	1	Cloudy.
24	7	0	57	63	29,89	10	54		W	1	Rain.
	2	0	58	63	29,78	6	64		SW	1.2	Cloudy.
25	7	0	60	64	29,76	4	57		W	1	Cloudy, dull weather.
	2	0	63	64	29,77	10	66		SW	1	Cloudy.
26	7	0	57	62	29,65	4	55		S	2	Cloudy.
	2	0	62	62	29,64	10	67		S	1	Cloudy.
27	7	0	59	59½	29,69	5	51		SSW	1	Cloudy.
	2	0	60	66	29,58	12	64		SW	1	Rain.
28	7	0	53	60	29,67	9	51		NW	1	Fair.
	2	0	59	63	29,70	23	63		NW	1	Cloudy. a storm of thunder, lightning, and hail at noon.
29	7	0	54	58	29,82	14	47½		NW	1	Fine.
	2	0	62	60	29,85	11	65		SSW	1	Fine.
30	7	0	58	60	29,75	5	55		W	1	Cloudy.
	2	0	64½	62	29,72	23	68		WNW	1	Fine.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
July 1	7	0	56	60	29,79	25	50½	Rain this Month 0,750 Inches.	W	1	Fine.
	2	0	65	61	29,82	30	65		NW	1	Cloudy.
2	7	0	56	60	29,88	13	51		SW	1	Fair.
	2	0	62	61	29,87	18	67		WSW	1	Rain.
3	7	0	60	61	29,78	9	56		SW	1	Cloudy.
	2	0	69	65	29,81	20	71		WSW	1	Fine.
4	7	0	61	64	29,76	19	60		S	1	Cloudy.
	2	0	74	72	29,81	27	81		WSW	1	Fine.
5	7	0	66	66	29,84	10	62		W	1	Fine.
	2	0	71½	71	29,85	30	78		S	1	Fine.
6	7	0	63	66	29,94	13	62		S	1	Thick and hazy.
	2	0	63	66	29 92	10	69		SE	1	Rain.
7	7	0	62	66	30,12	12	61		N	1	Hazy.
	2	0	67	67	30,15	30	69		W	1	Fine.
8	7	0	62	65	30,01	8	60		N	1	Rain.
	2	0	64	65	29,98	12	67		SE	1	Cloudy.
9	7	0	59	64	30,08	10	60		W	1	Cloudy.
	2	0	69	68	30,12	49	70		WNW	1	Fine.
10	7	0	59	63	30,12	20	56		W	1	Cloudy.
	2	0	66	67	30,06	30	70		WNW	1	Fine.
11	7	0	61	64	30,08	18	53		W	1	Cloudy.
	2	0	66	65	30,08	39	70		WNW	1	Fine.
12	7	0	66	65	30,09	20	57		W	1	Fine.
	2	0	71	68	30,09	23	74		WSW	1	Fine.
13	7	0	64	64	30,17	20	62		S	1	Cloudy.
	2	0	66	65	30,18	36	71		S	1	Fine.
14	7	0	59	64	30,16	27	56		SSE	1	Fine.
	2	0	64	65	30,12	19	68		S	1	Fine.
15	7	0	58	63	30,05	18	57		N	1	Dull and Cloudy.
	2	0	64	65	30,02	22	68		N	1	Fine.
16	7	0	58	63	30,00	23	56½		E	1	Cloudy.
	2	0	68	69	30,01	27	72		E	1	Fine.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
July 17	7	0	62	64	29,99	15	56	Rain this Month 0,750 Inches.	N	1	Cloudy.
	2	0	71	67	30,01	27	71		NW	1	Cloudy.
18	7	0	63	65	30,00	14	61		W	1	Fine.
	2	0	65	68	29,84	30	65		W	1	Fine.
19	7	0	62	64	29,68	15	58		SSW	1	Fine.
	2	0	61	71	29,59	48	76		W	1	Fine.
20	7	0	63	67	29,32	31	61		S	1	Cloudy, rain in the night.
	2	0	66	68	29.33	31	74		SW	1	Fine.
21	7	0	57	65	29,46	14	56		SSE	2	Cloudy.
	2	0	63	64	29,58	5	61		NE	2	Cloudy.
22	7	0	57	63	29,95	19	54		S	1	Fine.
	2	0	65	67	30,07	40	69		S	1	Fine.
23	7	0	59	63	30,14	17	55		W	1	Hazy.
	2	0	69	69	30,15	61	70		N	1	Fine.
24	7	0	65	65	30,18	23	61		N	1	Fine, but rather hazy.
	2	0	69	70	30,16	35	79		N	1	Fine.
25	7	0	66	65	30,10	16	59		E	1	Fine.
	2	0	68	72	30,07	29	74		SE	1	Fine.
26	7	0	65	67	30,02	14	59		NNE	1	Fine, rather hazy.
	2	0	70	72	30,03	35	74		SW	1	Fine.
27	7	0	58	66	30,12	20	63		N	1	Cloudy.
	2	0	72	72	30,13	42	73		N	1	Fine.
28	7	0	63	67	30,19	16	61		NbyE	1	Cloudy and hazy.
	2	0	70	73	30,21	40	74		S	1	Fine.
29	7	0	63	66	30,17	17	49		S	1	Cloudy, rather hazy.
	2	0	74	73	30,11	42	77		WSW	1	Fine.
30	7	0	67	67	30,10	25	61		N	1	Fine.
	2	0	75	77	30,09	37	79		E	1	Fine.
31	7	0	69	68	30,02	18	65		E	1	Fine.
	2	0	72	74	30,01	38	78		S	1	Fine.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Aug. 1	7	0	68	68	30,01	13	66		W	1	Cloudy and hazy.
	2	0	69	71	30,01	49	80		N	1	Fine.
2	7	0	62	66	30,00	15	61		SE	1	Cloudy.
	2	0	69	73	29,95	34	72		W	1	Fine.
3	7	0	63	68	29,91	29	61		N	1	Cloudy.
	2	0	65	67	29,92	48	71		N	1	Fine.
4	7	0	63	66	29,88	17	62		N	1	Cloudy.
	2	0	66	66	29,89	16	68		NW	1	Fine.
5	7	0	63	67	29,92	16	62		N	1	Cloudy.
	2	0	66	67	29,95	18	67½		NNW	1	Fine.
6	7	0	62	67	29,99	16	59½		NW	1	Fine.
	2	0	72	70	29,97	21	75		W	1	Cloudy.
7	7	0	66	66	29,98	13	62		NW	1	Cloudy and hazy.
	2	0	73	71	29,99	50	73		SW	1	Fine.
8	7	0	64	65	30,05½	24	61		NE	1	Cloudy.
	2	0	67	67	30,08	36	70		NE	1	Fine.
9	7	0	66	66	30,11	20	60		N	1	Fine.
	2	0	72	72	30,12	36	76		NNE	1	Fine.
10	7	0	64	66	30,11	16	61		E	1	Fine.
	2	0	68	72	30,08	24	73		SSE	1	Fine.
11	7	0	63	64	30,00		59		E	1	Cloudy and hazy.
	2	0	68	70	29,95	33	74		SE	1	Fine.
12	7	0	62	64	29,92	23	61		ESE	1	Cloudy and hazy.
	2	0	74	71	29,90	43	76		W	1	Fair.
13	7	0	64	65	29,92	15	62		W	1	Cloudy.
	2	0	72	69	29,92	24	74		WSW	1	Cloudy.
14	7	0	65	66	30,01	24	63		NE	1	Cloudy and hazy.
	2	0	69	68	30,03	34	72		E	1	Cloudy.
15	7	0	64	65	30,08	26	59½		N	1	Fine.
	2	0	72	70	30,08	37	73		SSW	1	Fine.
16	7	0	68	67	30,16	21	67		N	1	Fine.
	2	0	73	76	30,18	62	74		NW	1	Fine.

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	° *	Inches.				Points.	Str.	
Aug. 17	7	0	65	67	30,22	20	64	Rain this Month 0,250 Inches.	NNE	1	Fine.
	2	0	75	75	30,25	60	78		E	1	Fine.
18	7	0	62	65	30,29	10	65		ENE	1	Cloudy.
	2	0	71	73	30,31		74		NE	1	Fine.
19	7	0	64	65	30,23	14	63		E	1	Hazy.
	2	0	69	68	30,19	35	72		ESE	1	Fine.
20	7	0	63	64	30,18	20	61		N	1	Cloudy and hazy.
	2	0	69	69	30,19	44	71		NE	1	Fine.
21	7	0	63	66	30,22	13	61		NbE	1	Fine, rather hazy.
	2	0	71	73	30,19	48	73		NE	1	Fine.
22	7	0	64	64	30,15	18	62		NE	1	Cloudy and hazy.
	2	0	70	68	30,13	37	73		ENE	1	Fine.
23	7	0	66	66	30,11	10	64		NNE	1	Fine.
	2	0	72	75	30,11	57	74		S	1	Fine.
24	7	0	66	68	30,06	12	62		N	1	Fine.
	2	0	73	75	30,06	73	76		SE	1	Fine.
25	7	0	66	68	29,96	16	62		NbE	1	Fine.
	2	0	74	74	29,95	60	75½		S	1	Fine.
26	7	0	65	67	30,05	18	62		NbE	1	Fair.
	2	0	70	69	30,06	41	71		N	1	Cloudy.
27	7	0	63	67	30,05	14	60		NE	1	Cloudy.
	2	0	70	68	30,05	22	72		SE	1	Fine.
28	7	0	63	68	29,97	12	61		E	1	Cloudy.
	2	0	68	68	29,88	27	72		SbyE	1	Cloudy.
29	7	0	63	66	29,71	5	59		SW	1	Cloudy.
	2	0	71	72	29,56	29	74		S	1	Fine.
30	7	0	63	66	29,41	7	59		SE	1	Cloudy.
	2	0	69	67	29,20	10	70		E	1	Rain.
31	7	0	55	63	29,31	13	52		SW	1	Fair.
	2	0	67	67	29,39		67				

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1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Sept. 1	7	0	50	57	29,48	18	47	Rain this Month 1,320 Inches.	W	1	Fine.
	2	0	61	62	29,55	54	62		SW	1	Fine.
2	7	0	52	57	29,67	17	48		W	1	Fine.
	2	0	64	66	29,71	50	67			1	Fine.
3	7	0	63	64	29,61	2	63		W	1	Cloudy.
	2	0	68	68	29,72	40	71		WSW	1	Fine.
4	7	0	58	61	29,91	8	54		W	1	Fine.
	2	0	67	68	29,96	37	70		W	1	Fine.
5	7	0	63	64	29,77	7	62		NW	1	Rain.
	2	0	63	65	29,81	27	67		W	1	Fine.
6	7	0	55	60	29,91	16	51		W	1	Fine.
	2	0	62	67	30,01	48	65		W	1	Fine.
7	7	0	60	64	30,05	7	55		SW	1	Cloudy.
	2	0	68	65	30,06	56	69		SW	1	Fine.
8	7	0	65	65	30,09	16	62		WSW	1	Fine.
	2	0	71	69	30,10	28	73		W	1	Fine.
9	7	0	64	65	30,12	8	62		S	1	Cloudy.
	2	0	69	72	30,12	44	72		SW	1	Fine.
10	7	0	63	66	29,99	10	61		E	1	Cloudy and hazy.
	2	0	68	71½	30,06	39	70		S	1	Fine.
11	7	0	60	64	30,07	8	57		N	1	Thick hazy weather.
	2	0	60	64	30,12	30	66		N	1	Cloudy.
12	7	0	59	66	30,21	17	58		SW	1	Cloudy.
	2	0	68	70	30,23	37	76		S	1	Fine.
13	7	0	57	61	30,27	22	53		N	1	Hazy.
	2	0	66	70	30,27	51	66		E	1	Fine.
14	7	0	58	63	30,25	8	55		E	1	Thick and hazy.
	2	0	66	71	30,21	54	69		SE	1	Fine.
15	7	0	62	65	29,98	9	58		E	1	Cloudy, rather hazy.
	2	0	70	70	29,87	34	73		SSW	1	Fine.

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for September, 1819.

1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Sept. 16	7	0	57	63	29,69	11	57	Rain this Month 1,320 Inches.	N	1	Cloudy.
	2	0	63	64	29,74	38	67		NW	1	Fine.
17	7	0	54	59	29,96	14	48		N	1	Fine.
	2	0	63	64	30,06	27	69		N	1	Fine.
18	7	0	55	59	30,19	7	53		W	1	Cloudy.
	2	0	65	67	30,17	30	68		W	1	Fine.
19	7	0	57	61	30,19	28	52		SW	1	Fine.
	2	0	60	62	30,19	36	66		W	1	Fine.
20	7	0	52	56	30,29	15	47½		N	1	Fine
	2	0	70	63	30,34	40	70		NE	1	Fine.
21	7	0	51	56	30,44	18	47		NbyE	1	Fine.
	2	0	60	62	30,44	39	66		N	1	Fine.
22	7	0	51	57	30.45	22	48		N	1	Fine.
	2	0	60	61	30,36	27	66		E	1	Cloudy.
23	7	0	57	59	30,23	16	53		N	1	Fine.
	2	0	59	60	30,10	24	62		E	1	Cloudy.
24	7	0	54	58	29,87	20	53		ESE	1	Cloudy and hazy.
	2	0	61	63	29,78	40	63		SE	1	Fine.
25	7	0	56	59	29.59	5	52		E	1	Rain.
	2	0	58	61	29,52	7	62		E	1	Rain.
26	7	0	60	67	29,48	17	52		E	1	Fair.
	2	0	60	62	29,58	32	62		NW	1	Fine.
27	7	0	56	59	29.55	5	52		W	1	Fine. Rain in the night.
	2	0	63	62	29,60	13	64		W	1	Rain.
28	7	0	60	61	29,59	11	58		S	3	Rain.
	2	0	60	63	29,61	19	65		S	1	Fair.
29	7	0	59	61	29,47	3	56		S	2.3	Rain.
	2	0	61	63	29,61	11	65		W	1	Cloudy.
30	7	0	61	62	29,77½	11	56		W	1.2	Cloudy.
	2	0	60	63	29,79	19	64		W	1	Cloudy.

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for October, 1819.

1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Oct. 1	8	0	63	63	29,80	17	61	Rain this Month 1,069 Inches.	S	2	Fine.
	2	0	69	65	29,75	17	69		SW	1	Cloudy.
2	8	0	61	64	29,58	6	60		SbyE	1.2	Cloudy.
	2	0	68	68	29,75	42	68		W	1	Fine.
3	8	0	61	64	29,72	16	58		W	1	Cloudy.
	2	0	63	67	29,67	40	63		W	1	Fine.
4	8	0	58	63	29,54	12	54		S	1.2	Cloudy.
	2	0	57	63	29,56	15	57		SW	1	Rain.
5	8	0	46	59	29,81	21	44		NW	1	Fine.
	2	0	58	62	30,04	34	58		NW	1	Fine.
6	8	0	47	56	30,13	14	43		W	1	Cloudy.
	2	0	55	58	30,01	27	55		SW	1.2	Fine.
7	8	0	54	58	29,91	10	53		W	1	Cloudy, rather hazy.
	2	0	58	62	29,87	25	58		SW	1	Cloudy.
8	8	0	58	60	30,00	6	56		W	1	Cloudy.
	2	0	59	61	30,03	19	59		W	1	Fair.
9	8	0	57	60	29,93	8	57		SbyE	1	Cloudy.
	2	0	58	61	29,86	20	58		S	1	Cloudy.
10	8	0	60	61	29,78	11	58		WSW	1	Cloudy.
	2	0	60	67	29,81	44	60		W	1	Fine.
11	8	0	62	63	29,83	60	60		E	1	Fine.
	2	0	66	68	29,92	35	66		ESE	1	Fine.
12	8	0	59	63	29,98	13	56		NNE	1	Thick and hazy.
	2	0	70	70	29,99	25	70		SW	1	Fine.
13	8	0	61	64	29,97	2	59		S	1	Cloudy and hazy.
	2	0	64	65	29,95	20	64		SW	1	Cloudy.
14	8	0	54	60	30,08	13	51		SW	1	Cloudy and hazy.
	2	0	60	62	30,10	13	60		W	1	Rain.
15	8	0	57	60	30,27	12	51		NbyE	1	Fine.
	2	0	57	62	30,31	38	57		SE	1	Fine.
16	8	0	53	57	30,26	14	47		W	1	Cloudy and hazy.
	2	0	57	60	30,01	19	57		W	1	Cloudy.

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for October, 1819.

1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Oct. 17	8	0	44	54	30,11	19	42		NW	1	Fine.
	2	0	50	54	30,16	22	55	*	N	1	Rain.
18	8	0	43	52	30,20	8	42		N	1	Fine.
	2	0	50	54	30,21	30	54		SW	1	Fine.
19	8	0	39	52	30,13	5	37		W	1	Cloudy.
	2	0	47	55	30,01	25	60		W	1	Fine.
20	8	0	54	55	29,67	5	37	Rain this Month 1,069 Inches.	SW	1	Cloudy.
	2	0	53	55	29,57	10	57		SW	1	Rain.
21	8	0	42	49	29,48	8	44		NbW	1	Cloudy and hazy.
	2	0	42	51	29,52	0	42		NW	1	Cloudy, snow at 1 o'clock.
22	8	0	34	47	29,41	3	32½		NNE	1.2	Cloudy, a heavy fall of snow in the night.
	2	0	42	49	29,42	12	43		NW	1.2	Rain.
23	8	0	37	47	29,35	3	37		W	1	Fair.
	2	0	54	55	29,31	10	54		W	1	Cloudy.
24	8	0	47	48	29,32	7	38		W	1	Cloudy.
	2	0	46	48	29,35	4	49		W	1.2	Cloudy.
25	8	0	36	46	29,41	12	35		W	1	Cloudy.
	2	0	43	52	29,47	8	43		NW	1	Fine.
26	8	0	40	49	29,56	4	34		N	1	Cloudy and hazy.
	2	0	48	57	29,58	6	60		NW	1	Fine.
27	8	0	34	51	29,81	1	32		NbyW	1	Hazy and thick.
	2	0	43	55	29,79	6	45		SbyE	1	Rain.
28	8	0	37	50	29,80	3	36		NNE	1	Fine.
	2	0	46	56	29,79	7	45		N	1	Fine.
29	8	0	37	52	29,57	1	35		N	1	Cloudy and hazy.
	2	0	43	53	29,48	7	44		NbyE	1	Cloudy.
30	8	0	42	52	29,54	2	40		NE	2	Rain.
	2	0	45	55	29,57	8	45		SW	1	Rain.
31	8	0	43	52	29,78	0	44		E	1	Rain.
	2	0	45	50	29,23	7	48		E	1	Cloudy.

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for November, 1819.

1819	Time.		Therm. without.	Therm. within.	Barom.	Hygrometer	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Nov. 1	8	0	45	50	29,35	0	45	Rain this Month 0,917 Inches.	E	1	Rain.
	2	0	47	57	29,77	6	51		NbyE	1	Cloudy.
2	8	0	42	51	29,59	0	41		NE	1	Fine.
	2	0	46	59	29,61	10	47		WNW	1	Fine.
3	8	0	39	55	29,90	6	37		NW	1	Fine.
	2	0	45	59	30,09	25	46		SW	1	Fine.
4	8	0	43	53	29,96	12	39		W	1	Fine.
	2	0	51	58	29,91	19	56		SSW	1	Cloudy.
5	8	0	49	56	29,74	16	46		SW	1	Cloudy.
	2	0	52	59	29,82	19	60		S	1	Cloudy.
6	8	0	46	57	29,52	7	46		W	1	Fine.
	2	0	52	62	29,47	17	53		SW	1	Fine.
7	8	0	44	55	29,41	11	43		S	1	Fine.
	2	0	53	55	29,47	22	54		S	1	Fine.
8	8	0	39	53	29,55	3	38		N	1	Fine.
	2	0	46	59	29,59	11	56		N	2	Cloudy.
9	8	0	33	53	29,88	1	33		W	1	Fine.
	2	0	47	56	29,85	13	49		W	1	Cloudy.
10	8	0	44	56	29,34	1	34		W	1	Thick and hazy.
	2	0	49	60	29,32	17	51		NW	1	Cloudy.
11	8	0	45	57	29,58	4	45		N	2	Cloudy.
	2	0	46	57	29,75	11	48		S	1 2	Cloudy.
12	8	0	46	59	29,96	3	45		NE	1	Cloudy.
	2	0	48	59	29,95	12	48		N	1	Fine.
13	8	0	43	54	29,82	10	41		N	1	Cloudy.
	2	0	45	57	29,81	16	47		N	1	Cloudy.
14	8	0	43	54	29,79	9	42		S	1	Cloudy.
	2	0	47	56	29,79	15	48		S	1	Cloudy.
15	8	0	42	52	29,78	0	42		W	1	Foggy.
	2	0	46	61	29,70	3	49		S	1	Cloudy.
16	8	0	42	52	29,54	0	42		NW	1	Rain.
	2	0	56	64	29,48	12	57		SW	1	Cloudy.

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for November, 1819.

1819	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Nov. 17	8	0	41	51	29,61	2	39	Rain this Month 0,917 Inches.	E	1	Cloudy.
	2	0	44	58	29,80	10	45		N	1	Cloudy.
18	8	0	42	54	30,05	15	42		E	1	Cloudy.
	2	0	45	56	30,08	19	46		E	1	Cloudy.
19	8	0	35	49	29,99	14	34		NE	1	Fine.
	2	0	39	55	29,99		40		S	1	Fine.
20	8	0	35	48	29,74	9	34		NW	1	Cloudy.
	2	0	38	53	29,58	12	42		W	1	Cloudy.
21	8	0	38	49	29,11	3	35		W	1	Fine.
	2	0	42	49	29,14	8	44		W	1	Cloudy.
22	8	0	33	46	29,49	3	33		W	1	Fine, but hazy.
	2	0	40	54	29,54	11	40		N	1	Fine.
23	8	0	31	48	29,74	6	21		W	1	Fine, hoar frost.
	2	0	38	54	29,78	13	43		W	1	Fine.
24	8	0	29	47	29,88	2	28½		W	1	Fine.
	2	0	48	52	29,96	5	38		NW	1	Fine.
25	8	0	32	47	30,01	1	31		W	1	Fine, rather hazy.
	2	0	44	53	29,95	8	39		W	1	Fine.
26	8	0	37	48	29,66	3	35		NNW	1	Thick fog.
	2	0	41	52	29,73	0	42		NbyW	1	Dark and cloudy.
27	8	0	37	52	29,90	0	37		N	1	Damp, dark and hazy.
	2	0	40	55	29,93	12	44		SW	1	Rain.
28	8	0	34	49	29,89	0	30		N	1	Cloudy.
	2	0	33	54	29,79	9	35		W	1	Cloudy.
29	8	0	47	47	29,57	0	33		S	1	Cloudy.
	2	0	50	53	29,51	0	52		SW	2	Rain.
30	8	0	50	42	29,59	0	51		SSW	1.2	Rain.
	2	0	51	58	29,59	5	52		SW	1	Cloudy.

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for December, 1819.

1819	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Dec. 1	8	0	47	53	29,73	4	47		WbyN	1	Rain
	2	0	48	57	29,94		51		SW	1	Fine.
2	8	0	47	52	29,98	8	40		S	2	Cloudy.
	2	0	48	59	29,93	0	50		W	1	Rain.
3	8	0	35	53	30,17	0	34		S	1	Fine, rather hazy.
	2	0	46	57	30,17	11	43		S	1	Fine.
4	8	0	42	52	29,51	0	40	Rain this Month 1,873 Inches.	S	3	Rain.
	2	0	45	57	29,78	0	48		S	2	Rain.
5	8	0	40	52	29,87	7	41		N	2	Cloudy.
	2	0	47	51	29,92	12	49		N	1	Cloudy.
6	8	0	37	50	30,04	8	37		E	1.2	Cloudy.
	2	0	39	53	30,07		39		E	1	Cloudy.
7	8	0	35	58	30,03	3	35		N	1	Cloudy.
	2	0	47	55	30,01	3	52		N	1	Cloudy.
8	8	0	30	47	30,01	7	31		SSE	1	Cloudy.
	2	0	31	52	30,05	4	52		SSE	1	Fine.
9	8	0	27	43	30,07	3	26		NE	1	Cloudy.
	2	0	31	49	30,07	10	31		N	1	Cloudy. [the night.
10	8	0	30	46	29,89	2⅛	28½		N	1.2	Cloudy, and snow in
	2	0	32	49	29,84	6	32		W	1	Cloudy.
11	8	0	18	43	30,01		18		W	1	Fine, but hazy.
	2	0	30	48	30,01	15	29		W	.1	Fine.
12	8	0	34	43	29,96	4	32		W	1	Cloudy.
	2	0	36	43	29,88	9	36		N	1	Cloudy.
13	8	0	28	42	29,78	0	28		N	1	Cloudy.
	2	0	35	48	29,75	3	36		W	1	Cloudy.
14	8	0	26	43	29,74		25½		W	1	Fine, rather hazy.
	2	0	37	48	29,62	7	38		W	1	Cloudy.
15	8	0	33	44	29,46	4	31		W	1.2	Fine.
	2	0	43	48	29,46	13	39		W	1	Cloudy.
16	8	0	32	44	29,81	2	32		W	1	Fine.
	2	0	38	48	29,95	15	41		W	1	Cloudy.

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for December, 1819.

1819	Time.		Therm. without.	Therm. within.	Barom.	Hy-gro-me-ter.	Six's Therm.		Winds.		Weather.
	H.	M.	°	°	Inches.				Points.	Str.	
Dec. 17	8	0	37	49	29,71	1	34	Rain this Month 1,873 Inches.	E	2.3	Rain, a stormy night.
	2	0	43	51	29,45	3	43		E	2	Rain.
18	8	0	51	51	29,35	1	43		W	1	Cloudy.
	2	0	49	54	29,46	11	54		NW	1	Cloudy.
19	8	0	48	53	29,52	5	50		WNW	1	Cloudy.
	2	0	42	53	29.75	9	45		SW	1	Cloudy.
20	8	0	52	53	29,64	0	51		W	1	Rain.
	2	0	50	60	29,64	7	54		W	1	Cloudy.
21	8	0	44	53	29,93	9	44		W	1	Cloudy.
	2	0	45	57	29,89	0	48		SE	1	Rain.
22	8	0	51	56	29,49	5	43		W	2	Cloudy.
	2	0	55	60	29,71	17	54		W	1	Cloudy.
23	8	0	49	56	29,24	7	49		W	1	Cloudy.
	2	0	47	58	29,31	9	49		W	1	Cloudy.
24	8	0	33	52	29,37	1	33		W	1	Fine.
	2	0	49	58	29,33	11	53		W	1	Cloudy.
25	8	0	36	50	29,48	7	32		W	1	Cloudy.
	2	0	36	48	29,39	7	38		W	1	Cloudy.
26	8	0	27	46	29,49	3	28		W	1	Fine.
	2	0	36	47	29,51	18	39		W	1	Fine.
27	8	0	31	45	29.44	5	31		E	1	Fine.
	2	0	35	47	29.45	6	38		ESE	1	Fine.
28	8	0	35	45	29,47	9	31		E	1	Cloudy.
	2	0	33	50	29,48	4	34		E	1	Snow.
29	8	0	28	44	29,61	6	29		SE	1	Cloudy.
	2	0	32	46	29,63	3	34		W	1	Fine, rather hazy.
30	8	0	30	48	29,52	4	24		W	1	Cloudy.
	2	0	32	46	29,46	5	33		W	1	Cloudy.
31	8	0	30	43	29,34	3	29		W	1	Cloudy.
	2	0	38	47	29,35	11	41		W	1	Fine.

1819.	Thermometer without.			Thermometer within.			Barometer.*			Leslie's Hygrometer.			Six's Thermometer.			Rain.†
	Greatest height.	Least height.	Mean height.	Greatest height.	Least height	Mean height.	Greatest height.	Least height.	Mean height.	Greatest height.	Least Height.	Mean height.	Greatest height.	Least height.	Mean height.	
	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Inches.	Inches.	Inches.	Deg.	Deg.	Deg.	Deg.	Deg.	Deg.	Inches.
January	52	32	41,8	60	41	51,8	30,49	29,22	29,78	28	2½	11,0	50	32	42,5	0,789
February	51	30	41,9	59	47	52,2	30,04	29,02	29,65	34	3	13,0	58	30	42,5	1,638
March	57	36	46,0	63	46	54,0	30,30	29,21	29,87	50	4	17,1	60	36	46,3	1,010
April	65	39	50,7	69	52	59,1	30,22	29,11	29,78	52	3	19,8	67	38½	51,5	1,694
May	68	48	57,3	65	52	59,7	30,21	29,10	29,73	56	4	21,6	71	40	57,5	1,400
June	70	51	59,7	70	56	61,8	30,28	29,58	29,90	39	3	16,9	72	46½	59,5	1,017
July	75	56	64,6	74	60	66,4	30,21	29,32	29,88	61	8	24,2	81	50½	64,9	0,750
August	75	55	67,0	76	63	68,2	30,31	29,20	29,99	73	5	27,1	80	52	67,2	0,250
September	71	50	60,7	72	56	63,3	30,45	29,47	29,94	56	2	23,4	73	47	60,7	1,320
October	70	34	51,5	70	46	57,3	30,31	29,23	29,79	60	0	14,7	70	32	58,9	1,069
November	56	29	42,9	64	46	52,9	30,08	29,11	29,71	25	0	8,6	60	21	42,7	0,917
December	55	18	38,4	60	42	52,2	30,17	29,24	29,71	17	0	6,0	54	18	38,9	1,873
Whole year			51,9			58,2			29,81			16,8			52,7	13,727

* The quicksilver in the bason of the barometer, is 81 feet above the level of low water spring tides at Somerset-house.

† The Society's Rain Gage is 114 feet above the same level, and 75 feet 6 inches above the surrounding ground.

PHILOSOPHICAL
TRANSACTIONS,

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXX.

PART II.

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PHILOSOPHICAL TRANSACTIONS.

VIII. *On a new principle of constructing ships in the mercantile navy. By Sir* Robert Seppings, *F. R. S.*

Read March 9th, 1820.

So deeply is this country in particular interested in whatever may tend to give additional safety to the number of persons, and the immense sums of money employed in commerce, that I trust I shall stand excused for again bringing to the notice of the Royal Society, the subject of the construction of ships. The description of vessels of which I am now about to treat, are those employed in our mercantile service, in the construction of which, the imperfections are such as on a close examination will, I conceive, leave but little doubt that lives and property to an immense amount, must, from time to time, have been sacrificed by the present injudicious mode of constructing these ships; and under that impression I have presumed to lay this paper before the Society.

And first, as to the principle on which mercantile ships are at present built, and particularly as regards the putting together their ribs or frames, and the arrangement of the materials.

In forming the frames or ribs, half of the timbers only are united so as to constitute any part of an arch; every alternate couple only being connected together: the intermediate two timbers (termed fillings) being unconnected with each other, and merely resting upon the outer planking, instead of giving support to it. Now, it must be very evident that ships so constructed, can by no means possess equal strength with those that have the whole of their timbers formed into frames or arches.

This loose practice is, I believe, peculiar to the English merchant ship-builder; and indeed was pursued till very lately even in His Majesty's Navy, while the preferable system of connecting the ribs was common to other maritime powers.

The principle of uniting the frames, lately introduced in the construction of English ships of war, might, no doubt, be also introduced into the mercantile navy; which would give to the ships in that employ additional strength and increased durability, without adding to the expense of building.

But the present mode of joining together the several pieces of the same rib, is also highly objectionable. It is done by the introduction of a third piece, technically termed a *chock* or *wedge piece*, (Pl. VIII. Fig. 1, marked A) of which pieces the number amounts to upwards of 450 in a 74 gun ship, and not less than that number in an Indiaman of 1200 tons: (to which class of ships the drawings in this statement have reference.) Of these chocks not one in a hundred is ever replaced in the general repair of a ship; for they are not only found defective, but very generally to have communicated their own decay to the timbers to which they are attached. Besides this, the grain of the rib-pieces being much cut, to give them the curvature required, has a considerable share in weakening the general fabric. That they occasion a great consumption of materials, is obvious, as the ends of the two rib-pieces must first be cut away, and then be replaced by the chock.

This mode of putting together the frame, is also peculiar to the English ship-builder; and I find, from an old work in my possession, dedicated to George the First, that the practice was introduced in the construction of English ships about the year 1714; and having heard that so unfriendly to it was the builder (Mr. Naish) of the Royal William, that he refused to adopt it; and being desirous of ascertaining the fact, when that ship was taken to pieces at Portsmouth, in 1813, I found that she was built without the wedge pieces or chocks, to which, in a certain degree, I ascribe her strength and durability; her ribs being by her structure less grain-cut, and for want of chocks, less liable to decay in those parts where they are inserted.

The introduction of chocks, was no doubt to procure that curvature which is so necessary in the formation of a ship, when crooked or compass timber became scarce; as may be seen by Pl. VIII. Fig. 2. which describes the shape of a piece of timber in the converted form; and by which it will also be seen, that the introduction of the chocks assists in obtaining the required curve. But this curve may equally be obtained by a different combination of materials, and at a considerable less consumption of useful timber.

The frames of a mercantile ship (on the present mode of building) before they are placed and united to each other, may be seen in Pl. VIII. Fig. 3, with their chocks or wedge-pieces. To the evils already stated of the present practice, may be added that of imperfect workmanship, so that the surfaces of the chocks are seldom in contact with those of the timbers; and the ends of both are frequently reduced so thin, as to split by the fastenings that are necessary to secure the planks to the ribs; and thus the ship, in the event of grounding, or even in the act of rolling, derives little support from timbers united only, in fact, by two narrow edges.

Another great defect arising out of the present plan of constructing mercantile ships is, that the ends of the lower ribs or timbers, commonly termed the lower futtocks, (Pl. VIII. Fig. 3. B) are not continued across the keel C, so that no support is given in a transverse direction when the ship touches the ground; nor any aid to counteract the constant pressure of the mast. This great sacrifice of *strength* and *safety* is made for no other purpose than that of giving a passage for the water to the pumps.

The floor timbers, which by this mode of construction are the only timbers that cross the keel, are also weakened for the same purpose, as shown at D, Pl. VIII. Fig. 3. This mode also makes the conveyance of the water very uncertain, for the passage is not unfrequently choaked; and the pumps (from its not being practicable to continue them sufficiently down) always leave from 6 to 8 inches of water in the ship; so that these compartments constantly contain a certain quantity of putrid bilge water, offensive and injurious to the health of those on board.

The deficiency of strength causes also an alarming insecurity in the plank of the bottom, as shown at E, Pl. VIII. Fig. 9. termed the garboard strake; which consequently, has no other fastening to the general fabric, than its connection with the keel at F, Pl. VIII. Fig. 9. and a slight security at G, Pl. VIII. Fig. 9: hence it is obvious, that in the event of the keel being disturbed, the garboard strake, from its being attached to it, must share the same fate as the keel, and in that case the loss of the vessel would be inevitable.

To obviate these serious defects, is the principal object of this paper.

The principle I would recommend is explained in Pl. IX. Fig. 4; by which it will be seen, that the component parts of each rib are of shorter lengths and less curvature, and consequently less grain-cut; that they are more firm and solid by the substitution of coaks or dowels, for chocks or wedge-pieces; and that the mode of connecting the lower timbers is better adapted, in the event of a ship grounding, to give support and strength to the fabric, as will appear by the line marked H.

The plan of connecting the ends of the timbers by circular dowels or coaks (as at I,) is simply that which has, from time immemorial, been practiced to unite the fellies of carriage wheels; and we learn from Mr. Wood, that the same method has been observed in joining together the separate pieces of the shafts of the stone columns in the ruins of Balbec. " Little more of this great edifice (says this author) " remains, than nine lofty columns supporting their entabla- " ture. It is remarkable, that the shaft of these columns " consists of three pieces most exactly joined together with-

" out cement, which is used in no part of the buildings, they " being strengthened *with iron pins received into a socket.* " How much this method contributed to the strength of the " building, is remarkably seen in the most entire temple, " where a column has fallen against the wall of the cell with " such violence, as to beat in the stone it fell against, and " break part of the shaft, *while the joinings of the same shaft " have not been in the least opened by the shock.*"

That the frame of the Thunderer, (now Talavera) built on this principle, is superior in point of strength, to a frame constructed on the common system, is fully established by a report from the officers of His Majesty's Yard at Woolwich to the Navy Board, who directed them to compare the strength of the frames so united, with those of the Black Prince, constructed in the usual way with chocks or wedges.

It may be necessary to observe, that the frame of the Thunderer is composed of small timber, hitherto considered applicable only for the frames of frigates. I was prompted to attempt the introduction of the plan on which she is built, from there being a surplus store of small timber in the yard, and from a conviction, that a well combined number of small timbers, might be made equal, if not superior, both in strength and economy, to the large, overgrown, and frequently grain-cut materials, made use of in constructing the frames of large ships; and the result has shown the correctness of the principle; the adoption of which cannot fail to prove of great national advantage, in the application of sloop timber to the building of frigates, and of frigate timber to ships of the line, whenever larger timber cannot be procured. On this principle also, may frigates and small ships of war, or merchant vessels, be built of straight fir, without the assistance of oak or elm, which were formerly employed to give the necessary curvature of the sides. As it respects the general safety of the ship, it will be seen, by Fig. 5 and 6, Pl. IX. X, that the timbers uniformly cross the keel; that the frame of the ship is filled so as to form one compact body to the height marked K; and that only certain internal strakes of plank, or thick-stuff, as it is termed, are introduced, which are those on the joints of the timbers, for the purpose of giving strength where every alternate timber necessarily joins, as shown at L (Pl. X.) All the rest of the inner planking may be omitted; and dunnage battens, brought in a perpendicular direction, upon the timbers *between* the plank, as shown at M, forming regular spaces between each, as is usual at present *upon* the plank; thereby giving an increase of stowage in proportion to the thickness of the plank omitted. Water courses, as shown by dotted lines at N, are to be left in the joints of the timber under the plank, for the purpose of conveying the water to the pumps; which, by this plan, will reach below the water, instead of being some inches above, as is the case with the present mode, before described; consequently, by the proposed system, no stagnant water will remain; and farther, the limber passage, or water course, will be one smooth, uniform channel, which can be cleared with ease, should it be required, whenever the hold is unstowed; whereas at present it is inaccessible in places, and forms compartments for putrid water, without there being any means of removing it.

It is obvious, that a ship on the principle I have here recommended, may sustain the loss of certain planks of the bottom, and also the keel, (which has frequently been found to have happened to ships of war on their being taken into dock) and still reach the place of her destination; when the loss of *either*, would be the destruction of a ship built on the present mode. It will be evident also, that a ship constructed as now recommended, possesses greater stowage, and more space for leakage, than by the old plan; by the omission of the useless inner planking, and by laying the kentlage on dunnage, leaving a space for the water, which was formerly occupied by the inner lining. This dunnage in the bilge may be formed with the iron kentlage, and thereby serve as ballast, for which it is well calculated from its situation; and by its occupying a space heretofore forming part of the fabric of the ship, will give an increase of stowage, as before stated.

The best mode of closing the openings between the timbers, is by filling the intermediate spaces with pieces of wood, about three inches in depth, of such lengths as the inferior conversions will supply, abundance of which may be procured from the offal. These fillings are to be well caulked, after which the exterior plank is to be brought on. When the works are going on within board, similar pieces are to be fitted internally, and afterwards taken out for the purpose of filling the spaces between the pieces so fitted, with a mixture of PARKER's Roman cement and drift sand, in the following proportions, viz.

PARKER's Roman cement,	$\frac{2}{3}$
Drift sand, - - - -	$\frac{1}{3}$

previously paying the opening well with coal tar. Where there is sufficient space, a brick, or part of one, may be introduced, provided there is room for cement between it and the timbers. When filled in to within about two inches of the surface of the frame, the pieces of three inches already fitted and taken out, are to be well driven in and caulked, and by so doing, no space will be left unoccupied. If considered desirable, these pieces may be driven below the surface of the timber, thereby leaving water courses to convey the leakage to the pumps in channels. And prior to launching or undocking of ships, built on the principle I have recommended, it has been the practice to inject the part filled in with mineral tar, by means of a simple forcing pump, boring holes in the joints of the timbers for the introduction of the pipe. By following this method, the air will be excluded, which, as experience has shown, tends much to the durability of the fabric; confirming the assertions made by Doctor HALES on this subject, in his work on Ventilators, published in 1750, and also of Doctor HENRY, in his work on the Elements of Chemistry. If what is here recommended be attended to, and mercantile ships were built under roofs, as ships of war now are, durability would be obtained in addition to safety, from the mode of their construction.

The beams are to be attached to the sides, as shown at O, Pl. IX. Fig. 5. rendering wood knees unnecessary, and requiring only a small number of those of iron.

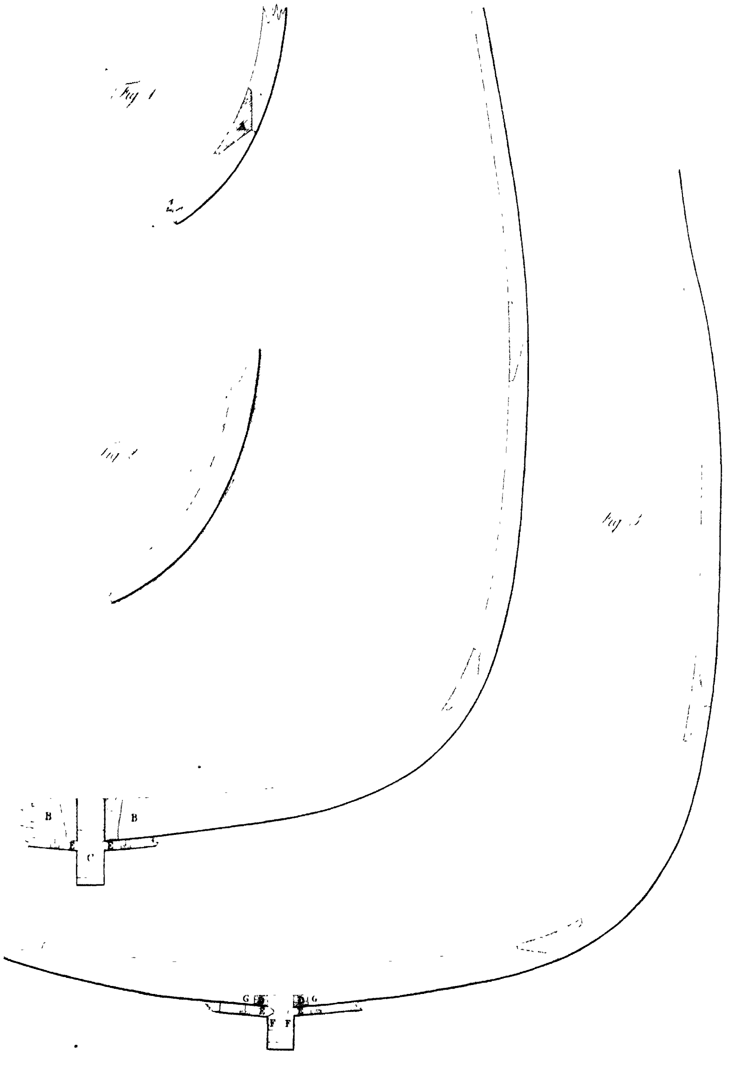
Pl. XI, marked P, describes the old principle of framing the stern with transoms. Q, the new principle, with timbers similar to the bow, omitting the transoms below the wing or upper transom; and by introducing the new principle on which the floors are made, the necessity of using valuable compass, or crooked timber, hitherto required, and with difficulty procured for these purposes, is avoided. Uniform support will thus be given, and also an increase of room for stowage.

In large mercantile ships above 500 tons, I would recommend that plate-iron be laid diagonally, as shown in Pl. X.

The principle now recommended will cause a decrease in the consumption of materials, and the difficulty of procuring the necessary curvature will be obviated. It also affords protection from worms externally, and vermin internally. Leaks may be more easily discovered and stopped than by the old method; and in point of additional strength, there can be no doubt. But were farther proof required, I need only state, that the Malabar, of 74 guns, built at Bombay, arrived at Portsmouth in October last, loaded to her upper deck with timber, and during her passage encountered four heavy gales of wind, without showing a symptom of weakness, as will appear by the following Extract from the Survey made by the Officers of Plymouth Yard, on that ship, by order of the Lords Commissioners of the Admiralty.

"When we consider the nature of the lading that this "ship has brought home, with the temporary security to "the beams of all the decks, except the orlop, and that on "her passage she encountered four very severe gales of "wind, it must, we presume, be very gratifying to your "Honorable Board to find, that she does not indicate any "past symptoms of weakness or straining in any part."

This ship had no other attachment for her beams than the internal hoops and thick waterway; the remainder of her security, the iron knees, being omitted, (from the difficulty of



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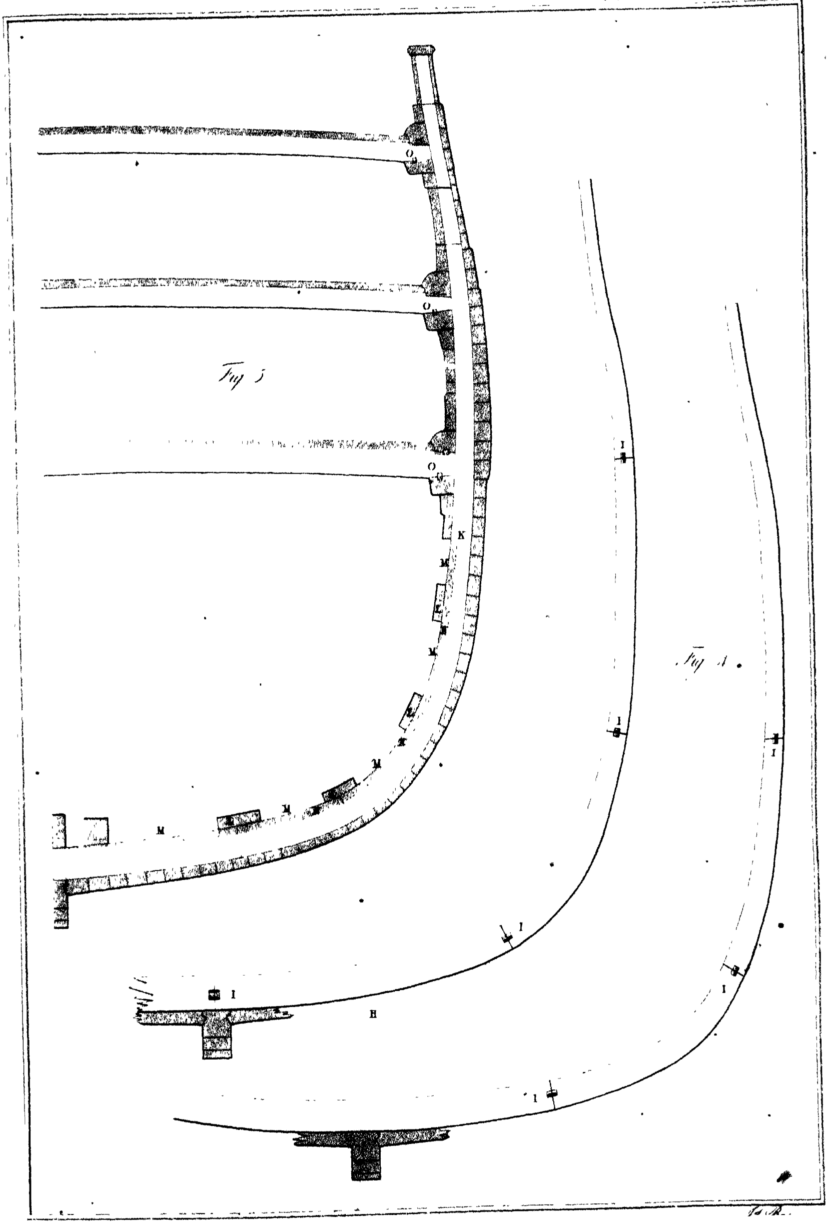
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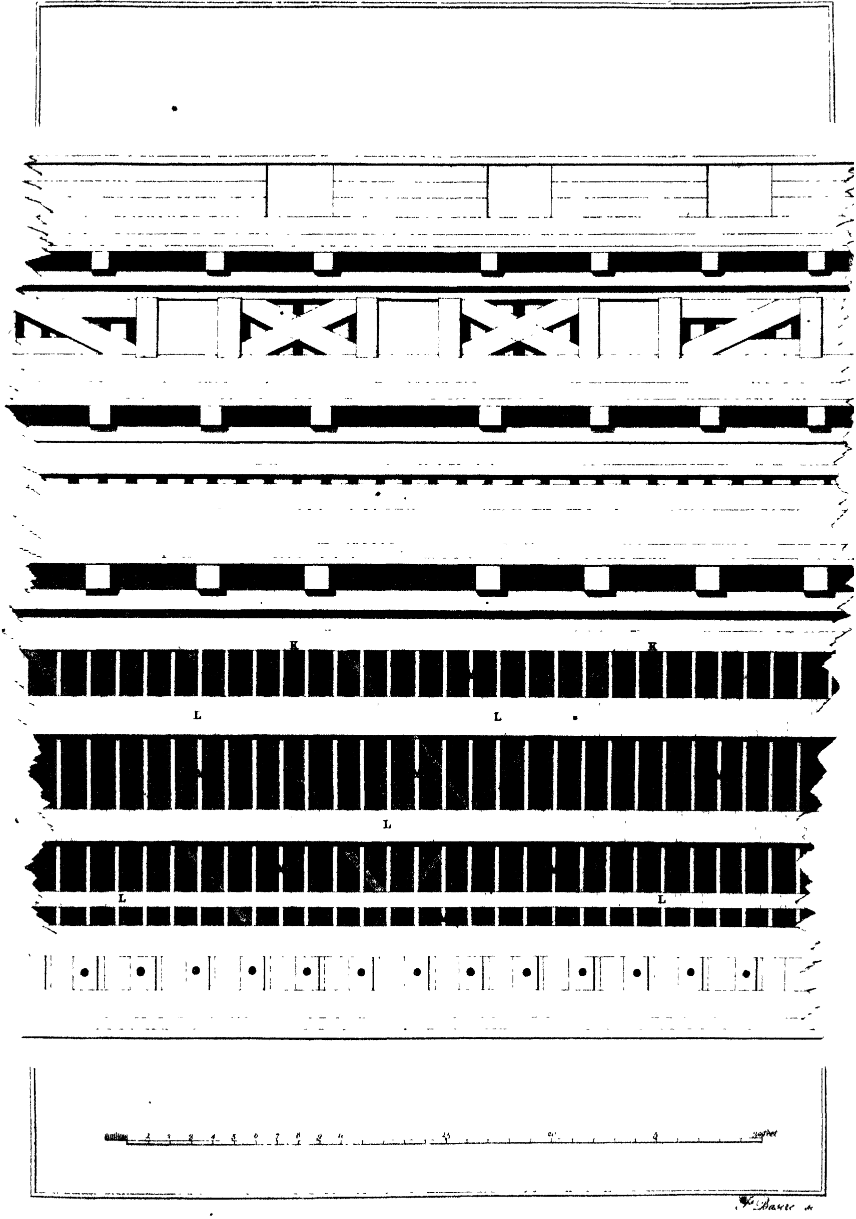
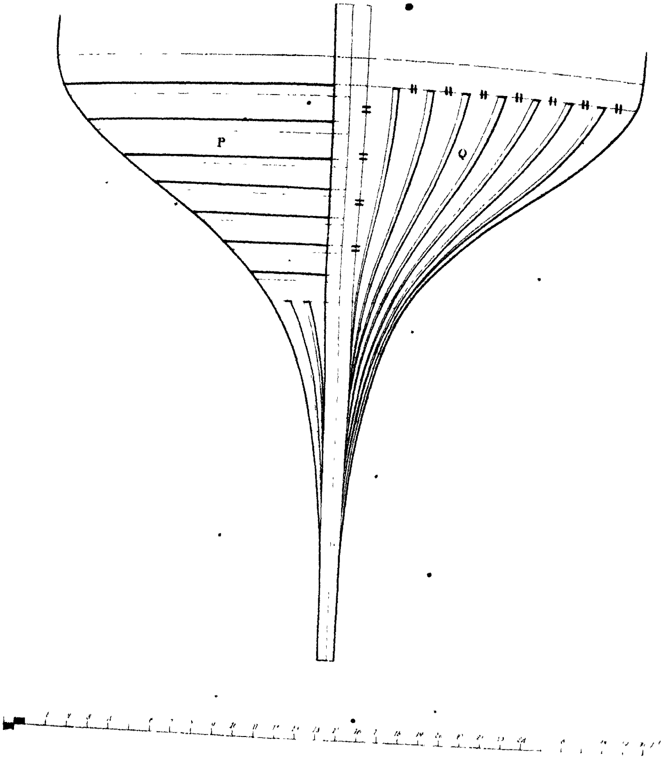
Philos Trans MDCCCXX Plate X p.112




P
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procuring them in India) until her arrival in this country; thus supporting her cargo without the aid of knees, either of wood or iron.

The advantages of this plain, but important subject, particularly in point of safety, has induced me to bring it before the Society, in the hope that its utility and importance to this great maritime nation, may plead an excuse for the absence of abstract science, to which the attention of the Society may, perhaps, be more peculiarly directed.

IX. *On the milk tusks, and organ of hearing of the Dugong.*

By *Sir* Everard Home, *Bart. V.P.R.S.*

Read April 13th, 1820.

I have found nothing that so much promotes the enquiry respecting those animals with which we are little acquainted, as laying before the public the materials, however scanty they may be, that have been already procured.

By making those materials generally known, naturalists have their minds awakened to the enquiry, and are not only more disposed, but better able to take advantage of such opportunities as come within their reach, to advance our knowledge of that subject.

This at least has happened in so great a degree with respect to the fossil bones of the Proteosaurus, of which very little was known at the time I laid my first observations upon them before the Royal Society, that I am induced to make a similar trial respecting the Dugong, which, I believe, has never been seen of its full size, by any one conversant in comparative anatomy.

On the present, as well as on former occasions, it is not the whole structure of the animal that has attracted my notice, but such parts of it as differed in form from similar parts in other animals most nearly allied to it; as I consider that all peculiarities of this kind are deserving of the notice of this Society, since they bring to our knowledge a new construction of parts, and therefore make us acquainted with animals that form intermediate links in the chain of gradation between those that are already known. The truth of this observation is very strongly illustrated in the proteosaurus.

Our knowledge of comparative anatomy has been so much encreased in extent within the last few years, that we can now better, than at any former period, attempt the arrangement of animals into a regular order, according to their internal mechanism; showing that they form so many parts of one great system of structures; and if the pursuit is followed with the same ardor, and attended with the like success, we may soon expect to have all the material links of this great chain made out.

To assist our progress towards this end, the great purpose for which comparative anatomy is cultivated as a science, I have in all my researches into the structures of animals, or their fossil remains, kept it in view.

The observations I now bring forward respecting the milk tusks, and organ of hearing of the dugong, have been made upon a skull of that extraordinary animal, in a very perfect state of preservation, sent to me, by my friend Sir THOMAS STAMFORD RAFFLES, from Sumatra.

There are three skulls which I may say have come under my observation, of animals nearly of the same age, but not entirely so, for although the milk tusks at their points in all of them, put on the same appearance, that is not the case with their other extremities. One of these skulls is in the possession of M. CUVIER, and a figure of it is engraved in the thirteenth volume of the *Annales du Museè d'Histoire Naturelle*; and also in the fourth volume of his work, entitled *Recherches sur les Ossimens Fossiles des Quadrupedes*. In this figure the bones of the ear are not represented; and as in that stage of the animal's growth they have no bony connection with the skull, there is sufficient evidence of their having belonged to a young animal. Another skull is in the collection of comparative anatomy belonging to Mr. Brookes, in London. In that also the organs of hearing are wanting. The third is in my possession, and is the subject of the present observations; in this the bones of the ear on both sides of the skull are preserved, although they have acquired no bony connection with the skull.

The first examination which it occurred to me to make, was respecting the length of the tusks, the points of which were the only parts exposed. For this purpose, one of the bony canals, composed of the bones of the face in which the tusk is contained, was laid open, and one of the tusks was removed from its socket, and a longitudinal section afterwards made of it. The whole of its substance was found to be solid, showing that it had arrived at its full growth, and was therefore only a milk tusk: at the posterior extremity there was a small shallow cup composed of the same materials, which appeared to be no part of the tusk itself, but, as it were, fixed to the end of it. This was contained in a corresponding cavity adapted to it, in the skull; but upon the upper surface, the bony table of the skull was entirely removed to some extent by absorption, so that the shallow cup at the root of the tusk was exposed externally, giving the skull at that part a very extraordinary appearance. In Mr. Brookes's specimen, the aperture through the external table of the skull is to a less extent, than in my specimen, but exactly of the same kind. In M. Cuvier's skull, there is no appearance of any breach in the upper table whatever, so that the process of absorption of the outer table of the skull, to make room for the formation of the permanent tusk, had not begun to take place.

When the section of this tusk was compared with a similar section of the milk tusk of the narwhal, and of the elephant, I found that in its internal structure it closely resembled them; and I noticed that the external surface in all of them was deficient in smoothness, when compared with the permanent tusks of those animals.

In the narwhal, the root of the milk tusk terminates in a rounded blunt end, behind which there is no appearance of any preparation for the formation of the permanent tusk; so that there is no evidence of that tusk having its origin in a cell similar to those of the dugong; but by comparing the place where the root of the milk tusk has its origin, with that of the permanent tusk in the same animal, for the two tusks in the narwhal come forward at such different periods of time, that this comparison may be made even when one permanent tusk appears to have arrived at its full growth, or nearly so; at which time the rudiments of the corresponding permanent tusk have not begun to be formed, and the depth in the skull of the milk and permanent tusk exactly correspond.

As the permanent tusk in the narwhal begins to form in a direct line immediately behind the origin of the milk tusk, the great purpose of the milk tusk is evidently to open the road for, and to direct the course of the permanent tusk, till it is completely pushed out by it. In the elephant, both of whose tusks appear at the same time, and whose skull encreases in size much more rapidly than that of the narwhal, the permanent tusk has its origin even farther forward than the milk tusk; but then it afterwards has an increase backwards in the space between the tables of the skull, which the tusks of the narwhal have not; and in this growth backwards, which is very slow, corresponding to that of the skull, absorption of the upper table previously formed, is produced, to make room for it. In this way the sockets of the elephant's tusks, which are shallow when the animal is young, acquire an increase of depth as the elephant grows up, to give it sufficient firmness in the skull to support them in the exertions that are made with them.

A similar absorption to what takes place in the upper table of the skull in the elephant, is shown to occur in the dugong; it is however probably at a different period of the animal's growth, as the milk tusk in the elephant is shed between the first and second year, and the absorption of the upper table many years after; but in the dugong, the absorption of the skull takes place just as the milk tusks begin to extend themselves beyond the gums, but the age at which the animal has at this time arrived is not known.

The use of the shallow cup, which appears to be an appendage peculiar to the milk tusk of the dugong, forming no part of the tusk itself, would appear to be for the purpose of receiving the point of the permanent tusk as soon as it is formed; so that as the milk tusk advances in the act of its being shed, the other may be directed forwards in the same course, which is different from that in which it set out.

The facts that were brought forward in a former paper upon the milk tusks of the narwhal, explained many circumstances in the natural history of that animal which were involved in obscurity; and the observations that are now made upon the tusks of the dugong, show that the milk tusks have been mistaken for the permanent ones, and that the real appearance of the permanent tusks of this animal is at present unknown. Having corrected so great an error in the natural history of this animal, we must wait till we are so fortunate as to procure the skull of a full grown dugong, before we attempt to form any analogies between its tusks and those of other tribes of animals.

The grinding teeth, as will be seen in the annexed plate, are not exactly similar to those of any known animal, but form an approach to the hippopotamus. They consist of two cones united together; but when a transverse section is made, there is no line of separation, the whole being uniform; the external crust is not enamel, nor is it the hardest part; a little within it, there is a narrow rim of a yellow colour, that describes an oval figure, much more dense than the rest of the tooth, although to the eye there is no apparent difference in its texture; all that is within this rim is soft ivory; so that these teeth in wearing down will always have the crown rendered concave.

On the organ of hearing.

In examining the organ of hearing of the dugong, which was done with more facility, as the skull had not been completely formed, I met with a peculiarity, that does not belong, as far as I am aware, to any other tribe of animals. The malleus and incus, which have nearly the same shape as in other animals, are fastened to the sides of the tympanum by a bony substance extending across the intervening space. The malleus is in this way connected to one side of the tympanum, the incus to the other, so as to render these ossicula in a great measure immoveable. The stapes is unconnected with the foramen ovale, to which it is opposed; nor is it anchylosed with the ramus of the incus. The handle of the malleus projects in the centre of the circle over which the membrana tympani had been spread, so as to leave no doubt that, in the living animal, it is attached to the centre of that membrane.

The nearest approach to this mechanism, is an attachment by bony union of the malleus to one side of the tympanum, which Mr. Hunter, in his paper on the Anatomy of Whales, states he had met with in some of that tribe; but does not, (which is unfortunate) mention in what species. He says not in the porpoise; and the author knows it is not the case in the balæna mysticetus; so that this structure does not belong to those genera that live principally upon the surface, nor to those that penetrate the unfathomable depths; and as Hunter describes the organ at length in the pike-whale, without mentioning this peculiarity as part of the description, it could not have occurred in that species. This renders it probable that he met with it in the grampus, or the bottle-nose, and it is very likely in both, as he insinuates that it is not confined to one genus.

The dugong feeds on the plants that grow at the bottom of the sea, and comes in great numbers to the harbours of uninhabited islands, and remains for many hours in the shoal water, where it finds food: at least this account is given of its habits by Mr. Leguat, who, with several companions, spent nearly three years in an island previously uninhabited, about fifty leagues from the Mauritius, and the dugong was a principal part of their food. It is from him we learn that their usual length is twenty feet; but that catching them full grown was a service of danger; nor were they so delicate as food for the table. This gentleman's account was published in 1720, and is written with so much apparent candour, that there is no reason for doubting the veracity of his narrative.

As these habits are allied to those of the hippopotamus, I inquired whether this peculiarity of structure exists in the ear of that animal, but find there is nothing of the kind; all the ossicula are separate, and very readily drop out of the cavity of the tympanum when the skull is deprived of the soft parts. There are peculiarities in the mechanism of this organ in the hippopotamus which deserve being mentioned. The projecting ear and meatus auditorius externus, have a higher situation upon the head than in most other animals; and the tube that passes down to the membrana tympani, is one straight line. The bony portion of it is four inches long, its direction from the external surface of the head down to the membrane, is at an angle of 45°, and its termination is upon a ledge which forms a slight projection beyond the bony ring to which the membrana tympani is attached; the space on the outer surface of the membrane, between it and the opposite side of the tube, is so small, that it cannot exceed the thickness of the membrane itself. The ossicula are small, when the size of the animal is considered; but the cochlea makes two turns and a half, which is by no means common; the semicircular canals have the usual appearance.

This animal I understand, from every enquiry I have been able to make, when it goes down into the water to feed at the bottom, allows itself to sink by its own weight, descending in a standing posture, so that as soon as the head is covered, the water would pass into the tube of the ears, were there not something like a valve at the orifice of the meatus externus to prevent it. In other animals that live in the sea, or are much under water, there are contrivances, very different from this, to prevent the water getting into the tube of the ear. In the whale tribe, the external orifice is so extremely small, as to exclude it. In the seal, the meatus externus makes a turn nearly circular, to answer the same purpose. In the ornithorhyncus paradoxus, the external opening is at a great distance from the organ; and the meatus, which is the size of a crow quill, and cartilaginous, winds round upon the temporal bone. The external meatus in the walrus, I have not had an opportunity of examining; the orifice in the bony tube corresponds in size with that of the hippopotamus, its termination at the membrana tympani is less oblique, and its direction horizontal. The ossicula in the hippopotamus are small, the stapes is imperforate, and the bones have no bony union.

The external orifice of the meatus externus in the dugong is extremely small, so as readily to exclude the water. The cochlea is very small, making only one turn and nearly one half. The semicircular canals are also exceedingly small. The peculiar bony connections to the tympanum, connecting the malleus and incus with the bones of the skull already described, lead to the idea that this animal is more indebted for its hearing, than any other that lives in water, to the vibrations received by the bones of the skull, being communicated, through the bony connections that have been mentioned, to the ossicula, and from thence to the cochlea, and semicircular canals.

This animal, although the tail is horizontal, possesses the sense of smell, since there are orifices in the cerebriform plate of the skull for the olfactory nerves.

While this paper was in the press, I have been so fortunate as to receive from Sir THOMAS STAMFORD RAFFLES, another skull belonging to a dugong, 8 feet long, in which the milk tusks had been shed, and the permanent ones had acquired sufficient length to show the degree they project beyond the bony sockets, and the manner in which they are worn down, by rooting up the plants while feeding. They are considerably broader than the milk tusks, even at their points, explaining the use of the cup at the end of the milk tusk being wider than the tusk itself, to admit of the point of the permanent one having this encreased breadth.

These tusks bear a near resemblance to those of the hog, the points being turned a little outwards, and they wear down in the same manner.

It is not a little remarkable, that in writing this paper upon the milk tusks, I should have an opportunity, during an accidental delay in the press, of being furnished with materials to show the appearance of the permanent tusks, which forms so essential a character of the animal, and also to prove that while young, there are incisors in the lower jaw; two of them in the jaw of this skull having been accidentally entangled (while shedding,) in the new bone that was closing up the socket, and consequently retained there. This is a curious additional fact, and is so far an approach to ruminating animals, whose incisors are only in the lower jaw. These incisors enable the young dugong to crop the tender plants, but are no longer wanted when the animal grows up.

EXPLANATION OF THE PLATES.

PLATE XII.

This plate consists of three figures.

Fig. 1. A side view of the first skull of the dugong, sent by Sir T. S. RAFFLES, in which the organ of hearing is in its place, and one of the tusks exposed through its whole length.

Fig. 2. The longitudinal section of the tusk.

These two figures are in the proportion of half an inch to one inch.

Fig. 3. An exact copy taken from Mr. CUVIER's representation of the dugong's skull, the proportions of which are not mentioned.

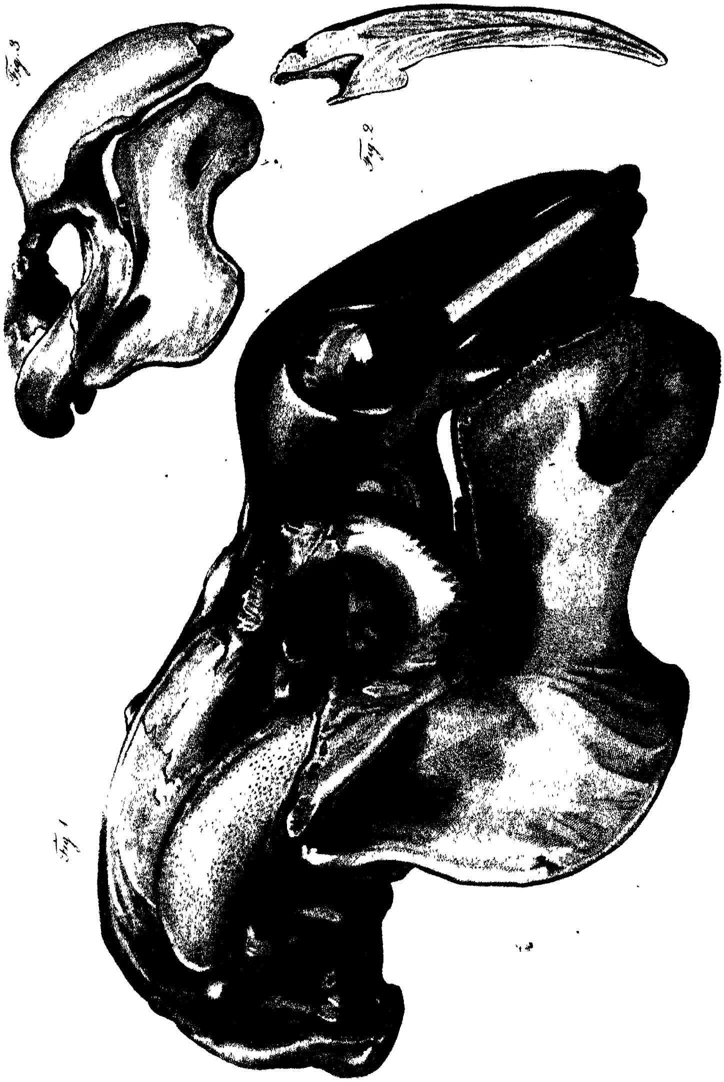
PLATE XIII.

This plate consists of three figures, in the proportion of half an inch to one inch. The first represents the basis of the skull of the dugong with milk tusks; the second, the appearance of the permanent tusks in situ; the third, the section of the permanent tusk.

PLATE XIV.

This plate consists of four figures.

Fig. 1. The lower jaw of the skull represented in Pl. XIII.



W. H. Clift del.

J. Basire sc.

Fig. 1

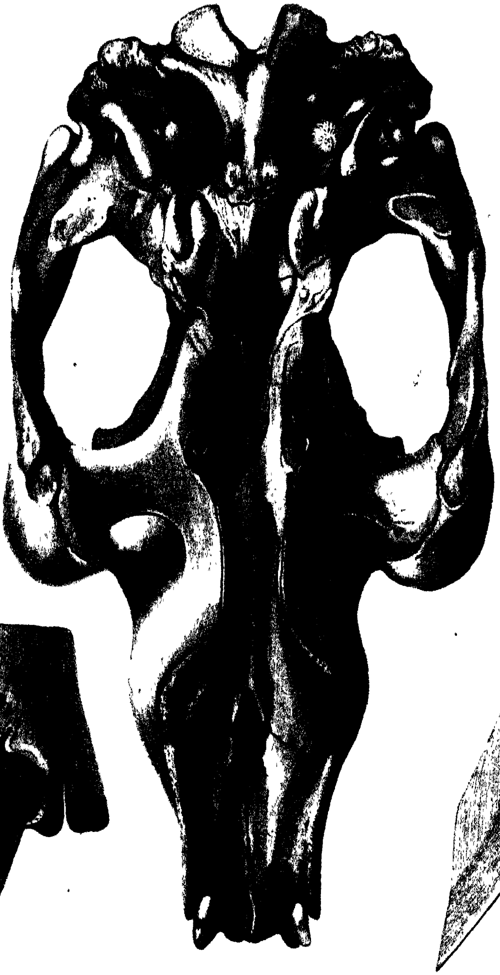


Fig. 2

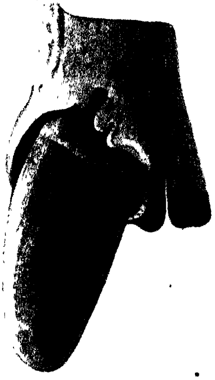


Fig. 3

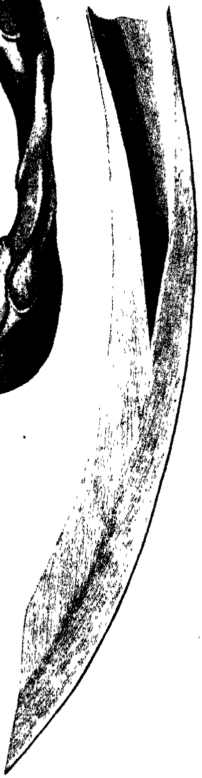


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Fig 4.

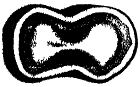

Fig 1

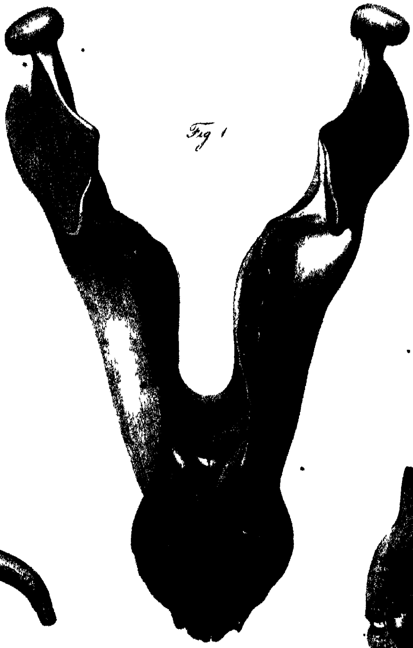

Fig 2

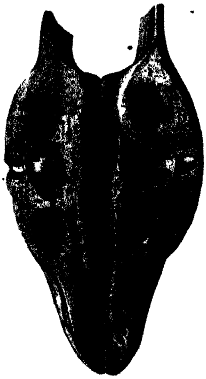

Fig 3

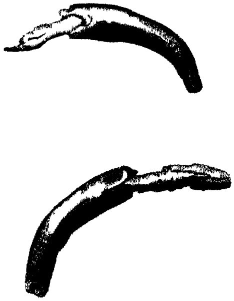

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Fig. 2. The remains of two incisors in situ; both these figures are in the same proportion.

Fig. 3. The two incisors in a shedding state, removed from the jaw, of the natural size.

Fig. 4. A transverse section of one of the dentes molares, represented also of the natural size.

X. *Upon the different qualities of the alburnum of spring and winter-felled oak trees.* *By* THOMAS ANDREW KNIGHT, *Esq. F.R.S.*

Read April 20, 1820.

THE timber of oak trees, felled in winter, was formerly very generally believed to be much superior in quality to that afforded by similar trees felled in spring; and the same opinion appears to be still rather extensively entertained; though the practice of felling in winter has wholly ceased, on account of the encreased value of the bark. But efforts have been made, and supposed to have been successful, to obtain the advantages of both seasons of felling, by taking off the bark in spring, and suffering the tree to stand till the ensuing winter. A good many facts, which had come within my own observation, and information which I received from other sources, had satisfied me that the durability of the alburnum, at least, of oak trees is considerably increased by this mode of management; and I was, consequently, led to make a few experiments (with the result of which I now take the liberty to trouble the Royal Society) with the hope of discovering the cause of this supposed superiority in the quality of the wood of winter-felled trees.

In the spring of 1817, two oak trees, of nearly the same age, and growing contiguously in the same soil, were selected, each being somewhat less than a century old. The one was deprived of its bark, to as great an extent as the inexperience of my workmen permitted me to have done without danger to them, and it was then suffered to remain standing. The other tree was felled, and, in the usual manner, immediately stript of its bark; and the trunk was then removed to a situation in which it was securely protected from the sun and rain. The following winter, in December, the other tree (which still retained life) was felled, and its trunk immediately placed in the same situation with that of the other tree; pieces of each, selected from similar parts, have been subjected to the following experiments at different subsequent periods.

Small blocks, of similar form and size, were taken from the alburnum of each tree; and after these had ceased to lose weight, in a very warm and dry situation, the specific gravity of each was ascertained; when that of the alburnum of the spring-felled tree was found to be 0,666, and that of the same substance of the winter-felled tree to be 0,565, taking the average of several pieces of each. I had anticipated a loss of weight to about this amount in the alburnum of the winter-felled tree, having inferred, from former experiments, that it must have given out a large quantity of matter in the spring and early part of the summer, to form the leaves and young shoots, which quantity could not have been restored to it during the summer, on account of the descending current of sap through the bark having been wholly intercepted.

Small blocks of equal weight of the alburnum of each tree were divided by cleaving into thin pieces; and these, after having become perfectly dry, were suspended together during ten days, in a somewhat damp room; when 1000 grains of the alburnum of the spring-felled tree were found to have gained 162 grains, and an equal weight of that of the winter-felled tree 145 grains; and I found that each substance permanently retained moisture nearly in the same proportion that it absorbed it. The alburnum of the oak, as of other trees, therefore, undergoes some change of properties in the spring; and I do not entertain any doubt but that, in all cases in which it is expedient to give durability to that substance, much advantage may be obtained by taking off the bark in spring, and suffering the trees to stand till winter. The durability of the alburnum of large oak-trees of British growth is not, however, generally an object of much consequence; because it almost always lies wholly exterior to the heart wood; but in the oak timber, which is imported from the North of Europe, the alburnum and heart wood are very often intermixed, the growth of ten or a dozen years, or more, of alburnum and heart wood composing, in alternate layers of unequal depth, the whole body of the tree; and the value of the timber of such trees, is probably much affected by the season of felling.

Many experiments, similar to the preceding, were made upon the heart wood, in which I found the disposition to absorb moisture, somewhat greater in that of the spring-felled, than in that of the winter-felled tree; and I scarcely entertain any doubt but that the winter-felled heart wood is the best, and most durable; but I do not think any conclusion can safely be drawn till the heart wood of many trees has been subjected to experiment; and therefore, as I have no evidence to offer which is in any degree conclusive, I shall not at present trespass farther upon the attention of the Society.

Downton,
March 29, 1820.

XI. *On the mode of formation of the canal for containing the spinal marrow, and on the form of the fins (if they deserve that name) of the Proteosaurus. By Sir* EVERARD HOME, *Bart. V.P.R.S.*

Read May 4th, 1820.

THE last communication respecting the bones of the Proteo saurus which I laid before the Royal Society, contained so many important facts connected with the skeleton, that there was no room left to hope, I should ever again call the attention of its members to this subject.

Yet such has been the exertion made by some persons employed by Colonel BIRCH, to explore the cliffs at Lyme, in search of fossil organic remains, from an expectation that they will receive the full reward of their labours, that many new specimens have been lately collected; in one of these which Colonel BIRCH has submitted to my observation, several bones are met with that have not been described, and which give a very important character to the vertebræ, and also form a more complete set of the bones of the fin. Of these I shall give a description, where their appearance is not sufficiently illustrated by the annexed Plate, (Pl. XV.)

Although the bodies of the vertebræ of this animal have been more frequently met with than any other bones, the manner in which the canal for the spinal marrow is formed, has not till now been made out. In this specimen we find in what manner that is done, and in this as well as in many other particulars, the skeleton differs from that of other animals.

In all the lizard tribe, the vertebræ, like those of other quadrupeds, consist of a body, two transverse, and one spinous process, surrounding a central hole, which forms the spinal canal; the vertebræ of the different tribes of Proteus have the same structure, as is shown in a Plate annexed to my former Paper upon that subject. In cartilaginous fishes, the mechanism is very different; the bodies of the vertebræ, properly speaking, form the spinal column; and the canal for the spinal marrow is surrounded by separate cartilages, united to the vertebræ, by projections fitted to corresponding notches in the side of each vertebra. Through the whole length of the tail, a similar canal is formed upon the under edge of the vertebræ, in which are contained and defended from injury, the great artery and vein. In the fossil remains of the shark tribe, the bodies of these vertebræ are very frequently met with, and have hitherto been readily distinguished from all others by these peculiarities.

The structure of the vertebræ of the Proteosaurus, is something intermediate between the two kinds which I have described. It is composed of bone, as in the lizard tribe, but consists of a body only, behind which is a canal for the spinal marrow, and the spinous process with two lateral branches which belong to it, quite distinct from the body of the vertebra, and between these there is no bony union, but a species of joint peculiar to themselves; the canal thus formed, defends the spinal marrow from injury, equally as when the whole vertebra is in one piece; it is unusually small. The bodies of these vertebræ, when met with without the spinous processes, have so close a resemblance to the vertebræ of the shark in a fossil state, that they have till now been mistaken for them; and I found several of them preserved in the HUNTERIAN Collection, marked as belonging to that fish.

The particular structure which I have endeavoured to describe, will be more readily understood by a reference to the annexed Plate, (Pl. XV.) in which it is very accurately represented.

In the same figure in which the manner of setting on the spinous process to the vertebra is shown, there is (what I have never before met with), one of the feet paddles or fins, (for it is difficult to say which of these names is the most proper), in which the bones are nearly complete; for although some of the smallest are wanting, there are quite enough to give a tolerably exact figure of the outline; of this no other specimen has afforded the most distant idea. This outline certainly has more resemblance to that of a fin, than to any of the fore feet of the tribe of lacerta, or the seal, or even the turtle; and, what is the most remarkable circumstance, there is nothing in any way correspondent to the thumb, or claw for laying hold, which distinguishes the animals that occasionally inhabit the sea, and come ashore for the purpose of laying their eggs, or depositing their young. It certainly has the appearance of a fin; but if in future it is to be designated under that name, it is always to be understood that this fin, contrary to every thing hitherto known in nature, is made up of bony materials; and it is from the great number of bones of which it is composed, and the correspondent number of joints by which these bones are united to each other, that it is enabled to perform the office of any thing resembling the fin of a fish.

In the whale tribe, they are not properly fins, (although they cannot deserve any other name) as they contain the same series of bones as in the seal; and yet no reason can be given why that is the case. In the Proteosaurus, which partakes more of the fish than of the whale tribe, there is probably a greater degree of nicety of adjustment of motion wanted than in the whale, and consequently a greater subdivision of parts, to compensate for the unyielding materials of which these parts are composed; and in this instance there are more than two hundred bones, which must greatly exceed the number of cartilages met with in the fins of the shark.

The facts shown in this specimen, it will be admitted, are of considerable importance, in making us more intimately acquainted with the skeleton of this most extraordinary animal. The second figure contained in the same Plate, (Pl. XV.) is of a less interesting nature, because the parts have been repeatedly shown upon former occasions; and although there is sufficient resemblance of parts to give the idea of this figure belonging to the same skull as the other, on which the facts that have been detailed have their dependance, if that is admitted, much intervening substance must have been lost. In this figure, the teeth are better seen than in any other that I have examined; they are distinctly grooved on their surface up to their points, and are firmly fixed in the jaw, so that they are all preserved in their proper place.

The section of the nose, represented in Pl. XVI. shows that the nostrils are continued on to the point of the nose, and the section of the teeth determine their mode of growth to be the same as in the crocodile.

At the close of this paper, I have much satisfaction in stating, that the bones of the pelvis, the only ones not yet described, have at last been brought under my observation, in a specimen found at Watchet, in Somersetshire, and lately purchased for the Museum of the Royal College of Surgeons in London. The specimen is four feet six inches in length, and in consequence of the bones having been little disturbed, is in all respects more complete than any other specimen that has been met with. The bones of the pelvis are, however, in this specimen too much pressed upon one another to admit of a satisfactory drawing being made of them; but they resemble so closely those of the crocodile, as to make any representation unnecessary.

All the fossil bones of the proteosaurus which I have described, belong to the same species; but I have seen bones of another species, which differs from this in the following particulars—the upper jaw is much longer, narrower, and comes more to a point; the teeth were so loosely fixed in the jaws, that hardly any are met with in their sockets; that part which had been enclosed by the gum was grooved; the other portion is very slender, has a polished surface, and comes to a sharp point.

The long bone belonging to the anterior fin in both species is as large again as that belonging to the posterior, but both these bones are flatter and longer in the new species; and where articulated with the two small bones of the fin, send a short process between them; there is also a deep notch on the outer or lower side of one of these two small bones.

EXPLANATION OF THE PLATES.

PLATE XV.

This Plate consists of two figures, representing portions of the skull, jaw, and other parts, of the natural size.

Fig. 1. A portion of the skull behind the orbit. The two principal objects in this figure are, the mode of articulation of the spinous processes with the bodies of the vertebræ of the back, forming the canal in which the spinal marrow is contained; and the fin of this animal in the most perfect state yet discovered, one hundred and seventy separate bones remaining in their natural situation.

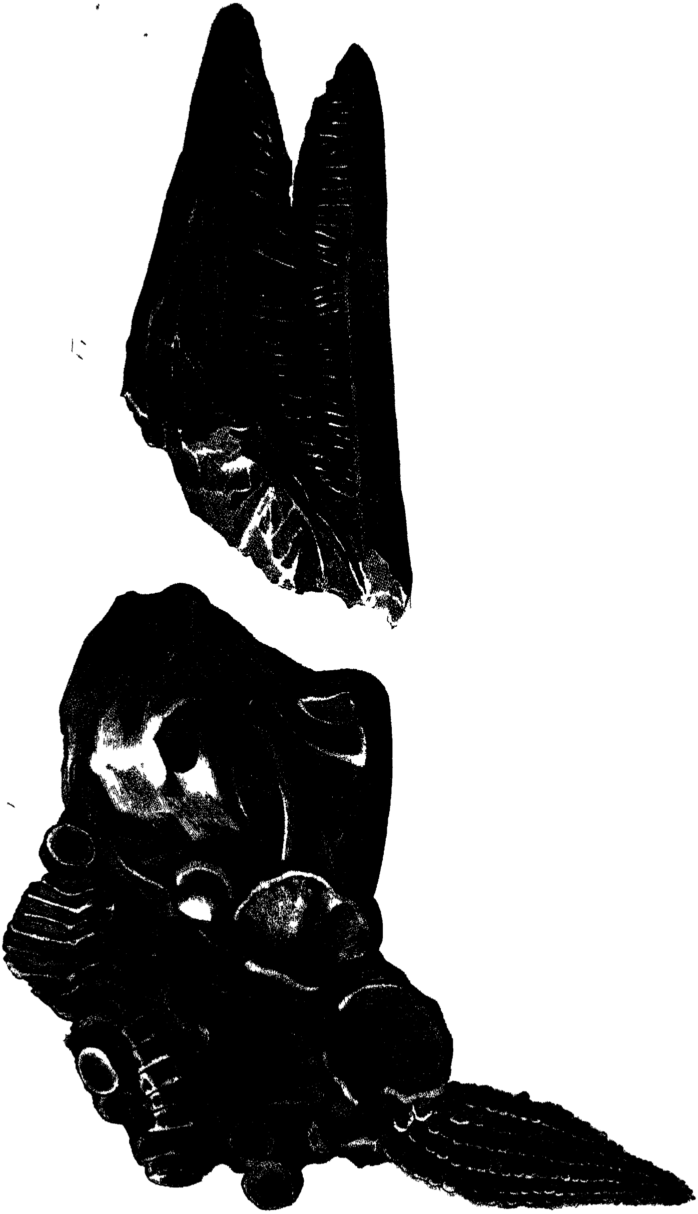
Fig. 2. Shows the anterior portions of both jaws, which appear from their size to have belonged to the same skull as Fig. 1, having been found in the same situation; allowance must however be made, for a considerable intervening portion having been lost.

PLATE XVI.

Represents a transverse section of the nasal bones, of the natural size.

This is the only opportunity that has offered of seeing the cavity of the nostrils.

The teeth and their sockets in both jaws were accidentally divided in making the section of the nose, and their internal structure exposed, from which it appears, that the mode of growth is similar to that of the crocodile; the succeeding tooth filling up the cavity of that which preceded it.



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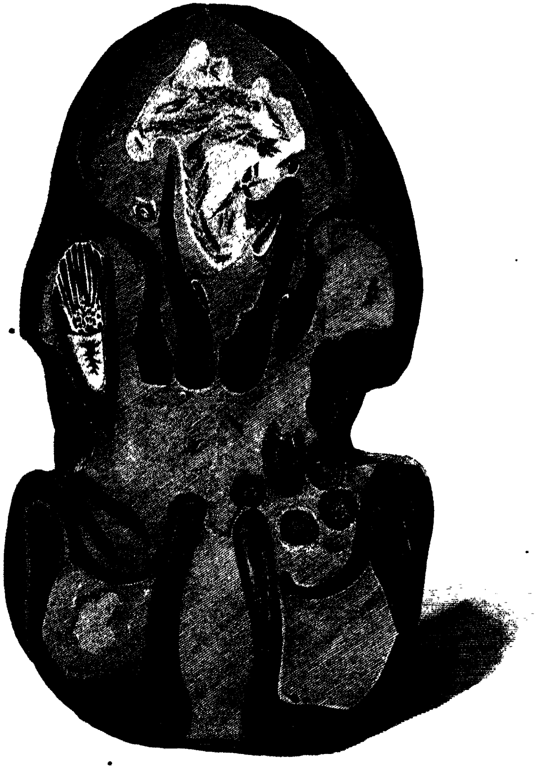

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XII. *Some experiments on the fungi which constitute the colouring matter of the red snow discovered in Baffin's Bay. By* FRANCIS BAUER, *Esq. F. L. S. In a Letter addressed to the Right Honourable Sir* JOSEPH BANKS, *Bart. G.C.B. P.R.S. &c. &c.*

Read May 11th, 1820.

SIR,

THE continuance of the severe cold weather, and the frequent falls of snow, during last winter, afforded me ample opportunity for investigating the nature and economy of the fungi, which constitute the colouring matter of the red snow, discovered in Baffin's Bay, on the 17th of August, 1818, during the Northern Expedition, under the command of Captain Ross; and I have now the honor of laying before you, the result of my investigation; and if you should consider it to merit the notice of the Royal Society, I should feel highly honoured by its being communicated to that learned Body, through your favor.

Since the month of March, 1819, I preserved in a small glass, a portion of the original sediment of the red snow, brought from Baffin's Bay, during which time the fungi, which constitute that sediment, had considerably increased in number; but the newly formed fungi remained ever perfectly colourless, and formed a kind of whitish crust, at the top of the red sediment.

With a view to ascertain whether those fungi really vegetate, and propagate in the snow, after draining off the water, I carefully separated the colourless crust, and taking out of the glass the greatest part of the red sediment (which I intended for another experiment, but which failed, and shall be described hereafter) I filled the glass again with water, and left it at rest for several hours; when the few fungi which were left in the glass, formed a slight sediment of about $\frac{1}{40}$ part of an inch in thickness, at the surface of which the colourless crust, about $\frac{3}{40}$ parts of an inch in thickness, remained. Fig. 1. in the annexed Plate (Pl. XVII.) is a correct representation of the glass, and the quantity and proportion of its contents.

On the 10th of December, 1819, when we had the first fall of snow, I marked with a diamond on the glass the precise height of the mass of sediment it contained, and after carefully draining off the water, I filled the glass with snow, which I pressed in as hard as I could, by which means the fungi became completely mixed and immersed in the snow. Thus prepared, I put the glass into a wire bird cage, to prevent its being disturbed by birds, or any thing else, and placed the cage in a N. W. aspect, in the open air. The weather was then very cold, and on the following morning, the 11th of December, the thermometer was 26 degrees below the freezing point. On the 13th of December the weather suddenly changed and became milder, and about noon the snow in the glass was entirely dissolved; on examination, after the glass had been fifty-two hours in the open air, I found the whole contents had formed a sediment, and had become of the same red colour as the original sediment was; but no increase in quantity was then perceptible; see Fig. 2.

I immediately poured off the water, and filled the glass again with snow, in the same manner as before, and exposed it in the open air, where it remained until the morning of the 17th of December, when a general thaw had begun; on examination I found the snow in the glass was not quite dissolved, but was a lump of ice, perforated, and full of cells, like a honeycomb, and the mass of fungi was raised in little pyramids; see Fig. 3.

From the 17th of December till the 28th, the weather continued very mild, and we had no snow till the morning of the 28th; on that day I filled the glass again with fresh snow; but perceiving a slight increase in a very small portion of fungi that were accidentally smeared on the corner of the mouth of the glass, when on the 19th of December I took out some of the sediment for examination, and sunk the small glass, with its contents, into a larger cylindrical glass, also filled and pressed with snow, to afford to that portion of fungi room for increasing and spreading in the snow; and in that state I exposed the whole in the open air.

From the 28th of December, 1819, to the 10th of January, 1820, the cold and frost continued very severe, but on the latter day, after the glass had been buried fourteen days under the snow, I took it out, to see what change might have taken place during that time; on clearing away the external snow and ice, I found the appearance of the contents of the small glass, exactly as represented in Fig. 4.

The snow was frozen to a hard mass of ice, which was beautifully perforated and full of small cells, and the whole mass of fungi was raised in a pyramidal form, almost to the mouth of the glass, filling and occupying the perforations and cells in the ice; and from the base of the sediment, innumerable small air bubbles were visibly and rapidly evolving, by whose means, I think, the fungi are raised in the manner they are found during the time that the snow is dissolving. The small portion of fungi in the corner of the glass had also considerably increased, and spread in the snow above the small glass: see Fig. 4.

After making a drawing of the above described appearances, I deposited the small glass again into the larger one, which was filled with snow, as before, and exposed in the open air, where it remained till the 19th of January, 1820, when the weather changed, and a sudden and general thaw set in; after all the snow and ice were dissolved, and the glass stood several hours at rest, I had the satisfaction to find the appearance of the contents of the small glass, exactly as represented by Fig. 5.

If the sediment in the glass, represented in Fig. 5, is compared with that of Fig. 2, it is evident that, (though a considerable portion of the sediment had at several times been taken out of the glass for examination, and by the frequent changing of the water, some loss was sustained) the increase of fungi, within less than two months, has been more than double.

The only difference I observed, on examining the individual fungi of the newly formed sediment, was, that the number of larger than ordinary sized full grown fungi was greater in the newly formed sediment, than I ever found in the original sediment, brought from Baffin's Bay, where the ordinary size of the full grown red fungi is about a $\frac{1}{1600}$ part of an inch in diameter, and the largest are about $\frac{1}{1200}$ part of an inch in diameter; the number of large fungi, in proportion to those of the ordinary size, is as 1 to 100; but in the newly formed sediment, after the above described experiments, I found many of the larger fungi to be as large as $\frac{1}{800}$ part of an inch in diameter, and the number of large fungi, in proportion to those of the ordinary size, as 1 to 10.

I also found that, notwithstanding the snow in the glass had so frequently been changed, and though the fungi had sometimes been not more than three or four days in the snow, the water, after the snow was dissolved, always acquired the same disagreeable smell and taste as the original water imported from Baffin's Bay.

The above described experiments I attempted to repeat on a larger scale, and for that purpose I employed a glass of about twelve cubic inches in contents. The original red sediment which it contained was about $\frac{1}{8}$ of an inch in thickness; and after marking on the glass with a diamond the precise height of the whole mass, I drained off the water, and divided the sediment in three equal parts; on the 12th of January, 1820, I put the different portions of sediment into three equally sized glasses, which I afterwards filled and pressed with snow, and exposed them in the open air, as in the former experiments.

The first few days, the weather continued very cold and favourable for the experiment, and the appearance in the glasses was the same as in the former experiment. The fungi in some glasses had risen and spread in rays and pyramids of near three inches in length, and the increase appeared considerable and rapid; but during the day of the

19th of January, the weather suddenly changed, and the snow in the glasses entirely dissolved; but the following night it froze again very hard, and two out of the three glasses burst and fell to pieces; the following morning, the 20th of January, it suddenly thawed again; and before I discovered the accident that had happened, a great part of the snow in the glasses was dissolved, and consequently the greatest part of the fungi they contained was lost; however, I collected all the sediment left in the unmelted snow and ice of the broken glasses, and put it into that which remained entire; after the whole was melted, the sediment which then was produced, certainly appeared considerably increased; but the original glass which had been marked with a diamond being destroyed, I could not form a correct estimate; judging however from the previous appearances, I have every reason to believe, that if the accident had not happened, the result of this experiment would have been equally satisfactory as that of the experiment on the small scale.

On the 22d of January I again filled the glass, containing the remains of the sediments of all the three glasses, with fresh snow, and exposed it in the open air as before; but the next day the general thaw began, and after that day we had no more frost nor snow; I left the glass in the open air till the end of the month, when I examined its contents again, and found no particular change in the substance; but within the glass, which was not quite full, round the surface of the water I observed a kind of crust, consisting of a light yellowish green substance, which on examination in the field of the microscope, proved to consist entirely of the same sized and shaped globules as the original red fungi. I immediately decanted the water and red sediment into another glass; when, on examining the empty glass, I found the appearance of the green substance was not only confined to the top of the glass, above the water mark, but I found many very minute clusters scattered, not only over the sides, but also over the bottom of the glass. I filled that glass with melted snow water, and closed it with a bladder, and in that manner I have kept it ever since.

I examined the glass this morning, and find that a considerable increase of the green fungi has taken place since the 31st of January, when I filled it with water; the green substance forms now an entire crust over the whole inner surface of the glass, particularly the bottom, where the new globules now form a sediment that is visible to the naked eye, and the whole glass is now of a greenish tinge. To what cause this change of colour is to be attributed, I cannot guess.

The experiment mentioned in the beginning of my letter as having failed, was owing to an idea I conceived, that the red fungi would vegetate on the surface of the snow; and with that view I filled on the 10th of December, 1819, a large glass cup with snow, pressing it hard, and giving it a perfectly smooth surface, on which I made an impression with a sixpenny piece; the cavity thus produced I filled carefully with the original sediment of red fungi, expecting that on the slightest increase taking place, the fungi must come over the limits thus set them; but the degree of cold at that time was so great, that, soaked in water as they then were, they froze almost before they came in contact with the snow. On examining them the next morning, when the thermometer was 26 degrees below freezing, I found them of nearly a black colour, and almost entirely raised up, and detached from the snow; and when on the 19th of January the snow dissolved, the mass of fungi sunk to the bottom of the glass, and gradually changed their colour to a dirty brown, being evidently dead.

The same experiment was repeated several times, and the result was always the same. I subsequently collected the different masses of fungi that had been made use of in these experiments, and immersed them in snow, in one glass, and exposed them to the open air, as in former experiments; but though there was no visible change in the appearance whilst they were in the glass, I found, the latter end of January, when I examined them in the field of the microscope, that many new fungi had been produced during the time the dead sediment had been under the snow; and that the newly produced fungi were not only small and colourless, but had also many full grown bright red fungi amongst them. From this circumstance I conclude, that though the excessive cold, and the exposure in the sharp air and wind, may kill the primitive fungi, their seeds still retain sufficient vitality to vegetate and propagate, if immersed in the snow, which appears to me to be undoubtedly the natural soil of those fungi.

The season having passed by, I was unable to make any farther experiments on this subject.

I have the honour to remain, &c. &c.

FRANCIS BAUER.

Kew Green,
May 7, 1820.

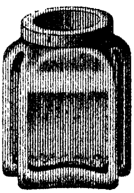


Fig. 1.

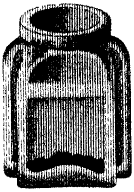


Fig. 2.

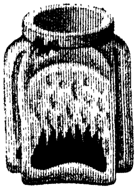


Fig. 3.

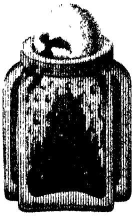


Fig. 4.

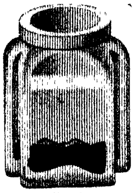


Fig. 5.

Fran. Bauer del.

Ja. Basire sc.

For the annexed meteorological diary, I am indebted to Mr. Aiton. It is extracted from the diary kept in the Royal Botanic Garden, at Kew, and within 300 yards of the spot on which I made the experiments.

December, 1819.					January, 1820.				
	Degrees of Fahrenheit's Thermometer.			Wind and Weather.		Degrees of Fahrenheit's Thermometer.			Wind and Weather.
	M 8	A 2	E 8			M 8	A 2	E 8	
December 1	48	46	40	SW Rain.	January 1	10	26½	35	SW Hazy.
2	47	49	38	S Do.	2	19	24	35	SE Rain.
3	26	42	37	SW Fine.	3	32	32	27	NW Snow.
4	41	48	36	SW Rain.	4	20	28½	19	W Cloudy.
5	39	39	36	SW Cloudy.	5	17	28	20	S Hazy.
6	38	36	36	NE Do.	6	22	37	33	S Cloudy.
7	35	34	33	E Sleet.	7	33½	32	26	E Do.
8	31	30	32	E Cloudy.	8	25	30	26	NE Do.
9	24	25	24	E Do.	9	22	28	29	NE Do.
10	30	31	29	E Snow.	10	19	27	26	N Snow.
11	6	24	20	W Fine.	11	27	28½	33	SW Sleet.
12	16	23	25	SW Do.	12	26½	25	12	NE Cloudy.
13	24	36	23	SW Do.	13	10	28	23	NE Do.
14	19	35	31	SW Do.	14	24	27	24	NE Do.
15	32½	38	34	SW Rain.	15	2	21	25	SW Clear.
16	32	39	30	SW Fine.	16	19	22	17	E Do.
17	38	44	46	W Rain.	17	32	35	31	NE Hazy.
18	52	54	50	W Cloudy.	18	27	31	33	NE Snow.
19	51	55	54	SW Rain.	19	47	47	33	SW Rain.
20	53	54	52	SW Cloudy.	20	29	33	32	NE Cloudy.
21	54	54	53	W Do.	21	37	35	33	S Sleet.
22	52	52	51	SW Do.	22	18	30	19	S Clear.
23	49	41	39	W Do.	23	28	33	35	SW Cloudy.
24	29½	36	30	NW Do.	24	38	42	43	SW Do.
25	31	33	28	E Do.	25	42	45	39	S Do.
26	21	34	28	E Do.	26	45	48	45	S Rain.
27	39	32	32	E Snow.	27	48	51	46	S Do.
28	33½	32	31	E Sleet.	28	42	48	38	NE Cloudy.
29	24	28	23	NE Fine.	29	38	44	38	NE Do.
30	14	31½	31	NE Do.	30	40	48	45	SW Do.
31	17	32½	17	NE Do.	31	40	48	42	S Fine.

XIII. *Some account of the Dugong. By Sir* THOMAS STAMFORD RAFFLES, *Governor of Sumatra; communicated in a Letter to Sir* EVERARD HOME, *Bart. V. P. R. S.*

Read May 18, 1820.

MY DEAR SIR,

I HAVE now the pleasure of communicating to you the desired information concerning the dugong. At Singapore, in June last, I had the good fortune to meet with one of these animals, and Messrs. DIARD and DUVAUCEL, two French naturalists, employed under my authority, undertook the dissection of it; and have sent a dissertation upon it to Sir JOSEPH BANKS. This does not interfere with my sending to you, as I promised, an account of it. I was present at the dissection; and the following observations, as far as they go, may be depended upon. I have read them over to Dr. WALLICK and General HARDWICKE, and they concur in opinion as to the correctness of the description. I have the pleasure to acquaint you, that General HARDWICKE has just now got a small dugong, four feet six inches long; which I have succeeded in persuading him to send home to you for dissection, and you will receive it by the next ships.

The dugong which we examined measured eight feet and a half in length, and afforded no less interest under the knife than satisfaction on the table, as the flesh proved to be most excellent beef. Our entertainment was truly marine;

for we had on the same day discovered those Neptunian sponges which General Hardwicke has since described, and which served us as goblets.

In form the dugong resembles the common cetacea, having, like them, a broad horizontal tail, and two pectoral fins without nails. The skin is smooth, thick, bluish above and whitish beneath, with a few remote and scattered hairs. The mammæ (in the two male individuals examined) are small, and situated on the breast, immediately below the pectoral fins.

Head small in proportion, obtuse, and of a peculiar form. Upper lip very large, thick, and obliquely truncated, forming a short, thick, and nearly vertical kind of snout. The surface of the truncated portion is covered with soft papillæ, and is also furnished with a few bristles. Two short tusks project straight forward from the extremity of the upper jaw, and are nearly covered by the upper lip, which is very moveable, and tumid at the margin. The lower lip is much smaller, and resembles a round or oblong chin. The margin of both lips is furnished with strong coarse bristles. There are no incisors in either jaw (the tusks above mentioned being more properly defences), their place being supplied by the rough bristly surfaces of the palate and jaws, which serve as rasps, to enable the animal to browse upon the algæ and other submarine vegetables. To facilitate this still farther, the anterior part of the jaw is bent downwards at an angle, in such a manner as to bring the mouth into nearly a vertical direction. There are no canine teeth. The molares are twelve in number, six in each jaw, placed far back on the horizontal part. They are cylindrical, with flat crowns; the first are somewhat oblique, and worn to a kind of point; the second are perfectly flat; but the last are composed of two parallel and adhering cylinders. They are short, and scarcely project from the gums. The tongue is small and short. The nostrils are situated on the summit of the upper jaw, where it makes its curvature downwards. They penetrate obliquely, in such a manner that the upper semilunar edge pressing upon the lower surface, forms a perfect valve. The eyes are small, and situated on the sides of the cranium. The aperture of the ears is so small as with difficulty to be perceived, and is situated at some distance behind the eyes.

Body rounded, diminishing to the tail, and without any vestige of dorsal or ventral fins. The place of the anterior extremities is supplied by fins, which offer no appearance of nails, but are somewhat verrucose on their anterior margin. They are thick and fleshy, and neither from their form nor size capable of supporting or assisting the animal out of the water.

Tail broad, horizontal, and of a crescent or semilunar form.

Dissection.

Skin three quarters of an inch thick, with little adipose matter, and yielding no oil.

The cavity of the abdomen large.

The stomach is large; and the relative position of the cardiac and pyloric orifices is nearly as in the human subject. It has two appendages, which open into it near the junction of the duodenum. Membrane of the stomach thick, internal surface smooth, and not corrugated into plicæ. The stomach and its appendages were distended with fucus or sea-weed, but little masticated or altered. Intestinal canal long. Small intestines uniform. Cæcum very large, somewhat curved, and containing a portion of partially digested sea-weed. Colon exceeding the small intestines in diameter by one third, very uniform, and with few or no contractions. Liver of moderate size, consisting of two large and distinct lobes, connected by a smaller one somewhat tongue-shaped, and a fourth which was very small, on the posterior side. Gall bladder little distended, and situated beneath the third and tongue-shaped lobe. Spleen very small, not exceeding three inches long and one inch thick, attached to the left side of the stomach. Pancreas lying below the duodenum. Kidneys in their usual place, and large. Bladder much contracted, not exceeding the size of an egg, but from the thickness of its coats, is probably capable of much greater distension.

Testicles situated a little below the kidneys, egg-shaped, flattened, partly embraced by a very perceptible epididymis.

Penis large; while collapsed entirely concealed within the prepuce. The glans consists of two lobes, separated or cloven above, in such a manner as to give the whole the appearance of the cloven foot of a ruminating animal. The urethra opens on a small tubercle or papilla between the lobes of the glans.

In the thorax, the thymus gland is particularly large, black and friable under the fingers, and occupying the space between the folds of the mediastinum.

Lungs two, distinct, of an elongated form, not lobulated, and situated posteriorly, one on each side; their substance of the usual mottled colour. The trachea bifurcates very high up, and the two branches diverge to their respective lungs.

Heart situated on the left side, double; that is to say, having the ventricles entirely separate at their points, and only connected at the upper part, or base. Each side possesses a ventricle and auricle, with the usual valves, and without any communication between the right and left sides. The left ventricle, which gives off the aorta, is stronger and more muscular than the right, whose cavity is larger, and coats thinner.

Of the skeleton, a few observations will suffice.—The skull is remarkable by the peculiar manner in which the anterior part of the upper jaw is bent downwards, almost at a right angle, so as to form a kind of beak. The lower jaw is truncated in such a manner as to correspond, and become parallel with the elongated portion of the upper jaw. This portion of the lower jaw has eight alveolar excavations, which are sometimes empty, and sometimes contain the rudiments of teeth.

The vertebræ are fifty two in number, seven to the neck, eighteen to the back, and twenty seven to the tail.

Ribs, eighteen on each side.

Sternum nearly a foot long, bifurcate at the apex, and articulated to the cartilages of the upper ribs.

There is no pelvis or posterior extremities, but there are found opposite to the eighth or tenth lumbar vertebra two bones, one on each side, lodged in the flesh, which are narrow and flattened, and not above five or six inches in length. Scapulæ broad and thick; humerus short and strong, as is also the radius and ulna. The whole of these are firmly articulated to each other; and though externally the fins offer no appearance of fingers, all the corresponding bones are found complete even to the last phalanges.

The food of the dugong appears to consist exclusively of fuci and submarine algæ, which it finds at the bottom of shallow inlets of the sea. The position and structure of the mouth, enables the animal to browse upon these vegetables, much in the same manner as a cow in a meadow; and the whole structure of the masticating and digestive organs show it to be truly herbivorous. The flesh resembles young beef, and is very delicate and juicy. The individual, of which the skeleton and intestines are now sent to England, was taken at Singapore, in June, 1819.

According to the information given by the natives, the dugong is never found on land, or in fresh water, but generally in the shallows and inlets of the sea, where the water is only two or three fathoms deep. During our short possession of Singapore, (not more than six months) four of these animals have been taken; but the greatest number is said to be caught during the opposite or northerly monsoon, when the sea is calmest, near the mouth of the Johore river, in the inlet of the sea between Singapore Island and the main. They are usually taken by spearing (at which the natives are particularly dexterous) during the night, when the animals give warning of their approach by the snuffling noise they make at the surface of the water. The first object is to secure and elevate the tail, when the animal becomes perfectly powerless, and at their disposal. They are seldom caught above eight or nine feet in length, but how much larger they grow is not ascertained, as when they exceed this size, their superior strength enables them to make their escape when attacked.

The Ikan dugong is considered by the Malays as a royal fish, and the king is entitled to all that are taken. The flesh is highly prized, and considered by them far superior to that of the buffalo or cow. They distinguish two varieties, the duyong *bumban*, and the duyong *buntal;* the latter much thicker and shorter in proportion. The breasts of the adult females are said to be large. The affection of the mother for its young is strongly marked; and the Malays make frequent allusion to this animal, as an example of maternal affection. When they succeed in taking a young one, they feel themselves certain of the mother, who follows it to the margin of the sea, and allows herself to be speared or taken with the greatest ease. The young have a short sharp cry, which they frequently repeat; and it is said they shed tears. These tears are carefully preserved by the common people as a charm, the possession of which is supposed to secure the affections of those to whom they are attached, in the same manner as they attract the mother to her young. This idea is at least as poetic, and certainly more natural, than the fable of the Syren's song.

I remain, my dear Sir,
yours truly,
THOMAS STAMFORD RAFFLES.

Dimensions.

	Ft.	In.
Total length of the animal - -	8	6
Greatest circumference - - -	6	0
Length of the head from the nostrils to the occiput	1	8
——— from the nostrils to the end of the snout - - -	0	3½
Width of the snout - - - -	0	9½
Depth of Do. - - - -	0	4½
Length of the chin - - - -	0	5
Breadth of Do. - - - -	0	5½
Distance from the nostrils to the eyes - -	0	6½
——— the eyes to the ears - -	0	6½
——— the eyes to the fin - - -	1	5½

	Ft.	In.
Length of the fins	1	4
Breadth of Do.	0	8
——— across the belly from fin to fin	1	11
Distance between the mammæ	1	5
Breadth of tail from tip to tip	2	7
Circumference of the root of the tail	1	9
Distance from the anus to the centre of the tail	2	9
——— from the anus to the penis	1	2
Total length of the intestines	115	0
—— Do. of small intestines including the cæcum	44	0
—— Do. of great intestines	72	0

With this account, Sir T. S. RAFFLES sent me a copy of some observations in French, by Messrs. DIARD and DUVAUCEL, upon the stomach of the dugong. Sir T. S. RAFFLES mentioned that these observations formed part of a Memoir written by those Gentlemen; under these circumstances, I have not felt myself authorized to lay them before the Society, along with those made by Sir THOMAS STAMFORD RAFFLES, which I consider of too much importance to be delayed.

EVERARD HOME.

XIV. *Observations on the Human Urethra, showing its internal structure, as it appeared in the Microscope of* F. BAUER, *Esq. By Sir* EVERARD HOME, *Bart. V. P. R. S.*

Read June 1, 1820.

THE smallest additions to our knowledge of Human Anatomy, have ever met with the most liberal reception from the Royal Society. Such communications must necessarily be of rare occurrence, the field having not only been gone over so often, but cultivated with the utmost labour and zeal, by men of the highest talents, and most acute discernment. Much, however, is still wanting to make us acquainted with the internal structure of all the organs of the human body; and some of those not yet thoroughly examined, are of very great importance, not only on account of the functions they perform, but the diseases to which they are liable; since by these diseases the natural functions are disturbed, and occasionally completely interrupted; and when that happens, the life of the patient is too often destroyed.

The structure of the human urethra, as far as it could be examined by ordinary vision, assisted by glasses of small magnifying power, has been long known to all the anatomists of the present day; and twenty-five years ago I thought myself thoroughly acquainted with it; but I now find that I was wholly ignorant of the more minute parts of its conformation.

For the knowledge I have now acquired, which it is the

the object of this paper to communicate to the Society, we are indebted to Mr. BAUER, who has given the most patient and laborious attention to the examination of these parts, magnified in different ways, and has faithfully represented them as they appeared to his view.

From Mr. BAUER's examinations, we find that the human urethra is made up of two parts, an internal membrane, and an external muscular covering.

The internal membrane is exceedingly thin, and no fibres are met with that can give it a power of contraction. When it is put on the stretch in a transverse direction, the circumference of the canal is no ways encreased; but when stretched longitudinally, a small degree of elongation is produced.

When a transverse section of the urethra is made while in a collapsed state, the internal membrane is found thrown into folds pressed together by the surrounding parts. This appearance is shown in the most satisfactory manner in the annexed Plates.

On the surface of the internal membrane, over which the urine passes, there are numberless small projections, or papillæ, the orifices of glands. The surface is covered with small blood-vessels, and the lacunæ leading to the deeper seated glands, are very numerous. These parts are beautifully represented in Plate XXI. Fig. 4, in a small portion of the lining of the urethra, near the external orifice, minutely injected, magnified ten times in diameter.

The muscular covering by which the membrane is surrounded, or enclosed, is made up of fasciculi of very short fibres, which appear to be interwoven together, and to be connected by their origins and insertions with one another; they all have a longitudinal direction. A muscular structure of this description, as far as I know, is not met with in any other part of the body. It is however to be observed, that there is no other instance in which fibres of such minuteness have been examined, and faithfully represented.

There is a greater thickness of this muscular structure upon the upper, than the under surface of the urethra, which is still more evident, as it approaches nearer to the external orifice. The fasciculi are united together by an elastic substance of the consistence of mucus. This muscular structure is represented in a longitudinal section of a portion of the corpus spongiosum urethræ, magnified fifteen times in diameter. The internal structure of these fasciculi is shown in Pl. XXIII. Fig. 2, 3, magnified twenty-five diameters, and in Fig. 4, magnified fifty diameters. In this Plate the fasciculi are thickened in consequence of inflammation. Immediately beyond the muscular portion of the urethra, is the cellular structure of the corpus spongiosum.

This mechanism, which forms the canal of the urethra, is very different from what it has hitherto been contemplated to be, and therefore a knowledge of it enables us to correct many errors that we had fallen into with respect to the actions of its parts, both in a healthy state, and when under the influence of disease.

Till now it was believed, that either the lining of the urethra was composed of circular fibres, possessed of a power of contraction, or that it was immediately surrounded by such fibres; and therefore, that the disease commonly known by the name of a stricture in the urethra, was produced by a contraction of some of these circular fibres; and that permanent stricture was a term applied to these parts, when, in consequence of inflammation, they became confined to that particular state.

We now find from the annexed Plates, that the lining of the urethra is never met with in a contracted state, but is thrown into folds by the action of the elastic ligamentous covering of the corpus spongiosum, and the swell of the longitudinal muscular fibres within it; and when these fibres have, by acting through their whole length, reduced the urethra to its shortest state, the pressure upon the internal membrane is so great, that there is not room for the urine to pass till these fibres are relaxed by elongating the whole canal.

A spasmodic stricture is in reality a contraction of a small portion of the longitudinal muscular fibres, while the rest are relaxed; and as this may take place either all round, or upon any one side, it explains what is met with in practice, and which could not be satisfactorily accounted for; the mark or impression of a stricture sometimes forming a circular depression upon the bougie, at others only on one side.

A permanent stricture is where, in consequence of inflammation, coagulable lymph is exuded, between the fasciculi of muscular fibres, and upon the internal membrane, in different quantities according to circumstances; and in the same proportion diminishing the passage for the urine at that part, or completely closing it up. The changes the parts undergo in the formation of a permanent stricture, are most satisfactorily shown in Pl. XXII. XXIII.

In this investigation, Mr. Bauer's examinations have not been confined to the canal of the urethra; he has, at my request, extended them to the corpus spongiosum, and to the corpora cavernosa; respecting the structure of which, even at this day, different opinions are entertained by some of the most celebrated physiologists in Europe.

When these bodies are entirely empty and collapsed, as we generally find them in the dead body, they contract so much from their elasticity, as to make it next to impossible to examine the structure of their different parts; and no inconsiderable art and contrivance becomes necessary, so to prepare them, that the internal structure, may be sufficiently displayed, to render the examination satisfactory. Every attempt to make the substances commonly used as injections, pass from the arteries into the cellular internal structure, proved ineffectual; that mode therefore of distending those cells was soon abandoned; injecting them directly by puncturing the surrounding coverings, could not give them their natural appearance; we therefore did not avail ourselves of it. The only mode of expanding the internal structure, at the same time that it was not disturbed, was to fill it through the medium of its own arteries, and then, by hardening the parts in spirit, preserve them in a distended state.

This was done in three different ways. In one instance they were found filled naturally, by the blood that had been retained there, and which, in the act of dying, had not been expelled, as in Plate XVIII.

In another, proof spirit was injected by the arteries till all the internal cellular structure was distended by it; and this spirit was retained there, as in Plate XIX.

In a third, the arteries were injected with quicksilver, which readily passed into the internal parts, and was retained there by plugging up the trunks with common injection. The parts were then dried, and put into oil of turpentine. When preserved some time in that state, a longitudinal section of one of the corpora cavernosa was made, as in Plate XX, and the quicksilver allowed to run out. The preparation thus made is in the Collection of the Royal College of Surgeons in London; one of the many valuable donations made to that Museum by Sir William Blizard. The others were made upon this occasion.

From these preparations, the fact is sufficiently established, that the cellular internal structure of the corpora cavernosa and corpus spongiosum, receives the blood into its cavities directly from the smaller branches of the arteries with which these parts are provided; but the nature of this structure remains to be explained.

It was found by Mr. Bauer, from an examination of these different preparations, that the cellular structure of the corpora cavernosa is made up of an infinite number of thin membranous plates, exceedingly elastic, so connected together as to form a kind of trellis work, the edge of which is firmly connected with the strong elastic ligamentous substance by which these bodies are surrounded, and which forms a septum between them, separating them from one another. This substance has an intermixture of muscular fibres. The septum has not the same appearance in different individuals; in some, it is much thicker than in others; and towards the anterior end, in one preparation, was almost entirely wanting; as in Plate XVIII, Fig. 2.

In the central line of each corpus cavernosum there is an open space. This is of considerable length, but is by no means equally so in different instances. This has no regular boundary, and appears to be formed simply by the elastic plates being wanting. These plates, from their extreme elasticity, are with difficulty examined; but as it appeared that their structure is the subject respecting which physiologists are not agreed, Mr. Bauer spared no pains, and exerted his ingenuity in putting them to the test of different modes of examination on the field of the microscope, and is perfectly satisfied that they are what he has represented them to be; namely, simple thin membranous elastic plates, not made up of more than one layer, not having a cavity between them, nor in any way forming regular cavities, but having irregular interstices: they vary in size considerably in different parts of the same body. It is into these interstices that the blood is occasionally received from the termination of the small arteries which are every where branching upon, and supported by, the elastic plates of which the trellis work is composed; as in Pl. XVIII, Fig. 1. Pl. XIX, Fig. 1. Pl. XX, Fig. 1.

The circulation of the blood in these parts being different at different times, is a most beautiful illustration of the fact, which upon a former occasion I communicated to the Society; namely, the great influence the nerves have upon the action of the smaller arteries, as well as on their larger trunks. When the nerves that supply the corpora cavernosa and corpus spongiosum, (and they are very numerous,) are not affected by the state of mind of the individual, the blood is not carried into this trellis work, but returns by the veins, as in other parts of the body; but when the mind receives such impressions as excite these nerves, the smaller arteries have their action so much increased, as not only to pour the blood into the interstices, and the open space which they inclose, but to distend them, overcoming the elastic power that, under ordinary circumstances, keeps them collapsed.

The corpus spongiosum differs in nothing from the corpora cavernosa, but in the parts being formed upon a smaller scale; there being no open space in the middle or central line of the trellis work; and the ligamentous elastic covering which surrounds it, having no muscular fibres intermixed with its substance. When a transverse section of the corpus spongiosum is examined in the microscope, the orifices of six or seven divided arterial trunks are distinctly seen in different parts of the section, as in Pl. XXI, Fig. 1.

As the corpus spongiosum may be said to be continued into, and to form the glans, the internal structure of which is of the same kind, I was desirous that Mr. Bauer should examine the covering of that body in the microscope, to ascertain whether there were any papillæ upon its surface, so different from those of the skin generally, as to account for this part having a sensation peculiar to itself, and as unlike the common feeling of touch, as the sense of taste, which is peculiar to the tongue; this last, I have explained upon a former occasion to the Society, is confined to the tip; since, when the sense of taste is destroyed by the nerves that produce it being injured, the common sensations of the rest of that organ, as well as its voluntary motions, are as perfect as before.

From the annexed magnified representations of a portion of the glans, showing the papillæ on its surface, it is evident that such a peculiar organization must be constructed to produce a more than ordinary delicate sensation. Pl. XXI, Fig. 5, Fig. 6.

To describe the appearance which is so beautifully represented in the drawing, would be trifling with the time of the Society; and I am afraid that I have already allowed myself to say much more upon several of the other subjects contained in this paper than was necessary, illustrated as they are by Mr. BAUER's drawings.

EXPLANATION OF THE PLATES.

PLATE XVIII.

Consists of two figures or transverse sections of the same penis, magnified four times in diameter.

Fig. 1. Shows the internal structure of the corpora cavernosa near the middle of the penis, where the central part is one general cavity, surrounded every where by the open trellis work; the interstices are filled with coagula of the blood they contained at the time of the person's death; the divided trunks of the arteries which supplied the blood are very distinct.

Upon the dorsum penis are seen the divided arteries which lie there, imbedded in the elastic ligamentous and muscular substance, by which the corpora cavernosa are covered, and which is continued between them, forming the septum; it also completely surrounds them.

The cellular structure of the corpus spongiosum forms a similar trellis work upon a smaller scale; the urethra passes through it rather above the centre. The lining is thrown into folds, and the opening, which is closed, is flattened, being more compressed from above and below than laterally.

Fig. 2. The same parts shown in a section nearer the glans. The trellis work here is more distended with blood.

In this section the septum is in part wanting, the muscular coat of the urethra is thinner, and there is no open space in the centre of the corpora cavernosa.

PLATE XIX.

Conſists of four figures, taken from a penis in which the corpora cavernosa and corpus spongiosum had been injected by the arteries with spirit, and then allowed to harden in spirit; the urethra had also been injected with spirit, and kept distended.

Fig. 1. This section was taken near the root. The parts are nearly the same as in the last Plate; they are only magnified two diameters.

Fig. 2. A very small portion of the trellis work of one of the corpora cavernosa of the same section; magnified fifteen times in diameter.

Fig. 3. Another transverse section nearer the glans, magnified two diameters. In this the cavity in the corpora cavernosa is seen, and the membrane of the urethra is thrown into folds.

Fig. 4. A very small portion of the last section; magnified fifteen times in diameter.

PLATE XX.

Consists of three figures, taken from a penis, the corpora cavernosa of which were injected with mercury by the arteries, and the mercury was retained there by plugging up the trunks with common injection. The parts were then dried, and a longitudinal section made so as to let the mercury escape and leave the trellis work empty. In this state the preparation is kept in oil of turpentine.

Fig. 1. A longitudinal section of one of the corpora cavernosa, showing the course of the artery, branching through the trellis work. The part at A, has the central cavity exposed. This section is magnified two diameters.

Fig. 2. A very small portion of the cellular substance of the same longitudinal section, taken from the part at A; magnified fifteen times in diameter.

Fig. 3. A small portion near B; magnified fifteen times in diameter.

PLATE XXI.

Consists of six figures, three of the corpus spongiosum urethræ, one of the internal membrane of the urethra, and two of the villi of the covering of the glans penis.

Fig. 1. A transverse section of the corpus spongiosum urethræ, taken from the same penis as Plate XIX, Fig. 1. magnified four diameters, to show the parts more distinctly, particularly the sections of the arteries going to the trellis work.

Fig. 2. A small portion of the transverse section, Fig. 1, magnified fifteen diameters, to show the trellis work in the corpus spongiosum.

Fig. 3. A longitudinal section of the same corpus spongiosum and muscular coat of the urethra; magnified fifteen times.

Fig. 4. The internal membrane with the glands upon its surface, and the lacunæ; magnified ten diameters.

Fig. 5. The covering of the glans penis for the papillæ; magnified ten diameters.

Fig. 6. A smaller portion; magnified twenty-five diameters.

PLATE XXII.

This Plate contains two figures, giving different views of a stricture in the urethra. The stricture had become so narrow, that a piece of gravel was prevented from passing, and had imbedded itself in a cul de sac immediately behind the stricture. This piece of gravel increased in size by receiving additions from the urine that passed over it; and the stream, every time of making water, turning it round in its bed, it acquired a spherical form. The patient never having had any attempts made for his relief, and being seized all at once with a complete suppression, died; after death, the calculus was found closing up the orifice of the stricture. Both figures are magnified five diameters.

Fig. 1 A transverse section of the urethra a little beyond the stricture, showing from behind the aperture which the stone closed up, and giving a side view of the cul de sac in which it had usually remained. This cavity is not made by a fold of the internal membrane, but by an exudation of coagulable lymph forming a sac.

The other parts are the same as have been already described in the transverse sections of the urethra, in a natural state.

Fig. 2. Represents the urethra laid open from above, in a longitudinal direction. The right hand portion exhibits the newly formed coagulable lymph produced by the irritation of the stone; the serrated processes of coagulable lymph projecting from the internal membrane, which is itself thickened, and pressed forward by the swell of the muscular structure surrounding it, are also distinctly seen. The na-

Fig. 1

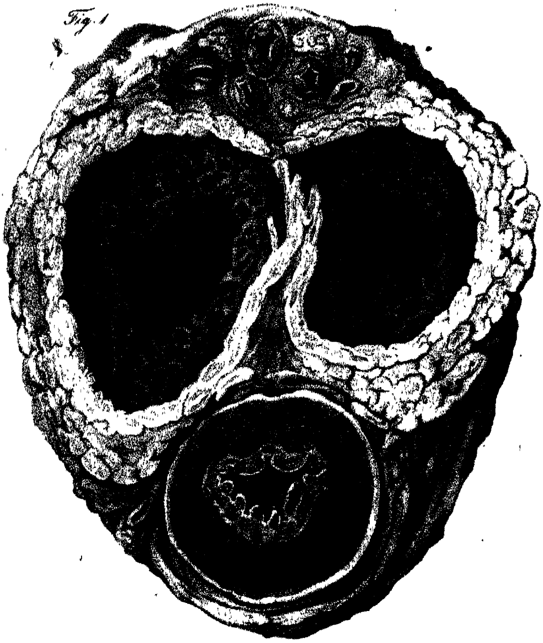
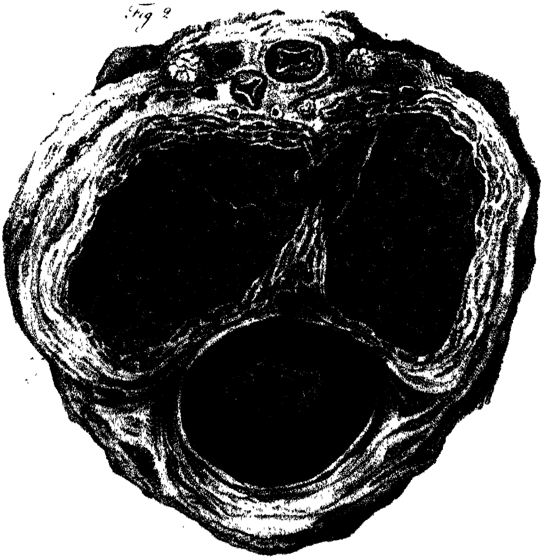


Fig. 2



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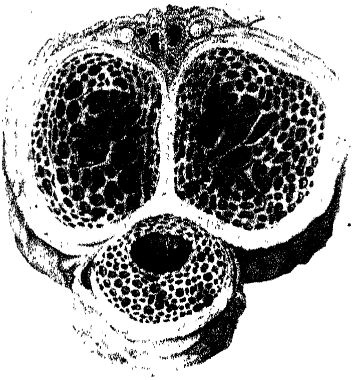


Fig 2

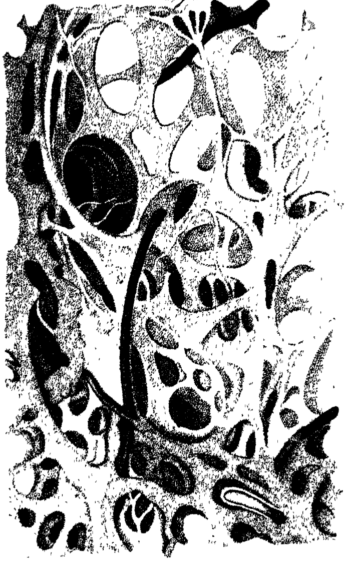


Fig 3

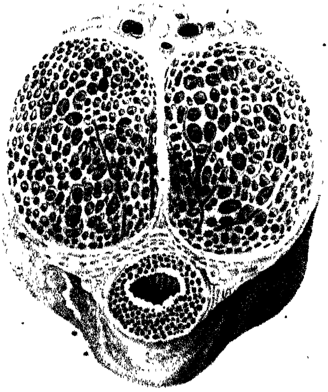
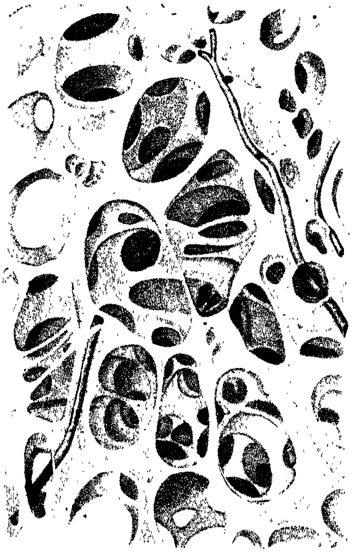


Fig 4



Franz Bauer del.

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Fig. 1.

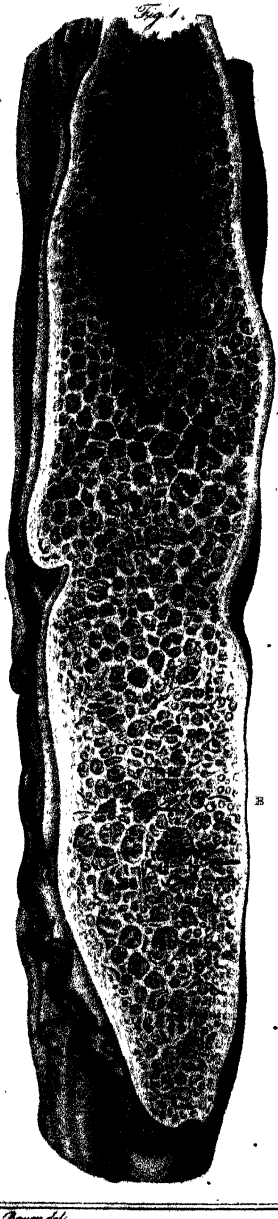


Fig. 2

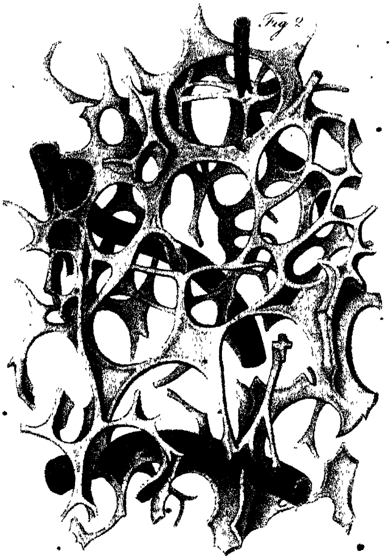
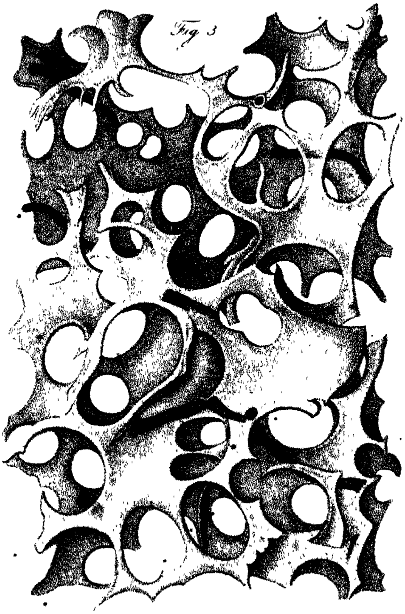


Fig. 3



Francis Bauer del.

Jas Basire sc.

Fig 1

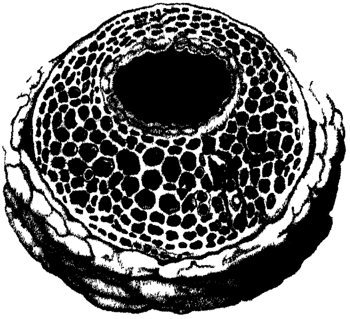
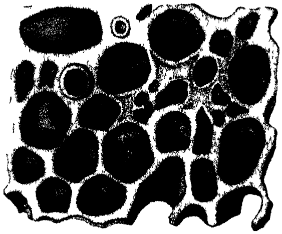


Fig 2



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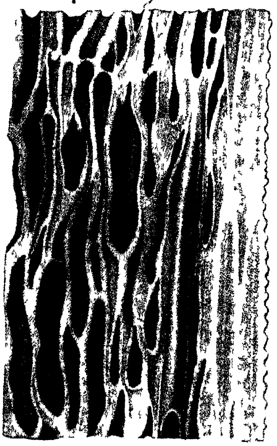

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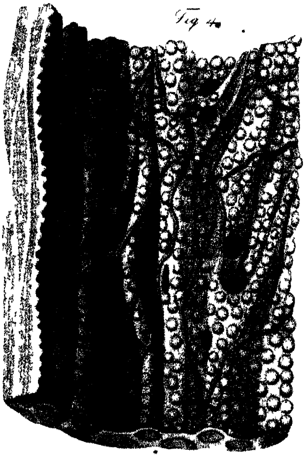

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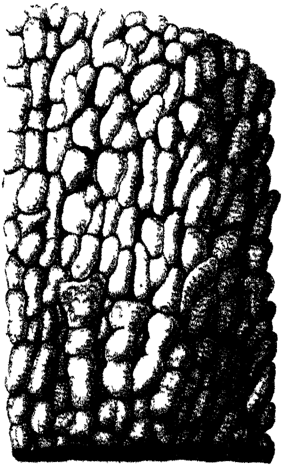

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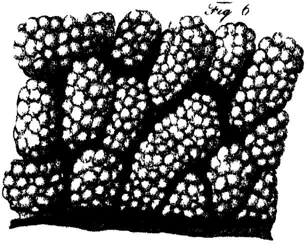

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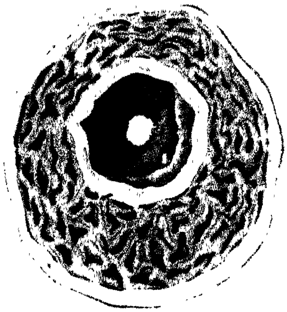
Fig 1.



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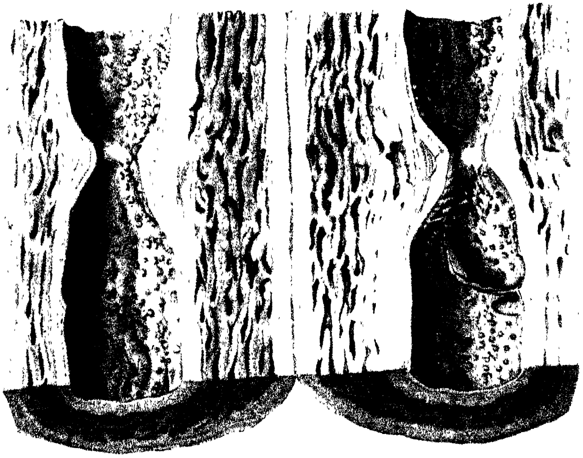

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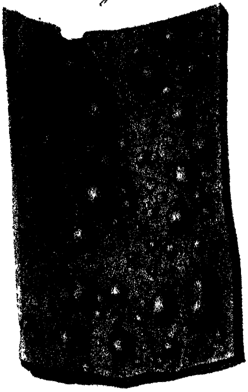
Fig 1



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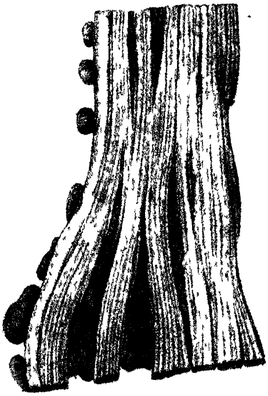

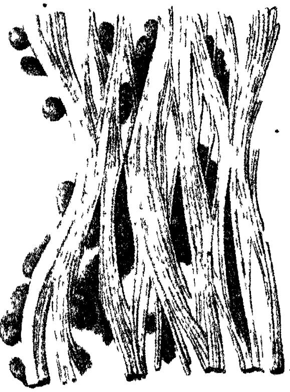

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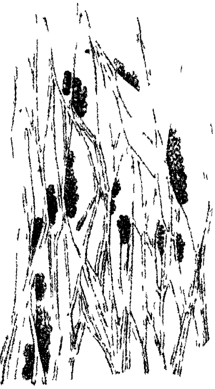

Fig 4

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tural granulated structure of the internal membrane is increased in size by the effects of inflammation. Besides which, there are small globular exudations of coagulable lymph, like studs, on different parts of the surface.

PLATE XXIII.

This Plate contains four figures, representing the internal membrane, and the muscular structure surrounding it, at the strictured part. The three first figures are magnified twenty-five diameters, the fourth fifty.

Fig. 1. Shows the exudations of coagulable lymph in masses of different forms and sizes, and the enlarged glands, nearly uniform in their shape, and of a smaller form.

Fig. 2. Shows the longitudinal muscular fibres, and the masses of coagulable lymph deposited between the fasciculi, increasing the volume so much, as to produce the narrow aperture or stricture.

Fig. 3. The same muscular fibres, having the fasciculi more separated from one another.

Fig. 4. The same fasciculi separated still more, and nearly into their ultimate fibres.

XV. *On the Errors in Longitude as determined by Chronometers at Sea, arising from the action of the iron in the ships upon the Chronometers.* *By* GEORGE FISHER, *Esq.* *Communicated by* JOHN BARROW, *Esq. F. R. S.*

Read June 8, 1820.

THE determination of the longitude at sea by timekeepers, is so exceedingly easy from the simplicity of the observations and calculations employed, and from the general practicability of the method, as to render chronometers, in the present improved state of navigation, almost indispensable articles in the equipment of ships for foreign service; and I shall feel happy if the following observations may, in any way, contribute to the more accurate determination of the longitude by this method.

The sudden alteration in the rates of chronometers when taken on board of ships, has been frequently observed by intelligent seamen; and is generally ascribed to the motion of the vessels. Before, however, I attempt to account for this alteration, I shall first prove that it actually takes place; and, in order to do this, shall relate the circumstances connected with the chronometers on board the Dorothea and Trent, commanded by Captain BUCHAN, which occurred during the late voyage to the North Pole.

Soon after the arrival of the ships on the coast of Spitzbergen, the chronometers on board the Dorothea (five in number) were found to be rapidly gaining on their former rates as determined in London previous to the ship's sailing; in consequence of which the land appeared considerably to the westward of its true position as determined by lunar observation, and they were found to be still gaining daily, which appeared not only from each subsequent set of lunars, but also by comparing the longitude of different points of land determined by the chronometers, with the longitude of the same points ascertained in the same way some time afterwards.

For instance; the longitude of a remarkable point of land on the north-west coast of Spitzbergen, called Cloven Cliff, was found by a mean of the observations taken with the chronometers on June 21, 1818, to be 10° 35′ 27″ E; but the longitude of the same point of land on July 31, was 10° 15′ 37″ E, making a difference of no less than about 20′ of longitude in five weeks; that is, estimating the longitude with the same rates and errors as determined in London before their departure; from this, therefore, it appears, they had been gaining on their former rates, or had been increasing their gaining rates, and diminishing their losing ones.

An opportunity soon afterwards occurred of observing the effect produced upon the chronometers by removing them on shore. On the 9th of August, the chronometers, nine in number, were landed on an island, where a temporary observatory had been erected for the purpose, and the latitude of which had been accurately determined with a repeating circle made by Troughton, when it was found that the acceleration immediately ceased; for the longitude of the place by chronometers, August 12, was 9° 42′ 36″ E, but on the 27th, it was 10° 1′ 0″ E, making a difference of 18′ 24″ of longitude in fifteen days, using the former rates.

Since, therefore, the chronometers were *getting easterly* by their removal on shore, the acceleration must have ceased; which will appear upon consideration.

A similar circumstance was observed by Lieut. Franklin to take place with the chronometers on board the Trent, which were four in number; and he observes, "It may be "worthy of remark, that the chronometers taken out by the "Hon. Captain Phipps, showed too great westerly longitude, "and consequently gained on these seas. The fact of so "many chronometers altering their rates the same way, is "curious, but I am not aware that any cause can be assigned."

The effect produced upon one or two of the chronometers by their removal to land, was very remarkable; a chronometer made by Baird was (by observations taken on shore near where the ships lay at anchor, by Lieutenants Franklin and Beechey, with false horizon, and eight inch reflecting circles of Troughton, from August 8, A.M. to 12, P.M.) losing 3,″4 daily when on board; but upon its removal on shore to the observatory, its rate per transit, from August 16, to 26, was observed by myself to be 18,″2 losing. Upon again removing it on board, it was found by observation, as before, to be losing 6,″5 daily; from which it appears the chronometer lost no less than about thirteen or fourteen seconds daily by its removal on shore.

Another chronometer in the Trent, made by Pennington, had been gaining rapidly on board; when taken on shore, it acquired immediately a losing rate of 1,″8, nearly the same as it had in London before the vessels sailed.

A chronometer of my own, by Arnold, was affected likewise nearly as much, losing about 9″ daily by its removal on shore.

In the other chronometers the alteration was less sudden, but was ultimately not less considerable; and they were several days on shore before they acquired a steady rate, as will appear by the following table of their rates, during the interval of the respective dates.

Table of Rates of Chronometers immediately when landed, August 9th, 1818.

	No. 1. Earnshaw.	2. Arnold.	4. Barraud.	5. Arnold.	Clock.
August 9	+12″,0	−0″,2	+8″,1	−33″,5	
11	+10,2	0,	+7,9	−34,2	
12	+ 7,98	−1,85	+0,8	−36,2	+70″,60
16	+ 6,2	−5,1	+7,1	−40,9	+69,03
20	+ 5,63	−5,2	+2,8	−37,7	+ 69,12
23	+ 4,02	−6,14	+4,3	−41,5	+69,29
26					

The rates from August 12 to 26, were determined by the sun's transit, and those from August 9th (the day on which the chronometers were landed) to the 12th, by a comparison with the clock, supposing its mean rate +69,″5, as no observation occurred during this interval; and by this table it appears, that the chronometers when landed were rapidly diminishing their gaining rates, and increasing their losing ones. In the others the effect was almost immediate.

The clock and chronometers were likewise landed upon a small island in Fair Haven, on the north coast of Spitzbergen, on the 30th June; and, as the same circumstances occurred, it will be needless to detail them.

The following table is intended to show the difference between the rates on board the ships and what they would have been had they been on shore.

Chronometers.	Error, Greenwich time.	Difference.	Interval.	Mean Rates at Sea.	Mean Rates on Shore.
No. 1. Earnshaw.	April 11, + 7′.40″.1 Aug. 25, + 26.32.2	+18′.52″ 1	days. 136	+ 8″.0	+ 3″.8
2. Arnold.	April 11, —0′.42″ Aug. 25, —4.15	—3′.33″	136	—1″.5	— 5″.2
3. Arnold.	May 7, —1′.44″ July 2, —3.36	1′.52″	56	— 2″.0	— 6″.5
4. Barraud.	April 1, — 0′. 1″ Aug. 25, + 5.55.5	+ 5′.56″.5	146	+ 2″.5	+1″.2
5. Arnold.	April 15, — 0′.38″ Aug. 25, — 28.48	— 28′.10″	132	—12″.8	—22″.4
6. Earnshaw.	April 11, + 1′.13″ Aug. 25, + 0. 2.2	—1′.10″.8	136	—0″.5	—0″.93
7. Pennington.	April 11, + 0′. 53″ Aug. 25, + 13.57	+13′.3″.5	136	+ 5″.8	— 0″.63
8. Arnold.	April 13, —0′.24″ Aug. 25, —15.53.5	+16′.17″.5	134	+ 7″.3	—2″.5
9. Baird.	April 15, + 0′.25″.1 Aug. 25, + 5.12.5	+ 4′.47″.4	132	+ 2″.2	— 5″.15

The errors of the chronometers in April, were those obtained in London before the ships sailed; those on the 25th August were determined at the Observatory on Dane's Island, Spitzbergen, the longitude of which was determined by a great many observations of the distances of the sun and moon for several days with TROUGHTON's eight inch sextants and reflecting circles. The rates in the column entitled "Mean Rates at Sea," are deduced by dividing the difference of the errors by the interval.

The rates in the column entitled "Mean Rates on Shore," or more properly what they *would have had*, are means between the rates of chronometers on shore before leaving England, and those obtained at Spitzbergen; and although a mean between the rates of chronometers obtained at different times, may not accurately be the mean rate they would have had during the interval of those times, from the continued variation to which they are subject; yet, upon comparing the two last columns together, of the rates thus deduced, it will be perceived, that in all the chronometers their gaining rates had either been increased, or their losing ones diminished on ship-board, or in other words, they had all been accelerated.

Nor is this acceleration peculiar to high latitudes; it was observed very soon after the chronometers were put on board in the River, particularly in Nos. 3 and 8, which, upon arriving at Shetland, were found to have gained instead of losing rates, which they had in London.

This acceleration was very soon perceptible in the chronometers taken out by the Hon. Captain PHIPPS, made by

Kendal and Arnold. Mr. Lyons, who accompanied him, landed at Sheerness Fort, and found the longitude by them to be 30'0" E, which is about 13' W of the true longitude, as determined in the Trigonometrical Survey.

The same occurrence took place last summer (1819). The longitude of a place in one of the Orkney Islands, as determined by three chronometers made by Arnold, two of them belonging to myself, the other to Lieut. E. Home, R. N. who accompanied me, was 6' 40" W. of the longitude determined by the difference of Æ of stars E and W of ☽.

Again, in the trial of Mr. Harrison's timekeeper, in 1764, the longitude of Barbadoes by the watch was 10' 45" more to the westward than that determined by astronomical observations made by the persons sent out for that purpose.

Soon after this trial, the commissioners of longitude agreed with Mr. Kendal, one of the watchmakers appointed by them to receive Mr. Harrison's discoveries, to make another watch on the same construction, which went considerably better than Mr. Harrison's. Mr. Kendal's watch was sent out with Captain Cook in his second voyage towards the South Pole and round the world, in the years 1772-3-4 and 5, "when the only fault found in the watch was, that its rate of "going was continually accelerated."

It now remains, therefore, to determine what this acceleration arises from. That it does not arise from the motion of the vessels, is evident in the case of the nine chronometers on board the Dorothea and Trent; since the acceleration was observed when the ships were firmly beset with ice; also in the case of the alteration in the rates of the chronometers upon landing, and taking them on board again at Dane's Island, the ships were riding at anchor close in shore without any perceptible motion.

An account was also kept on board and on shore, of the state of the temperature and barometer, every two hours, both night and day; and upon comparing them together, there does not appear to be the least correspondence between the change of rates and the temperature at the time.

It appears therefore to me, that this acceleration arose entirely from the magnetic action exerted by the iron in the ship on the inner rim of the balance, which is made of steel.

That the iron in the ships becomes magnetic, is plain, from the polarity which exists in it; the whole forming altogether one large magnet, having its south pole on deck nearly amidships, and its north pole below. This is seen from the constant deviation of the north end of the compasses placed on deck towards the centre of the ship, as appears from recent observations, which I have mentioned elsewhere.

Nor is it surprising that the force exerted by the ship's iron (thus become magnetical) on the balance of the chronometers, should be sufficient to cause a very sensible alteration in the rate of going, when we consider how easily, in other cases, the presence of any thing magnetical is detected by the alteration of the rate of a chronometer; and when we consider the great influence exerted by this iron upon the binnacle compasses at very considerable distances, and in situations where the utmost precaution is used to remove every piece of iron from them, by using copper-bolts, fastenings, &c.

It remains only to determine, how far this alteration in the rates of the chronometers, can be reconciled with that observed in chronometers when under the influence of magnets placed in different positions with respect to their balances.

To determine this, two watches were used, with steel balances and horizontal escapements, one by Earnshaw, the other by Allan and Caithness; also two chronometers made by Arnold. To each of these watches were applied, at a distance of two inches from the balance, magnets of twelve inches in length, in four different positions, and in the planes of the balances.

The following Table will show the rates of the watches in twenty-four hours, deduced by comparing them with an excellent clock with Graham's dead beat escapement, and regulated by transit.

	No. 1.	No. 2.			No. 3.	No. 4.
N	+9′.15″	+5′. 0″	3.	N	+0′ 34″	+0′.41″
S	+8.12	+21.0		S	+2.18	+0.44
N	+ 8′.0″	+18′.0″	6	N	+1′.28″	—1′.43″
S	+48.0	+ 8.0		S	+5.27	—0.36
N	+47′.10″	+17′. 8″	9	N	+5′.22″	+1′. 3″
S	+72. 0	+ 8 44		S	—0.14	+0.41
N	+4′.14″	+ 4′.32″	12	N	+2′.47″	+1′.12″
S	—2. 0	+15. 0		S	—1. 2	+1.24

The first column in this table shows the pole of the magnet applied to the watch; the second and third, the rate or effect produced on each watch; the fourth column shows the figure on the face of the watch opposite to which the magnet was applied.

The watch, No. 1, gained with both poles, and in every position of the magnet but one. No. 2, gained with both poles in every position. Nos. 3 and 4 gained in every position but two; and the quantities lost in the positions were far exceeded by the accelerations caused by the opposite poles, excepting one case in that of No. 4.

The magnets were likewise placed in different positions out of the planes of the balances; the results were very *similar* to those above, but differing in quantity, according to the distance of the magnets from the planes of the balances.

Upon placing the magnets very near to the rim of the balances, a very rapid acceleration took place with both poles, and in every position of the magnets, particularly in the watches Nos. 1 and 2. Upon too near an approach of the magnets, the watch No. 1, and chronometer No. 3, were rendered useless; the former, when the magnets were taken away, gaining no less than about 1¼ hour, and the chronometer losing about 50″ in 24 hours; and in again repeating the experiments in the plane of the balances, the rates of the chronometers (without the magnets) were so variable, that it was necessary to determine their rates before and after each application of the magnets; the following, however, is a Table of the results upon the chronometers Nos. 3 and 4.

	No. 3.	No. 4.	
N S	+0′ 2″.7 +1′ 24″	+0′ 43″.5 +0′ 46″	3
N S	−5′ 32″ +6′ 15″	−1′ 40″ −1′ 37″	6
N S	+5′ 19″ −1′ 53″	+1′ 18″ +1′ 12″	9
N S	+1′ 59″ +2′ 7″.3	+1′ 11″.7 +1′ 29″	12
N S	+0′ 31″ +2′ 8″	+1′ 14″ +0′ 43″	3
N S	−5′ 9″ +4′ 25″.7	−1′ 24″ −0′ 59″	6
N S	+5′ 25″ −1′ 53″	+0′ 55″ +0′ 42″	9
N S	+2′ 33″ −2′ 23″	+1′ 5″ +1′ 6″	12

Each of these results in this Table, is the difference of the rates when the magnets were applied, and a mean of the rates of the chronometers before and after the application of the magnets. The rate of the chronometer No. 3, is very different from that given in the former Table; that of No. 4 is nearly the same, and does not appear to have been affected, as No. 3 was, by the close approach of the magnet.

Upon the whole, however, it appears that chronometers will be generally accelerated (particularly if their balances have not received polarity by the too near approach of any thing magnetical) on ship-board. It appears probable, likewise, that the force of the balance springs is affected in the same way; since it is well known that chronometers having gold balance springs, although more difficult to adjust, yet keep better rates at sea than the others.

However this may be, these observations show the necessity of not trusting to the rates of chronometers ascertained during the time they are on shore; and if the rates are ascertained on board, the chronometers should always be kept in the same place, and also in the same position with respect to the ship; for I have but little doubt that, upon an accurate trial, a chronometer will be found to change its rate, more or less, according as these circumstances are attended to. If these precautions are not taken, land will appear to be considerably to the westward of its true position; this is particularly exemplified in the observations of the Hon. Captain Phipps; from which, nearly the whole line of coast on the west side of East Greenland has been placed nearly 1½° too much to the westward, by reason of the acceleration of his chronometers; the same circumstance would have occurred with the chronometers in both of the ships Dorothea and Trent, in the late voyage, had not the longitude been otherwise determined. It is therefore highly requisite that attention should be paid to a circumstance so much connected with the improvement of geography as well as the safety of the seaman.

As an Appendix to these observations, I beg leave to add the following Table of Rates, furnished by Mr. Coleman, to which I have prefixed his letter as an introduction.

157, Leadenhall-Street,
5th June, 1820.

Sir,

In compliance with the wish of our common friend Lieutenant Everard Home, I have much pleasure in sending you a table of rates of chronometers. My profession as Teacher of Navigation, conjointly with Mr. Norie, which connects me so intimately with the Officers of the Honourable East India Company, employed on board the ships trading to India, has enabled me to present you with this table upon an enlarged scale. While engaged in the same service eighteen years, this subject occupied my attention, and it affords me much satisfaction that you are bringing the subject before the Royal Society. My remarks correspond very much with those, I understand, you are now bringing forward.

I am, Sir,

Your obedient servant,

George Coleman.

TABLE I. A.

Date.	Persons' Names.	Ships' Names.	Burthen in Tons.	Copper or Iron fastened.	Chronometers.	No. of Days to wind up.	Rates given.	Rates found.	Difference of Rates.	REMARKS.
1802 & 1803	Capt. Dunsford	H. C. S. Ceres	1200	Iron	Arnold	1	″ 3.4 gaining	″ 5.0 gaining	″ 1.6 gained	These Chronometers went remarkably steady during the voyage, with the rates found.
					Margetts	8	5.0 gaining	5.2 gaining	0.2 gained	
					Margetts	8	2.5 losing	2.0 losing	0.5 gained	
					Earnshaw	1	6.2 losing	7.5 losing	1.3 lost	
1804 & 1805	Capt. Dunsford	H. C. S. Ceres	1200	Iron	Arnold	1	4.5 gaining	6.0 gaining	1.5 gained	These Chronometers gained on board the same as on the preceding voyage.
					Margetts	8	5.0 gaining	5.2 gaining	0.2 gained	
					Earnshaw	1	6.4 losing	8.0 losing	1.6 lost	This also *lost* in *similar proportion*
1806 & 1807	Capt. Dunsford	H. C. S. Ceres	1200	Iron	Arnold	1	2.5 gaining	3.8 gaining	1.3 gained	The same Chronometer as above, same alteration.
					Arnold	1	4.6 losing	4.0 losing	0.6 gained	
					Margetts	8	5.0 gaining	5.3 gaining	0.3 gained	This Chronometer went five years at 5″ on shore, and 5.3 on board.
					Earnshaw	1	1.4 losing	1.5 gaining	2.9 gained	The same Chronometer as above, altered after cleaning.
1808 & 1809	Capt. Dodds	H. C. S. Walmer Castle	1260	Iron	Barraud	1	7.3 gaining	6.0 gaining	1.3 lost	
					Brockbank	8	4.0 losing	2.8 losing	1.2 gained	
					Arnold	1	3.6 gaining	4.0 gaining	0 4 gained	
1810 & 1811	Capt. Mayne	H. C. S. Batavia	750	Iron	Earnshaw	1	6.7 losing	3.2 losing	3.5 gained	This Chronometer went very irregular during the voyage.
	Mr. Coleman				Arnold	1	3.7 gaining	5.4 gaining	1.7 gained	
					Brockbank	2	11.8 losing	13.8 losing	2.0 lost	
					These Chronometers were on shore at Calcutta upwards of three months, and then had the following rates given and found.					
1812	Capt. Mayne	H. C. S Batavia	750	Iron	Earnshaw	1	10.5 gaining	18.0 and 23″ gaining	…………	NO reliance on this Chronometer at any time.
	Mr. Coleman				Arnold	1	6.7 gaining	2.4 gaining	1.7 gained	
					Brockbank	2	15.4 losing	18.6 losing	3.2 lost	This Chronometer gradually lost during the whole voyage.
1813 & 1814	Capt. Blanchard Capt. Money Mr. Coleman Mr. Decharme	H. C. S. James Sibbald	700	Iron	The Book containing the various changes of these Chronometers is in Captain Blanchard's possession, whose arrival in England is daily expected. N. B. The above Chronometers were all under my entire charge; I therefore can vouch for the accuracy of the above statements.—G. COLEMAN.					

TABLE I. B.

Date.	Persons' Names.	Ships' Names.	Burthen in Tons.	Copper or Iron fastened.	Chronometers.	No. of Days to wind up.	Rates given.	Rates found.	Difference of Rates.	REMARKS.
1819 & 1820	Capt. Tennent	H. C. S. Apollo	700	Iron	Barraud Barraud Brockbank & Co.	1 8 2	″ 3.0 losing 3.5 gaining 9 5 gaining	″ 2.5 losing 4.2 gaining 12.5 and 16 gaining	″ 0.5 gained 0.7 gained 6.5 gained	Barraud's Chronometers went steady several months before they altered their rates; but Brockbank's altered its rate in a few weeks.
1818 & 1819	Capt. S. Lee	H. C. S. Moffatt	720	Iron	Brockbank Barraud	1 2	4.5 losing 2.4 gaining	3.2 losing 3.6 gaining	1.3 gained 1.2 gained	These Chronometers went steady within 2 or 3 tenths, during a voyage of fourteen months.
1819 & 1820	Capt. Stewart.	F. T. Sappho	360	Copper	Barraud Hatton Parkinson & Co.	8 8 1	6.8 losing 2.4 gaining 10.0 gaining	5.0 losing 7.0 gaining 16.5 and 22 gaining	1.8 gained 4.6 gained 12.0 gained	These went pretty regular with ship rates. This Chronometer proved a very bad one.
1819	Capt. Pitcher	F. T. Bloxbornbury	720	Copper	Morris	1	2.2 gaining	3.5 gaining	1.3 gained	
1818 1819	Capt. Foord	S. S. Phœnix	460	Iron	Brockbank & Co.	2	3.5 losing	6.3 losing	2.8 lost	This Chronometer went steady at 6.3 till let down.
1815 & 1816	Capt. Forbes	H. C. S. James Sibbald	700	Iron	Hatton Barraud Earnshaw	8 1 p. 1	14 0 gaining Going at mean time 15 0 losing	14.0 gaining 0.3 gaining 18.5 losing	.... 0.3 gained 3.5 lost	No difference was discovered in the going of this Chronometer outward. This Chronometer going well at 0.3, was sold at Calcutta. Altered its rate after leaving Madeira.
		On the Ship's returning home, the above Chronometers had rates as under, after being eleven weeks on shore at Calcutta.								
					Hatton Earnshaw	8 1	14.5 gaining 19.2 losing	16.0 gaining 20.7 losing	1.5 gained 1.5 lost	The Chronometers went very steady during the homeward passage with the rates found.
1818 & 1819	Capt. Forbes	H. C. S. James Sibbald	700	Iron	Hatton & Harris Hatton & Harris Barraud	8 1 p. 1 p.	9 5 losing 3.0 gaining 4.5 losing	8.5 losing 5.0 gaining 6.0 losing	0.5 gained 2.0 gained 1.5 lost	These Chronometers were rated at Madeira, and afterwards went steady at those rates.
		On the homeward bound passage, the same Chronometers had the following rates, after being three months on shore.								
					Hatton & Harris Hatton & Harris Barraud	8 1 p. 1 p.	9.0 losing 5.0 gaining 7.0 losing	9.0 losing 5.0 gaining 7.0 losing		It therefore appears the Chronometers did *not vary* their rates homeward in the most trifling degree.
1819 1820	Capt. Doveton	T. F. Lotus	580	Copper	Barraud	8	6.8 losing	8.0 losing	1.2 lost	This Chronometer did not alter its rate till at Calcutta.

TABLE II. A.

To follow Table I.

Date.	Persons' Names.	Ships' Names.	Tonnage.	Iron or Copper fastened.	Makers of Chronometers.	No. of Days to wind up.	Box or Pocket.	Rates given.	Rates found.	Difference of Rates.	REMARKS.
1802 & 1803	Capt. Dunsford	H. C. S. Ceres	1200	Iron	Arnold	1	Pocket	" 3.4 gaining	" 5.0 gaining	" 1.6 gained	These Chronometers went remarkably well during the voyage, with the rates found.
					Margetts	8	Box	5.0 gaining	5.2 gaining	0.2 gained	
					Ditto	8	—	2.5 losing	2.0 losing	0.5 gained	
					Earnshaw	1	Pocket	6.2 losing	7.5 losing	1.3 lost	
1804 & 1805	Capt. Dunsford	H. C. S. Ceres	1200	Iron	Arnold	1	Pocket	4.5 gaining	6 0 gaining	1.5 gained	These Chronometers gained during this voyage in the same proportion as in the preceding one.
					Margetts	8	Box	5.0 gaining	5.2 gaining	0.2 gained	
					Earnshaw	1	Pocket	6.4 losing	8.0 losing	1.6 lost	This Chronomerer *lost* in similar proportion.
1806 & 1807	Capt. Dunsford	H. C. S. Ceres	1200	Iron	Arnold	1	Pocket	2.5 gaining	3.8 gaining	1.3 gained	In the two former voyages there was nearly the same alteration.
					Ditto	1	—	4 6 losing	4.0 losing	0 6 gained	This Chronometer a very excellent one.
					Margetts	8	Box	5.0 gaining	5.3 gaining	0.3 gained	This Chronometer went five years & a half at 5". on shore, and 5".3 on board.
					Earnshaw	1	Pocket	1 4 losing	1.5 gaining	2.9 gained	Same Chronometer as mentioned on a former voyage.
1808 & 1809	Capt. Dodds	H. C. S. Walmer Castle	1200	Iron	Barraud	1	Pocket	7.3 gaining	6 0 gaining	1.3 lost	These Chronometers went very steady with the rates given them.
					Brockbank	8	Box	4 0 losing	2.8 losing	1 2 gained	
					Arnold	1	Pocket	3 6 gaining	4 0 gaining	0.4 gained	
1810 & 1811	Capt Mayne	H. C. S. Batavia	750	Iron	Earnshaw	1	Box	6.7 losing	3.2 losing	3.5 gained	This Chronometer went irregular during the whole voyoge.
					Arnold	1	Pocket	3 7 gaining	5.4 gaining	1.7 gained	
	Mr. Coleman				Brockbank	2	Box	11.8 losing	13.8 losing	2.0 lost	
The above Chronometers were on shore at Calcutta upwards of three months, and then had the following rates given and found.											
	Capt. Mayne				Earnshaw	1	Box	10.5 gaining	18.0 and 23 gaining	....	No reliance on this Chronometer at any time.
1812					Arnold	1	Pocket	6 7 gaining	8.4 gaining	1.7 gained	
	Mr. Coleman				Brockbank	2	Box	15.4 losing	18.6 losing	3.2 lost	This Chronometer gradually lost during the whole voyage.
1813 & 1814	Capt. Blanchard	H. C. S. James Sibbald	700	Iron	Hatton	8	Box	7.2 gaining	7.2 gaining	No difference	These Chronometers went remarkably well with the ship, except Capt. Money's box one, which at times veered and hauled exceedingly.
					Barraud	2	—	2.4 gaining	3.2 gaining	0.8 gained	
					Ditto	1	Pocket	3 5 losing	2 0 losing	1.5 gained	
	Capt. Money				Brockbank	1	Box	10.8 losing	12.6 losing	1.8 lost	
	Mr. Coleman				Ditto	1	—	4.4 gaining	5.8 gaining	1.4 gained	
	Mr. Decharme				Barraud	1	Pocket	2 6 gaining	2.6 gaining	No difference	
1819 & 1820	Capt. Tennent	H. C. S. Apollo	700	Iron	Barraud	1	Box	3.0 losing	2.5 losing	0.5 gained	These Chronometers went extremely well during the whole voyage.
					Barraud	8	—	3.5 gaining	4.2 gaining	0 7 gained	
					Brockbank	2	—	9.5 gaining	12.5 then 16 ditto	16.5 gained	This was a very indifferent one throghout.
1818 & 1819	Capt. S. Lee	H. C. S. Moffatt	720	Iron	Brockbank	1	Box	4.5 losing	3.2 losing	1.3 gained	These scarcely varied from the rates found during the whole voyage.
					Barraud	2	—	2.4 gaining	3.6 gaining	1.2 gained	
1815 & 1816	Captain Forbes	H. C. S, James Sibbald	700	Iron	Hatton	8	Box	14.0 gaining	14.0 gaining	No difference	Chronometer was sold at Calcutta.
					Barraud	1	Pocket	On mean time.	0.3 gaining	0.3 gained	
					Earnshaw	1	Box	15.0 losing	18.5 losing	3.5 lost	
These Chronometers were on shore about eleven weeks, and then the following rates were both given and found.											
					Hatton	8	Box	14 5 gaining	16.0 gaining	1.5 gained	These went very steady on the passage home with the rates found.
					Earnshaw	1	—	19.2 losing	20.7 losing	1.5 lost	

TABLE II. B.

Date.	Persons' Names.	Ships' Names.	Tonnage.	Iron or Copper fastened.	Makers of Chronometers.	No. of Days to wind up.	Box or Pocket.	Rates given.	Rates found.	Difference of Rates.	REMARKS.
1818 & 1819	Capt. Forbes	H. C. S. James Sibbald	700	Iron	Hatton Ditto Barraud	8 1 1	Box Pocket —	9.5 losing 3.0 gaining 4.5 losing	8.5 losing 5.0 gaining 6.0 losing	0.5 gained 2.0 gained 1.5 lost	These Chronometers were all rated at Madeira, and went steady at those rates on the passage out.
			On the Homeward passage the above Chronometers had the following rates given and found, after being on shore three months and upwards.								
					Hatton Ditto Barraud	8 1 1	Box Pocket —	9.0 losing 5.0 gaining 7.0 losing	9.0 losing 5.0 gaining 7.0 losing	No difference.	It therefore appears that these Timekeepers did *not vary* from the rates given them on shore in the smallest degree.
1819 & 1820	Capt Stewart	Ship Sappho	360	Copper	Barraud Hatton Parkinson	8 8 1	Box — —	6.8 losing 2.4 gaining 10.0 gaining	5.0 losing 7.0 gaining 16.5 then 22 gaining	1.8 gained 4.6 gained 12.0 gained	These went regular with the rates found. This very indifferent during the whole voyage.
1818 1819	Capt. Pitcher	H. C. S. Bloxbornbury	720	Copper	Morris	1	Pocket	2.2 gaining	3.5 gaining	1.3 gained	
1818 1819	Capt. Foord	S. S. Phœnix	460	Copper	Brockbank	2	Box	3.5 losing	6.3 losing	2.8 lost	This Chronometer went steady at 6.3 till let down.
1819 1820	Capt. Doveton	Lotus	580	Copper	Barraud	8	Box	6.8 losing	8.0 losing	1.2 lost	On being landed, it returned to its first rate—6.8 losing.
1818 & 1819	Capt. W. Morgan	Layton	400	Iron	Barraud	2	Box	6.0 losing	7.2 losing	1.2 lost	
					Ditto - Homeward bound -			7.0 losing	6.2 losing	0.8 gained	
1819 & 1820	Capt. Ogelvie	Juliana	448	Iron	Barraud	1	Box	6.0 losing	6.0 losing	No difference	
					Ditto - Homeward bound -			6.0 losing	5.2 losing	0.8 gained	
1819 1820	Capt. Scott	H. C. S. Charles Grant	1200	Copper	Barraud	8	Box	6.1 losing	5.4 losing	0.7 gained	
1814 1815	Capt. Hope	H. C. S. Surat Castle	1200	Iron	Barraud	8	Box	1.8 gaining	0.0	1.8 lost	
1816 1817	Capt. Hope	H. C. S. Surat Castle	1200	Iron	Same Chronom.	8	Box	5.2 gaining	5.5 gaining	0.3 gained	
1813 1814	Capt. Franklin	H. C. S. Northumberland	640	Iron	Barraud	8	Box	0.4 losing	0.0	0.4 gained	
1814 1815	Capt. Cameron	H. C. S. Ann	610	Iron	Barraud	8	Box	2.0 gaining	1.6 gaining	0.4 lost	
1816 1817	Capt. B. Laing	H. C. S. Royal Charlotte	1200	Iron	Barraud	8	Box	0.0	0.4 gaining	0.4 gained	
1818	Capt. M'Kellar, R. N.	H. M. S. Pique			Barraud	8	Box	6.5 losing	6.2 losing	0.3 gained	This Chronometer returned to the maker's rate on being brought on shore.
					Next six months on ship its rate was			6.5 losing			
1818 & 1819	Capt. Taylor	Cæsar	510	Copper	Barraud	8	Box	2.7 losing	1.9 losing	0.8 gained	
					Next twelve months on shore, its rate was			0.1 gaining			

TABLE II. C.

Date.	Persons' Names.	Ships' Names.	Tonnage.	Iron or Copper fastened.	Makers of Chronometers.	No. of Days to wind up.	Box or Pocket.	Rates given.	Rates found.	Difference of Rates.	REMARKS.
1817	Capt. Sever	West India S. Fanny	384	Iron	Barraud	8	Box	0.5 losing	0.9 gaining	1.4 gained	
					Ditto	Next four months on shore.......			2.0 gaining		
1818	Ditto	Ditto			Ditto			0.2 gaining	1.4 gaining	0.6 lost	
					Ditto	Next two months on shore			0.7 gaining		
1819	Ditto	Ditto			Ditto			0.7 gaining	1.4 gaining	0.7 gained	In this instance it returned to the same rate as on the *preceding* voyage.
1818	Capt. James Kay	H. C. S. Marchioness of Exeter - -	980	Copper	Barraud	8	Box	4.0 losing	4.1 losing	0.1 lost	
					Ditto	Next four months on shore........			3.3 losing		
1817	Capt. R. S. Dalrymple	H. C. S. Vansittart	1240	Copper	Barraud	8	Box	0.6 gaining	0.2 gaining	0.4 lost	It therefore appears this Chronometer varied precisely the same quantity and the same way, two succeeding voyages: this ship, it may be necessary to remark, is copper fastened throughout.
1818					Ditto	Next six months on shore			0.7 losing		
1819	Ditto	Ditto			Ditto	8	Box	0.4 losing	0.8 losing	0.4 lost	
1820					Ditto	Since the ship's return to England..			0.2 gaining		
1817 & 1818	Capt. Nairne	H. C. S. General Kyd	1240	Iron	Barraud	2	Box	1.5 gaining	1.7 gaining	0.2 gained	
					Ditto	Next six months on shore			3.0 gaining		
1816	Capt. R. Drew	West India Ship			Barraud	2	Box	9.3 gaining	10.3 gaining	1.0 gained	
1817					Ditto	Next five months on shore			8.4 gaining		
1817	Ditto	Ditto			Ditto			8.4 gaining	8.9 gaining	0.5 gained	
1818					Ditto	Next three months on shore			8.0 gaining	and then gradually decreased to 4.5, the rate when taken on board.	
1818	Ditto	Ditto			Ditto			4.5 gaining	6.5 gaining	2.0 gained	
1819					Ditto	Next thirty-two days on shore			5.0 gaining	and was then cleaned.	
1819	Capt. Cookesley	Ship Malida	480	Copper	Barraud	2	Box	6.5 losing	6.5 losing	No difference	This Chronometer, since its return on board ship, has taken up again the rate of 6.5, which favours Mr. Fisher.
					Ditto	Next forty days on shore..........			8.0 losing		
1818 & 1819	Capt. Johnson	H. C. S. Camden	490	Iron	Barraud	2	Box	Going mean time	9.3 gaining	9.3 gained	This increase of rate took place very gradually during the first four months, after which it kept that rate, making an error of *only* 16″, the following fifteen *months*, and continued the *same rate* for five months following *on shore*; was then cleaned.
1816	Capt. Barwick				Barraud	2	Box	0.0 to	9.5 gaining	in the course of thirteen months at sea, and the following two months.	
					On shore this Chronometer still increased to 11.5 gaining				12.2 gaining	0.7 gained	And went at 12.2, gaining next threee months on shore.
1816 & 1817	Capt. Tremenhere	H. C. S. Asia	890	Iron	Barraud	8	Box	2.0 gaining	2.3 gaining	0.3 gained	
					Ditto	Next six months on shore			3.7 gaining	and was then cleaned.	
1817	Capt. Auber	E. I. S. Lady Raffles	610	Iron	Barraud	2	Box	5.0 gaining	7.8 gaining	2.8 gained	And went next four months on shore at 5.5 gaining
1816	Capt. Druce				Barraud	1	Pocket	2.5 losing	3.4 losing	0.9 lost	And went next eleven months on shore at 0.3 losing.
1815 1816	Capt. Mortlock	H. C. S. Lowther Castle	1200	Copper	Barraud	1	Pocket	1.8 losing	1.7 losing	0.1 gained	And went next forty days on shore at 2.1 losing; then cleaned.
1819 & 1820	Capt. Gribble	H. C. S. Princess Charlotte of Wales -	978	Iron	Barraud	8	Box	11.0 losing	12.2 losing	1.2 lost	Since the ship's arrival it has been going at 14.8 losing.
						8	Box	2.0 gaining	2.3 gaining	0.3 gained	.... 1.9 gaining.

XVI. *An account of a new mode of performing the High Operation for the Stone.* *By Sir* EVERARD HOME, *Bart. V. P. R. S.*

Read June 15th, 1820.

HAVING performed the high operation for the stone in a manner less severe and less dangerous to the patient than that now in use, and by which the stone is more easily extracted, I am desirous of having it put upon record in the Philosophical Transactions, that at the same time it is made public, my claim to the first adoption of this mode may be established, which could not be so well done, were I to postpone the present communication.

For the previous information that led me to adopt this method, I beg to refer to the History of the High Operation for the Stone, published by Mr. CARPUE, to which publication I am indebted for the principle, and only claim the merit of having made, what must be allowed to be, a considerable improvement.

Case, and Operation.

JOHN RIVINGTON, aged 16, who had suffered as long as he could remember from pain in the act of making water, and immediately after the whole quantity was evacuated, came into St. George's Hospital on the 19th of May, 1820. His complaint was found to be the stone, and I performed the operation on the 26th of the same month, in the following manner.

The boy was laid with his back upon a table, his feet supported on a chair. An incision was made in the direction of the linea alba between the pyramidales muscles, beginning at the pubes, and extending upwards four inches in length: it was continued down to the tendon. The linea alba was then pierced close to the pubes, and divided by a probe-pointed bistory to the extent of three inches. The pyramidales muscles had a portion of their origin at the symphisis pubis detached to make room. When the finger was passed down under the linea alba, the fundus of the bladder was felt covered with loose fatty cellular membrane. A silver catheter, open at the end, was now passed into the bladder by the urethra, and when the point was felt by the finger in the wound pressing up the fundus, a stilet that had been concealed was forced through the coats of the bladder, and followed by the end of the catheter. The stilet was then withdrawn, and the opening through the fundus of the bladder enlarged toward the pubes by a probe-pointed bistory sufficiently to admit two fingers, and then the catheter was withdrawn. The fundus of the bladder was held up by one finger, and the stone examined by the fore-finger of the right hand. A pair of forceps with a net attached was passed down into the bladder, and the stone directed into it by the finger: the surface being very rough, the stone stuck upon the opening of the forceps, and being retained there by the finger, was extracted. A slip of linen had one end introduced into the bladder, and the other was left hanging out of the wound, the edges of which were brought together by adhesive plaster. A flexible gum catheter, without the stilet, was passed into the bladder by the urethra, and kept there by an elastic retainer surrounding the penis. The patient was put to bed and laid upon his side, in which position the urine escaped freely through the catheter.

The calculus was of the roughest mulberry kind, was nearly spherical, and weighed one ounce. As no blood had been lost in the operation, twelve ounces were taken from the arm. Outlines of the different instruments are annexed. Pl. XXIV.

May 27. The patient had rather a restless night, but had some sleep; the wound very tender: all the urine escaped by the catheter; no tenderness on the abdomen. The slip of linen was withdrawn, which gave great relief. The author begs to remark, that leaving any such slip in the bladder is unnecessary, as no urine escaped through the upper wound; it need only be left in the external wound, to prevent the formation of abscess.

May 28. The patient had some quiet sleep in the night; pulse quick, bowels open, the wound less tender; the urine escapes freely. Had his bed completely changed: in the evening the catheter was plugged up, and till cleared, the urine came by the wound. The pulse was quick and full, the tongue white. He lost ten ounces of blood, which upon standing became cupped and buffy.

May 29. The wound still tender. Had not much sleep in the night. The catheter being again plugged up, it was altogether withdrawn, none being at hand of a proper size to replace it; and the time for the urine to diffuse itself being past.

The catheter had been made to retain its curve; to do which its inside had less varnish, so that when soaked in urine it became spongy, and was readily clogged. The catheters used in this operation should have the inside of the tube polished like the outside.

May 30. The patient had a good night. The urine escaped by the wound, which looked healthy; he is quite free from pain, and in all respects better. A little urine came by the penis. He was moved into another bed.

May 31. His night not so good, but the pulse natural; the tongue losing its whiteness; the urine came for the most part by the wound, but in gushes of two or three table-spoonfuls at a time. Sat up for ten minutes in a chair.

June 1. He sat up an hour. Urine still comes principally by the wound.

June 2. The patient is gradually gaining strength; sat up twice in the day.

June 3. He sat up in a chair for upwards of two hours; and while in the effort of having a stool, a considerable quantity of urine came by the urethra.

June 4. Some urine still came by the wound; the abdominal muscles not acting, excepting when at stool. After sitting up some hours a soft bougie was passed, and immediately withdrawn, to clear the urethra: it was followed by a few drops of urine.

June 5. Sat up nine hours; and twice in a full stream made eight ounces of water; from that time none escaped by the wound.

June 9. The wound has become superficial, and the bladder retains eight or ten ounces at a time.

It is to be remarked, that the stone had been an unusual number of years in the bladder, was of the roughest possible

Fig 1

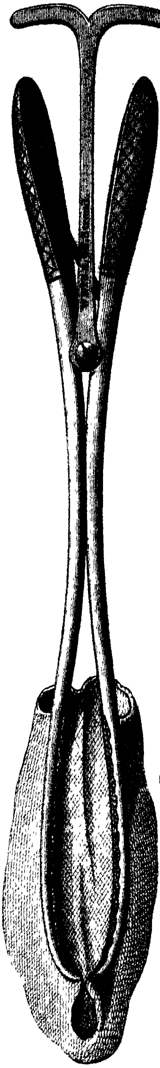


Fig 3

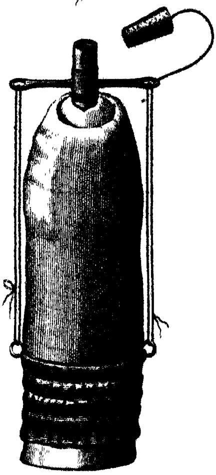


Fig 2

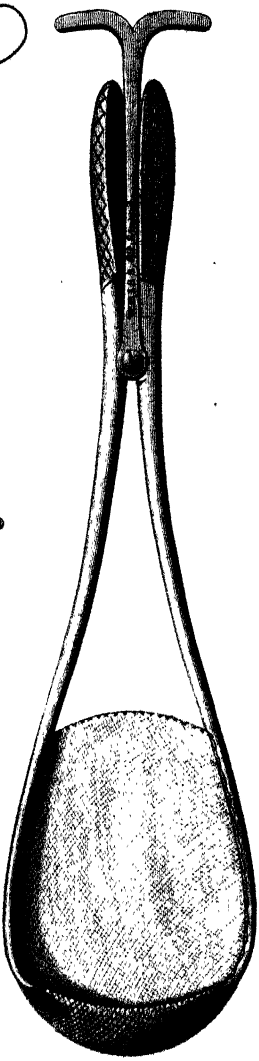


Fig 4

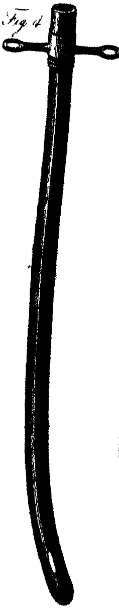
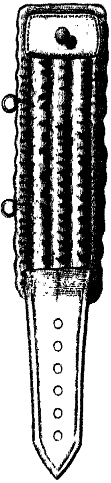


Fig 5



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kind, and yet so mild were the effects of the operation, that in ten days the bladder completely recovered its healthy actions.

EXPLANATION OF PLATE XXIV.

This Plate contains five figures $\frac{2}{3}$rds of their real size.

Fig. 1. The forceps closed by means of a spring, in which state they are to be introduced into the bladder; the silk net-bag in a collapsed state.

Fig. 2. The forceps opened by pressing the spring handles, and the silk net-bag extended to receive the stone, upon which the forceps are allowed to close, by removing the pressure from these handles The stone by falling to the bottom of the bag, becomes lower than the bent point of the forceps, and does not require an opening larger than itself to admit of being extracted. The stone is to be pulled out by the upright handle.

Fig. 3. The representation of a portion of the penis with the catheter in the urethra, which is prevented from coming out by the retainer applied round the penis.

Fig. 4. The flexible gum catheter of the proper length to reach the lower posterior part of the bladder.

Fig. 5. The catheter retainer detached from the penis.

XVII. *A Sketch of an Analysis and Notation applicable to the estimation of the value of Life Contingencies.* *By* BENJAMIN GOMPERTZ, *Esq. F. R. S.*

Read June 29, 1820.

THE estimation of the value of property, connected with life contingencies, has for many years occupied the attention of the philosophers, as well as the monied and landed interest of this country. The many institutions now existing in England, for the purpose of granting annuities and assurances on lives, are a sufficient evidence that they are conceived to yield advantages to the community:—advantages which seem to present themselves in two points of view; the one, the gain accruing to the parties granting the assurance or other object; the other, the benefit to be received by persons purchasing those grants. In a political point of view, it appears a question of great importance to decide, what ought to be the demands of those companies, so that the public may reap the greatest benefit from them? And the only means of answering this question, is the possession of the mathematical and philosophical principles, by which those institutions ought to be guided. In the present improved state of the science of life assurances, it is not sufficient for a proper regulation to follow old customs, and calculations, drawn from a less perfect experience than we have now the means of obtaining; but every company, to reap the advantage of the progressive state of the science, should not only possess every knowledge relative to this science, which it may be within its immediate power to acquire, but it should promulgate its individual information; that the actuaries of the different societies may, by their mathematical skill, collect for the common good of all, from multiplied resources, that which they cannot obtain from a less general observation. I am induced to venture this hint, as it is my firm belief, that the tables generally adopted, might, by this means, receive many extremely necessary corrections; for those tables should be as accurate as they can possibly be made, and the interest should be calculated at that rate which shall appear to be the average interest to be made for money; but such additional demands should be made by the company or institution, as to leave an adequate portion for its security, profit, and expenses; for it does not seem possible, in the various beneficial applications which can be made from a proper knowledge of this branch of the mathematics, to judge universally how to adopt tables of mortality,. which are not correct in themselves, connected with a rate of interest which is not the average rate made in reality, so that the advantage may tend to any one direction. In granting assurances on lives, it is a practice to use a certain table of mortality, and to calculate at a certain rate of interest, without making any additional charge; in the presumption, that the tables are in themselves incorrect, but that their deviation from the truth, is in favour of the society; and that the interest of money is less than that which they can really make: but such a plan does not appear to me sufficiently scientific, to be followed by companies concerned with life contingencies generally. It is not my intention in the paper I have at present the honour to present to this profoundly learned Society, to offer my opinion on the rates of mortality; but my object is to propose a plan of analysis and notation, which I conceive may be applied with utility to most problems likely to occur, and capable of suggesting a variety of new speculations in the pursuit of this science. Many accurate and ingenious men have occupied their hours in the improvement of this subject. Messrs. MORGAN, BAILY, and MILNE, of the present age, are among the number of mathematicians to whom this department is greatly indebted; the works of the former two gentlemen have been long before the public; that of Mr. MILNE, was not earlier than about the commencement of this year in my possession; but I felt much gratified with the able manner in which he has treated his different subjects; and particularly pleased with his notation, in his sixth chapter, referring to the different orders of survivorship.

I feel thankful for the information I have received from the labours of those who have preceded me; and I hope that this sketch may be received as a wish to aid science, but not as a medium to censure those whose steps may sometimes have faltered in the paths to knowledge. To a true philosopher, it will ever be much more pleasing to grant even more praise than is actually due, than to pluck the laurel from the deserving brow. This is an observation which might frequently be addressed to authors, but I profess no particular point of application in my remark.

BENJAMIN GOMPERTZ.

14th June, 1820.

Section I. Art. I. A function, that is an expression made up of certain quantities, has been often very usefully expressed by some letter with those quantities written underneath, of which that function or expression is made up. And frequently those quantities only are placed under, which it may be the particular object to bring into notice; thus, if in an analysis we had the frequent occurrence of some particular expression, $\sqrt{a^2 + x^2 + \sqrt{x + y}}$, for instance, it might be convenient to put some letter to represent it; and if we had two or more expressions of the same form made up of different letters, such for instance as $\sqrt{a^2 + x^2 + \sqrt{x + y}}$, and $\sqrt{a^2 + z^2 + \sqrt{z + w}}$, it may be more convenient to express them by some generic character, which shall still involve the peculiarity of each; thus, by writing for the one $M_{x,y}$, and for the other $M_{z,w}$, and this or a similar mode of notation; becomes more necessary when we are ignorant of the form of the expressions to which our analysis is to be applied.

Art. 2. If for x in the expression M_x signifying some function of x, we write separately $x = n$, $x = n + p$, $x = n + 2p$, $x = n + 3p$, &c. x increasing by the continual addition of p; then the sum of the terms commencing with $x = n$ and finishing with $x = m$, is $M_n + M_{n+p} + M_{n+2p}$ &c. . . . M_m; and to express this operation on x, I use the symbol $\begin{array}{c|} x \\ \hline p \\ n \\ m \end{array}$ prefixed to the function of x; that is, I should write $\begin{array}{c|} x \\ \hline p \\ n \\ m \end{array} M_x$ for this sum. This is the same as what is called the finite integral of

M_{x+p}; from $x=n$ to $x=m$; that is, the first value of the increment M_{x+p} of the series being M_{n+p}, and the last M_{m+p}, or which is the same thing, the first term of the series being M_n and the last term of the series M_m. And thus would

$\overset{x}{\overline{\substack{p\\ n\\ m}}}\Big| M_{\epsilon+x}$ express $M_{n+\epsilon}+M_{n+\epsilon+p}+M_{n+\epsilon+2p}\ldots\ldots M_{m+\epsilon}$,

and also would $\overset{x}{\overline{\substack{p\\ n+\epsilon\\ m+\epsilon}}}\Big| M_x = M_{n+\epsilon}+M_{n+\epsilon+p}+M_{n+\epsilon+2p}\ldots\ldots$ $M_{m+\epsilon}$ as is evident by writing $n+\epsilon$ and $m+\epsilon$ respectively in the room of n and m in the first of the series mentioned; consequently, $\overset{x}{\overline{\substack{p\\ n\\ m}}}\Big| M_{\epsilon+x} = \overset{x}{\overline{\substack{p\\ n+\epsilon\\ m+\epsilon}}}\Big| M_x$; they both being expressive of the sum of the same series. Moreover, because when x becomes n, $x+\epsilon$ becomes $n+\epsilon$, and when x becomes m, $x+\epsilon$ becomes $m+\epsilon$; therefore the symbols $\overset{x}{\overline{\substack{p\\ n\\ m}}}\Big|$ and $\overset{x+\epsilon}{\overline{\substack{p\\ n+\epsilon\\ m+\epsilon}}}\Big|$ when prefixed to the same function mean the same thing, that is

$$\overset{x}{\overline{\substack{p\\ n\\ m}}}\Big| M_x = \overset{x+\epsilon}{\overline{\substack{p\\ n+\epsilon\\ m+\epsilon}}}\Big| M_x.$$

Also because $\overset{x}{\overline{\substack{p\\ n\\ m}}}\Big| M_{p+x}$ is the symbol for $M_{n+p}+M_{n+2p}$ $+$ &c. .... M_{m+p}, and $\overset{x}{\overline{\substack{p\\ n\\ m}}}\Big| M_x$ is the symbol for M_n+M_{n+p} $+_{n+2p}+$ &c. ..... M_m, therefore $\overset{x}{\overline{\substack{p\\ n\\ m}}}\Big| M_x = M_n - M_{m+p} +$

$\begin{array}{c|}x\\ \hline p\\ n\\ m\end{array}\ M_{p+x}$, and also $= M_n + \begin{array}{c|}x\\ \hline p\\ n\\ m-p\end{array}\ M_{p+x}$ or $= \begin{array}{c|}x\\ \hline p\\ n-p\\ m-p\end{array}\ M_{p+x}$.
This last is also evident from above.

Art. 3. It may be proper to state that the symbol $\rceil$ is not necessarily written with the letters x, p, n, m; but that with other letters it will, *mutatis mutandis*, have a similar meaning: the first or highest letter within the symbol expressing the constant increment of the variable quantity above the symbol; the second, the value of that variable quantity at the commencement; and the last or lowest letter the value thereof at the end. If this letter be infinite, we omit its notation and write the symbol thus $\begin{array}{c|}x\\ \hline p\\ n\end{array}$. If p, or the first letter within be infinitely small, or o comparatively with finites, then the symbol will stand $\begin{array}{c|}x\\ \hline o\\ n\\ m\end{array}$ and will express the integral of a differential expression of x between the limits n and m of x, and will be the same as the fluent of a fluxional expression between the same limits, *mutatis mutandis.*

Art. 4. L_a, L_b, L_c, &c. are put to denote the number of persons living at the ages a, b, c, &c. in a given table of mortality, and $L_{a,\, b,\, c,\, \&c.}$ is put for an abbreviation of $L_a \times L_b \times L_c$, &c. also $L_{x\,:\,a,\, b,\, c,\, \&c.}$ is put for an abbreviation of $L_{a+x}, \times L_{b+x} \times L_{c+x}$; $L_{(x\,:\,a,\, b,\, c,\, \&c.)\, d,\, e,\, f,\, \&c.}$ for an abbreviation of $L_{a+x} \times L_{b+x} \times L_{c+x}$ &c. $\times L_d \times L_e \times L_f$ &c.

Hence the chances of nominated persons of the ages a, b, c, &c. living respectively, the times x, y, z, &c. are $\frac{L_{a+x}}{L_a}$, $\frac{L_{b+y}}{L_b}$,

$\frac{L_{c+z}}{L_c}$ &c. considered independently of each other; and the chance that the event shall take place conjointly, that is, that every one of all the events shall take place is $\frac{L_{a+x}}{L_a} \times \frac{L_{b+y}}{L_b} \times \frac{L_{c+z}}{L_c}$ &c. or $\frac{L_{a+y,\, b+y,\, c+z,\, \&c.}}{L_{a,\, b,\, c,\, \&c.}}$; and if y, z, &c. are each equal to x, the expression may be further abbreviated thus, $\frac{L_{x\,:\,a,\, b,\, c,\, \&c.}}{L_{a,\, b,\, c,\, \&c.}}$. Note, that one year is considered the unite of time.

And if r be the present worth of one pound due in one year certain, $r^{x} . \frac{L_{x\,:\,a,\, b,\, c,\, \&c.}}{L_{a,\, b,\, c,\, \&c.}}$ will be the present worth of one pound to be received in the time x in case nominated persons of the ages a, b, c, d, &c. should be all living at that time.

Corollary. Hence, $\left.\begin{array}{c} x \\ \overline{p} \\ n \\ m \end{array}\right| r^{n} . \frac{L_{x\,:\,a,\, b,\, c,\, \&c.}}{L_{n\,:\,a,\, b,\, c,\, \&c.}}$ will denote the present value of a periodic income of one pound, payable at equal intervals p of time, the first payment being to be made in the time n, and the last in the time m, on the contingency of the persons whose present ages are a, b, c, &c. being jointly living at those intervals; interest being reckoned at the rate per cent. which makes the present value of one pound certain due in one year equal to r; and if there be no danger of mistaking the variable radical, we omit it in the prefixed symbol, and would write the last expression $\left.\begin{array}{c} \overline{p} \\ n \\ m \end{array}\right| r^{x} . \frac{L_{x\,:\,a,\, b,\, c,\, \&c.}}{L_{n\,:\,a,\, b,\, c,\, \&c.}}$ and this we would write still shorter when a more developed expression is not necessary, thus $\left.\begin{array}{c} r \\ \smile \\ p \\ n \\ m \end{array}\right| \underline{a, b, c, \&c.}$ to signify as before. The present value of a periodic income of one pound payable on the joint lives of persons of the present ages a, b, c, &c. at equally distant intervals p, the first payment being due in the time n, and the last in the time m, the present value of one pound certain due in one year being r.

If there be not any character in the place of the r, in the last symbolic form, the interest is supposed to be the same as in some previous consideration, which shall be evident from the text; thus, if we had been conversing of an interest of 5 per cent. per annum, we should write $\begin{array}{c|l} \smile & \\ 1 & \underline{a, b, c,} \\ 1 & \end{array}$ to express the present value of an annuity of one pound (that is one pound payable yearly) on the joint lives of the present ages a, b, &c. the first payment to be made in one year, because in this case $p=1$, $n=1$, and m is infinite, or extends to the longest possible duration of life.

Art. 5. If the income is on the present ages a, b, c, &c. between the terms &c., on the contingency of v or more of them being living, we write as a symbol of its present value $\begin{array}{c|l} \overset{r}{\smile} & \\ p & \\ n & \underline{a, b, c, \&c.} \\ m & v \end{array}$, and according to this notation we should have in the following particular cases for the present value of the income $\begin{array}{c|l} \overset{r}{\smile} & \\ p & \\ n & \underline{a, b, c,} \\ m & 3 \end{array}$ the same as $\begin{array}{c|l} \overset{r}{\smile} & \\ p & \\ n & \underline{a, b, c\,;} \\ m & \end{array}$ $\begin{array}{c|l} \overset{r}{\smile} & \\ p & \\ n & \underline{a, b, c} \\ m & 2 \end{array}$, when the income for the term depends on two or more of the three persons whose present ages are a, b, c, being living; $\begin{array}{c|l} \overset{r}{\smile} & \\ p & \\ n & \underline{a, b, c} \\ m & 1 \end{array}$, when it depends on one or more of the three being living; that is, what is termed on the longest of the lives; and $\begin{array}{c|l} \overset{r}{\smile} & \\ p & \\ n & \underline{a, b, c} \\ m & 0 \end{array}$, when it depends on none, or any of them being living, and is therefore the income certain and quite independent of the ages a, b, c, and therefore this case may be written without those letters, thus $\begin{array}{c|l} r & \\ \smile & \\ p & \\ n & \underline{\quad} \\ m & o \end{array}$, and will be expressive of the present value of an income certain for the term, &c.; *o* inserted in the angle signifying that it does not involve any contingency; and this presents us with a ready notation for the purpose of expressing an income for a term, a rate of interest, and a periodic interval of payment given on any nominated contingency specified by any particular character, by placing that character in the angular point; thus if the contingency involved were designated by the character C, then the income for the term, rate of interest, &c. depending on that contingency, we should denote by $\begin{array}{c|l} r & \\ \smile & \\ p & \\ n & \underline{\quad} \\ m & C \end{array}$.

Art. 6. Moreover, as $\begin{array}{c|l} r & \\ \smile & \\ p & \\ n & \underline{\quad} \\ m & o \end{array}$ expresses the value of the income certain for the term, and $\begin{array}{c|l} r & \\ \smile & \\ p & \\ n & \underline{a,\, b,\, c,\, \&c.} \\ m & \nu \end{array}$ expresses the value of the income for the term on the contingency of ν or more of the persons of the present ages a, b, c, &c. being living; the excess of the former above the latter, will express the value of the income for the term on the contingency of $\mu - \nu + 1$, or more of the persons being dead; if μ be the number of persons in all: “ because it is certain that out of μ persons living at any “ time, there will after that time either be ν or more of them “living, or $\nu - 1$ or less of them living including *o*; but $\nu - 1$ “ or less of them being living, is the same as $\mu - \nu + 1$ or more

" of them dead; therefore the chance of ν of them being liv-
" ing + the chance of $\mu-\nu+1$ of them being dead, is equal
" to unity, that is certainty," and the said excess will be written $\begin{array}{c} r\\ \smile\\ p\\ n\\ m\end{array}\Big|\frac{\qquad}{0} - \begin{array}{c} r\\ \smile\\ p\\ n\\ m\end{array}\Big|\frac{a,\ b,\ c,\ \&c.}{\nu}$, and is expressive of the chance of π or more being dead, π being equal to $\mu-\nu+1$; or μ, that is, the number of persons in all being $=\pi+\nu-1$; and as the sign $\begin{array}{c} r\\ \smile\\ p\\ n\\ m\end{array}\Big|$ of summation and the interest indicated by r, the present value of one pound certain to be received at the years end, affect both contingencies alike, we may join those contingencies and use but one symbol, and we shall have $\begin{array}{c} r\\ \smile\\ p\\ n\\ m\end{array}\Big|\frac{-,\ a,\ b,\ c,\ \&c.}{0,\ \nu}$ for the value of the income for the term on the contingency, that out of $\pi+\nu-1$ persons, there shall be π or more of them dead.

Accordingly would $\begin{array}{c} r\\ \smile\\ p\\ n\\ m\end{array}\Big|\frac{-,\ a,\ b,\ c}{0,\ 3}$, $\begin{array}{c} r\\ \smile\\ p\\ n\\ m\end{array}\Big|\frac{-,\ a,\ b,\ c}{0,\ 2}$, and $\begin{array}{c} r\\ \smile\\ p\\ n\\ m\end{array}\Big|\frac{-,\ a,\ b,\ c}{0,\ 1}$ respectively signify the value of the income for the term on the contingency of one or more being dead, of two or three being dead, or of all being dead. And these besides being an appropriate abbreviation of the expressions, likewise indicate, that if from the value of the income certain for the term we subtract separately the value for the income for the term on the contingency of all three being living, on the contingency of two or more being living, and on the contingency of one or more being living, of three persons, we shall have respectively the value of the income for the term on the contingency of one or more being dead, two or more being dead, or of all

being dead of three persons. Again $\begin{matrix}\overbrace{p}^{r}\\ n\\ m\end{matrix}\left|\frac{a, b, c, \&c.}{\nu-\varrho}\right. - \begin{matrix}\overbrace{p}^{r}\\ n\\ m\end{matrix}\left|\frac{a, b, c, \&c.}{\nu}\right.$, or, abbreviated, $\begin{matrix}\overbrace{p}^{r}\\ n\\ m\end{matrix}\left|\frac{a, b, c, \&c. - a, b, c, \&c.}{\nu-\varrho \qquad\qquad \nu}\right.$ would express the value for the term of the income on the contingency of there not being more living than $\nu - 1$, and not less living than $\nu - \varrho$; and taking $\varrho = 1$, we have $\begin{matrix}\overbrace{p}^{r}\\ n\\ m\end{matrix}\left|\frac{a, b, c, \&c. - a, b, c, \&c.}{\nu-1 \qquad\qquad \nu}\right.$, the value on the contingency of there being precisely $\nu - 1$ of them living; and if there are μ lives in all, the same symbol will also express the value on the contingency of there being precisely $\mu - \nu + 1$ dead.

Art. 7. If the income for the term depended on ν, or more nominated persons of the present age a, b, c, &c. being living, on ν' or more of the present ages a', b', c', &c. on ν'' or more of the persons of the present ages a'', b'', c'', &c. being living, we should express its present value, thus

$$\begin{matrix}\overbrace{p}^{r}\\ n\\ m\end{matrix}\left|\frac{a, b, c, \&c. : a', b', c', \&c.\ a'', b'', c'', \&c.}{\nu \qquad\qquad \nu' \qquad\qquad \nu'', \&c.}\right..$$

If the income for the term depended on some contingency designated by C, provided there were ν or more living of the persons of the present ages a, b, c, &c. we should designate it thus $\begin{matrix}\overbrace{n}^{r}\\ p\\ m\end{matrix}\left|\frac{: a, b, c, \&c.}{c, \quad \nu}\right.$. And therefore, if there were μ persons of the ages a, b, c, &c. in conformity with our plan, we should denote by $\begin{matrix}\overbrace{p}^{r}\\ n\\ m\end{matrix}\left|\frac{- : a, b, c, \&c.}{\mathrm{C} \quad \nu}\right.$, the present value of the income on the contingency C, proved $\mu - \nu + 1$, or more of the persons of the present ages, a, b, c, &c. were dead.

Art. 8. If one pound is to be received at the expiration of the first of the equal periods p, after the expiration of the time $n-p$, which shall happen after the failure of any of the above conditions of joint or single existence of the persons in question, provided that that failure should take place between the intervals $n-p$ and m; the present value of this sum will be the present value discounted for the period p of the income on the contingency of the conditions not failing, the first payment to be made at the period $n-p$, and the last at the period $m-p$, less the present value of the income on the contingency of the conditions not failing, the first payment to be made in the time n, and the last in the time m. Thus suppose the income, payable at every interval p, of one pound on the contingency of certain lives remaining, the first payment to be made at the time n, and the last at the time m, be represented by $\begin{matrix}\overbrace{p}^{r}\\ n\\ m\end{matrix}\Bigg|\frac{\quad}{C}$, and therefore a similar regulated income of which the first payment is to be made in the time $n-p$, and the last in the time $m-p$, will be $\begin{matrix}\overbrace{\quad p\quad}^{r}\\ n-p\\ m-p\end{matrix}\Bigg|\frac{\quad}{C}$, and the present value of one pound to be received at the first period p which shall happen after the failure of the condition, &c. is

$$r^{p} \times \begin{matrix}\overbrace{\quad p\quad}^{r}\\ n-p\\ m-p\end{matrix}\Bigg|\frac{\quad}{C} - \begin{matrix}\overbrace{p}^{r}\\ n\\ m\end{matrix}\Bigg|\frac{\quad}{C}.$$

Because $\begin{matrix}\overbrace{p}^{r}\\ \pi\\ \pi\end{matrix}\Bigg|\frac{\quad}{C}$, is the value of one pound to be received at the period π in the contingency of the conditions not having failed, on or before that time; $\begin{matrix}\overbrace{p}^{r}\\ \pi\\ \pi\end{matrix}\Bigg|\frac{\quad}{C} \div r^{\pi}$ is the chance of its not having failed on or before that time;

in the same manner $\overbrace{\begin{smallmatrix}p\\ n-p\\ m-p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}\div r^{\pi-p}$ is the chance of its not having failed on or before the time $\pi-p$; and the excess of the latter above the former, is the chance of its failing between the intervals $\pi-p$ and π, which multiplied by r^{π} gives $\overbrace{\begin{smallmatrix}p\\ \pi-p\\ \pi-p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}\times r^{p}-\overbrace{\begin{smallmatrix}p\\ \pi\\ \pi\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}$, for the present value of one pound, to be received at the expiration of the time π, in case the condition should fail between the time $\pi-p$ and π; and if π be successively interpreted by $n, n+p$; $n+2p$, &c. m, the sum of the resulting expressions, will be the value of one pound to be received at the first of the periods p from the time $n-p$ that shall happen after the failure of the condition, provided that that failure takes place between the periods $n-p$ and m; but because $\overbrace{\begin{smallmatrix}p\\ n\\ m\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}=\overbrace{\begin{smallmatrix}p\\ n\\ m\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}+\overbrace{\begin{smallmatrix}p\\ n+p\\ m+p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}+\overbrace{\begin{smallmatrix}p\\ n+2p\\ n+2y\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}+\text{\&c.}\ldots\overbrace{\begin{smallmatrix}p\\ m\\ m\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}$, and similarly $\overbrace{\begin{smallmatrix}p\\ n-p\\ m-p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}=\overbrace{\begin{smallmatrix}p\\ n-p\\ n-p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}+\overbrace{\begin{smallmatrix}p\\ n\\ n\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}+\overbrace{\begin{smallmatrix}p\\ n+p\\ n+p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}+\text{\&c.}\ldots\overbrace{\begin{smallmatrix}p\\ m-p\\ m-p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}$; therefore the said value is equal to $\overbrace{\begin{smallmatrix}p\\ n-p\\ m-p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}\times r^{p}-\overbrace{\begin{smallmatrix}p\\ n\\ m\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}$ as above asserted.

From the two equations first cited, it appears that $\overbrace{\begin{smallmatrix}p\\ n-p\\ m-p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}=\overbrace{\begin{smallmatrix}p\\ n\\ m\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}+\overbrace{\begin{smallmatrix}p\\ n-p\\ n-p\end{smallmatrix}}^{r}\Big|\frac{\quad}{C}-\overbrace{\begin{smallmatrix}p\\ m\\ m\end{smallmatrix}}^{r}\Big|$; the value of the above contingency therefore becomes $(r^p-1)\,.\,\begin{array}{r|l}\overbrace{\;p\;}^{r}&\\ n&\overline{\qquad}\\ n&C\end{array}+\left\{\begin{array}{r|l}\overbrace{\;\;p\;\;}^{r}&\\ n-p&\overline{\qquad}\\ n-p&C\end{array}-\begin{array}{r|l}\overbrace{\;p\;}^{r}&\\ m&\overline{\qquad}\\ m&C\end{array}\right\}.\,r^p.$

If we look to the expression $\begin{array}{r|l}\overbrace{\;\;p\;\;}^{r}&\\ n-p&\overline{\qquad}\\ m-p&C\end{array}\times r^p-\begin{array}{r|l}\overbrace{\;p\;}^{r}&\\ n&\overline{\qquad}\\ m&C\end{array}$, we shall discover that if the increment of some variable quantity x be equal to p, and that be the only variable quantity concerned in the functions $\begin{array}{r|l}\overbrace{\;\;p\;\;}^{r}&\\ n-x&\overline{\qquad}\\ m-x&C\end{array}\times r^x$, that our quantity, the value of the assurance, will the increment thereof after x be made equal to *o*, and the assurance, or the increment of $\begin{array}{r|l}\overbrace{\;\;p\;\;}^{r}&\\ m-x&\overline{\qquad}\\ n-x&c\end{array}\times r^x$ thus modified, I shall denote by $\underline{\begin{array}{r|l}\overbrace{\;p\;}^{r}&\\ n&\overline{\qquad}\\ m&C\end{array}}$. And in conformity with this notation, I should write $\begin{array}{r|l}\overbrace{\;p\;}^{r}&\\ n&\underline{a,b,c,}\\ \underline{m}&\end{array}$ &c. for the assurance of one pound to be received at the first of the equal periods p from the time $n-p$ which shall happen after the time $n-p$, and not after the expiration of the time m; after the extinction of the joint lives of the persons now aged a, b, c, &c. also by $\begin{array}{r|l}\overbrace{\;p\;}^{r}&\\ n&\underline{a,\,b,\,c,\,\&c.}\\ \underline{m}&\end{array}$. I should denote the present value of the assurance on similar conditions with respect to the failure of the last v survivors, &c. When m is infinite, or is supposed to be the greatest possible limit, we, in conformity with the notation hitherto used, would write those expressions respectively $\substack{\overset{r}{\smile}\\ p\\ n}\underline{\Big|}\frac{\quad}{\mathrm{C}}$, $\substack{\overset{r}{\smile}\\ p\\ n}\underline{\Big|}\frac{a,\,b,\,c,\,\&c.}{\quad}$ $\substack{\overset{r}{\smile}\\ p\\ n}\underline{\Big|}\frac{a,\,b,\,c,\,\&c.}{,}$, when $p=o$, and the time m is not the utmost limit, they will stand $\substack{\overset{r}{\smile}\\ o\\ n\\ m}\underline{\Big|}\frac{\quad}{\mathrm{C}}$ $\substack{\overset{r}{\smile}\\ o\\ n\\ m}\underline{\Big|}\frac{a,\,b,\,c,\,\&c.}{\quad}$ $\substack{\overset{r}{\smile}\\ o\\ n\\ m}\underline{\Big|}\frac{a,\,b,\,c,\,\&c.}{,}$: and when m is the utmost limit, and $p=o$, they will stand $\substack{\overset{r}{\smile}\\ o\\ n}\underline{\Big|}\frac{\quad}{\mathrm{C}}$ $\substack{\overset{r}{\smile}\\ o\\ n}\underline{\Big|}\frac{a,\,b,\,c,\,\&c.}{\quad}$, &c. These denote the assurances of one pound to be received after the failure takes place; and if they be multiplied by r^p, they will denote the value of the assurance, if the money be to be received at the time p after the event takes place, whenever that may happen between the times n and m; and this is *properly what should be called the assurance* of the sums, though writers call what I denote by $\substack{\overset{r}{\smile}\\ p\\ n\\ m}\underline{\Big|}\frac{\quad}{\mathrm{C}}$, $\substack{\overset{r}{\smile}\\ p\\ n\\ m}\Big|\frac{a,\,b,\,c,\,\&c.}{\quad}$, &c. the assurance. See Scholium.

Art. 10. It is often necessary in calculating from tables, to have a method of interpolation for the discovery of terms not explicitly contained in the tables; if, for instance, we had the function $\mathrm{M}_{x,\,y,\,z,\,\&c.}$ calculated in a table, for values of x, y, z, &c. taken in as many series proceeding in arithmetical progression, according to certain scales of differences, and we wish to have the value of the function, when some only, or neither of them are in those series. And for this purpose the method of finite differences, when the differences converge, is applied with great advantage, as is well known, By the prefixed symbols $\underset{x\,\iota}{\Delta}$, $\underset{x\,\iota}{\Delta^2}$, $\underset{x\,\iota}{\Delta^3}$, &c., I mean the first, second, and third differences of the function to which it is prefixed, arising from the series formed by writing x, $x+\iota$, $x+2\iota$, $x+3\iota$, &c. in the place of x in that function. Hence from the method of differences we have

$$\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a+\alpha} = \left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a\;} + \frac{\alpha}{\iota}\cdot\underset{a\,\iota}{\Delta}\cdot\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a\;} + \frac{\alpha}{\iota}\times\frac{\frac{\alpha}{\iota}-1}{2}\cdot\underset{a\,\iota}{\Delta^2}\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a\;} + \text{\&c.}$$

$$\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a+\alpha,\, b,\, c,\, \text{\&c.}} = \left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a,\, b.\, c,\, \text{\&c.}} + \frac{\alpha}{\iota}\underset{a\,\iota}{\Delta}\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a,\, b,\, c,\, \text{\&c.}} + \frac{\alpha}{\iota}\frac{\frac{\alpha}{\iota}-1}{2}\underset{a\,\iota}{\Delta^2}\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a,\, b,\, c,\, \text{\&c.}} + \text{\&c.}$$

$$\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a+\alpha, b+\beta, c, \text{\&c.}} = \left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a,\, b,\, c,\, \text{\&c.}} + \frac{\alpha}{\iota}\underset{a\,\iota}{\Delta}\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{\text{[illegible]}\, b,\, c,\, \text{\&c.}} + \frac{\alpha}{\iota}\cdot\frac{\frac{\alpha}{\iota}-1}{2}\underset{a\,\iota}{\Delta^2}\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a,\, b,\, c,\, \text{\&c.}}\,,\ \text{\&c.}$$

$$+\frac{\beta}{\varpi}\underset{a\,\varpi}{\Delta}\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a,\, b,\, c,\, \text{\&c.}} + \frac{\alpha}{\iota}\cdot\frac{\beta}{\varpi}\underset{a\,\iota\; b\,\varpi}{\Delta\Delta}\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a,\, b,\, c,\, \text{\&c.}}\,,\ \text{\&c.}$$

$$+\frac{\beta}{\varpi}\frac{\frac{\beta}{\varpi}-1}{2}\underset{b\,\varpi}{\Delta^2}\cdot\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a,\, b,\, c,\, \text{\&c.}}\,,\ \text{\&c.}$$

And in a similar manner are other expressions written. It may be often of advantage, where the text is sufficiently explicit, to use the differencing symbols and omit the others, leaving them to be understood, and to write, for instance,

$$\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a+\alpha,\, b+\beta,\, c,\, \text{\&c.}} =$$

$$\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{a,\, b,\, c,} + \frac{\alpha}{\iota}\underset{a\,\iota}{\Delta}: + \frac{\alpha}{\iota}\cdot\frac{\frac{\alpha}{\iota}-1}{2}\underset{a\,\iota}{\Delta}: + \text{\&c.}$$

$$+\frac{\beta}{\varpi}\underset{b\,\varpi}{\Delta}: \frac{\alpha}{\iota}\cdot\frac{\beta}{\varpi}\underset{a\,\iota\; b\,\varpi}{\Delta\Delta}: + \text{\&c.}$$

$$\frac{\beta}{\varpi}\cdot\frac{\frac{\beta}{\varpi}-1}{2}\underset{b\,\varpi}{\Delta}: + \text{\&c.}$$

Section II. Art. 1. If there be any function M_x, which decreases as x increases, and for x we write $x+n$, the function will be transformed into M_{x+n}; and may be developed into $M_x - n M'_x + n^2 M''_x + n^3 M'''_x$, &c. where M'_x is positive; and if m be taken sufficiently small we shall have $M_{x+n} = M_x - nM'_x$ sufficiently near. And nM'_x will be nearly the decrement produced in M_x by writing $x+n$ for x. And this *approximative* decrement is proportional to n. Hence it appears, that the number of persons living in any table of mortality indicating the number of persons living at every possible age, that is to any fraction of a year or unity of time, the intervals of age may be taken so small, that whatever the law of mortality may be, during any portion of any the same interval, the decrements may be considered proportional to the time. Observation informs us, that this proportionality of decrement may be admitted as affording a tolerable degree of accuracy during very long intervals, and in that respect, it gives us some idea of the nature of the function of mortality; but independently of observations from known results, we see that we may approximate to any degree of accuracy contained in the tables, by dividing long intervals into shorter intervals, and taking, whatever may be the functions of mortality or of living, the decrements proportional to the portions of time between the separate intervals, and thus if we wished to find the value of $\begin{matrix} r \\ \smile \\ 1 \\ n \\ m \end{matrix}\left|\underline{\,a, b, c, \&c.\,}\right.$, that is the value of an annuity of one pound on the joint lives whose present [illegible]es are a, b, c, &c.; the first payment to be made in the time n, and the last in the time m, at a rate per cent. indicated by the present worth r of one pound to be received, certain in one one year; because $\overbrace{\substack{1\\n\\m}}^{r}\Big|\underline{a, b, c, \&c.} = (r^n . L_{n:a,b,c,\&c.} + r^{n+1} . L_{n+1:a,b,c,\&c.} + \&c. \ldots\ldots\ldots r^m L_{m:a,b,c,\&c.})$ divided by $L_{a,b,c,\&c.}$, it follows that if the interval be taken sufficiently small, the decrements being considered proportional to the time, and ϱ be taken not greater than $m-n$, we shall have $r^{n+\varrho} . L_{n+\varrho:a,b,c,} = r^{n+\varrho} \times \overline{L_{n+a} - \varrho L'_{n+a}} \times \overline{L_{b+n} - \varrho L'_{b+n}} \times \overline{L_{c+n} - \varrho . L'_{c+n}} \times \&c.$, by using, *mutatis mutandis*, the notation above, and this is $= r^{n+\varrho} . (L_{n:a,b,c,\&c.} - \varrho\, (L'_{n+a} . L_{n:b,c,\&c.} + L'_{n+b} . L_{n:a,c,\&c.} + \&c.) + \varrho^2 . (L'_{n+a} . L'_{n+b} . L_{n:c,\&c.} + \&c.) - \&c.) = r^{n+\varrho} . L_{n:a,b,c,\&c.} \times (A_n \varrho + B_n \varrho^2 - \&c.)$; A_n, B_n, &c. being put for the coefficients of the different powers of ϱ, are constant during the interval of uniform decrement, and consequently, by writing 0, 1, 2, 3, &c. for ϱ, we see how we may obtain the value of annuities approximately for portions of time sufficiently small, and that we therefore have

$$\overbrace{\substack{1\\n\\m}}^{r}\Big|\underline{a, b, c, \&c.} = r^n . \frac{L_{n:a,b,c,\&c.}}{L_{a,b,c,\&c.}} \times \left\{\begin{array}{lll} 1 & & \\ r & -A_n r & +B_n . r - \&c. \\ r^2 & 2r^2 & 4r^2 \\ r^3 & 3r^3 & 9r^3 \\ \&c. & \&c. & \&c. \\ r^{n-m} & \overline{n-m} . r^{n-m} & \overline{n-m}\; r^{n-m} \end{array}\right\}$$

and if T_{n-m} be put $= 1 + r + r^2 \ldots\ldots r^{n-m}$, $T'_{n-m} = r + 2r^2 + 3r^3 + 4r^4 \ldots\ldots \overline{n-m} . r^{n-m}$, $T'' = r + 4r^2 + 9r^3 \ldots\ldots \overline{n-m}|^2 . r^{n-m}$, &c., numerical values of which may be arranged in a small table, we shall have $\overbrace{\substack{1\\n\\m}}^{r}\Big|\underline{a, b, c, \&c.} = r^n . \frac{L_{n:a,b,c,\&c.}}{L_{a,b,c,}} \times (T_{n-m} - A_n T_{n-m} + B_n T''_{n-m} - \&c.)$ and by repetition and addition we may obtain $\overbrace{\substack{1\\n}}^{r}\Big|\underline{a, b, c, \&c.}$, that is the value of the annuity on the whole possible joint existence: and where there are many lives concerned, perhaps this may answer with as little trouble as by the common methods by the limited tables; and any accuracy may be obtained that the tables of mortality will afford, by using sufficiently small intervals; which is not the case when the common method is used, of searching in the tables of two joint lives, and then of a single life, and then of another life with this single life, &c. till we have comprehended the whole number of lives.

Art. 2. Moreover any functions M_{x+m}, may be developed into the form $M_x . \pi^{mM'_x + m^2 M''_x + \&c.}$ and, consequently, if n be sufficiently small to admit of all powers of n above the first being omitted, we shall have $M_{x+m} = M_x . \pi^{mM'_x}$; and if this remark be applied to the function L_{a+n} of the living at the age $a+n$, we see that we may take an interval m, from n so small, whatever be the real constitution of the function, that the number of the living, during that interval, shall decrease so as to form a geometrical progression very nearly, whilst the portions of time increase in an arithmetical progression; and that the decrements of living are also in consequence very nearly in geometrical progression: and we may moreover in this, as in the former case, accommodate the function so as to be accurate at the two extremes. Thus, for instance, if we wish to have an *approximative* expression in a geometrical progression, for the number of living for ages from 20 to 30 years, which shall be exact for the age 20 and 30, according to the Northampton tables of mortality. Assume π any convenient number at pleasure, say 10, take $a = 20$, and $a + n = 30$; $\therefore n = 10$, and we have $L'_a = L'_{20} = \frac{\text{Log. of } L_{30} - \text{Log. of } L_{20}}{10} = -{,}0068317$ according to the Northampton tables. And when n is any thing from 0 to 10, $L_{20+n} = L_{20} . 10^{-.0068317n}$, and the logarithm of the numbers of living at the age $20+n = 3{,}7102866$ $-006317n$. Hence we have respectively

Living at the ages	20	21	22	23	24	25	26	27	28	29	30
According to the Northampton Table	5132	5060	4985	4910	4835	4760	4685	4610	4535	4460	4385
Ditto, Geometrical progression - -	5132	5051	4972	4895	4819	4744	4670	9457	4525	4454	4385
Deficiency of the Geometrical progression	0	9	13	15	16	16	15	13	10	6	0

Art. 3. Hence, if we consider $L_{x+\tau} = L_x . \overline{10}|^{\tau . L'_x}$, whilst τ is not greater than $m-n$ as a sufficient approximation; and we wish to find the value of $\overset{\overset{r}{\smile}}{\overset{p}{\underset{m}{n}}}\Big|\underline{a, b, c, \&c.}$ we have by writing ϱ for the common logarithm of r, and putting $\varrho + L'_{a+n} + L'_{b+n}$ $+ L'_{c+n} + \&c. = \mu$, $\overset{\overset{r}{\smile}}{\overset{p}{\underset{m}{n}}}\Big|\underline{a, b, c, \&c.} = \overline{10}|^{n\varrho} \frac{L_{n:a,b,c,\&c.}}{L_{a,b,c,\&c.}} \times (1 + \overline{10}|^{p\mu}$ $+ \overline{10}|^{2p\mu} + \overline{10}|^{3p\mu} + \ldots\ldots \overline{10}|^{\overline{m-n}\,p\mu}) = \overline{10}|^{n\varrho} . \frac{L_{n:a,b,c,\&c.}}{L_{a,b,c,\&c.}} \times$ $\frac{1-10^{(m-n+p)\mu}}{1-10^{p\mu}}$; and by restorations this may be written

$$r^n \frac{L_{n:a,b,c,\&c.}}{L_{a,b,c,\&c.}} \times \frac{1 - r^{m-n+p} \frac{L_{m+p:a,b,c,\&c.}}{L_{n:a,b,c,\&c.}}}{1 - r^p . \frac{L_{n+p:a.b,c,\&c.}}{L_{n:a,b,c,\&c.}}}$$

Art. 4. If the number of living, corresponding to times in arithmetical progression, form a series in geometrical progression, we should have

$$\overset{\overset{r}{\smile}}{\underset{m}{\overset{p}{p}}}\Big|_{\underline{a,b}} = r^p . \frac{L_{p:a,b}}{L_{a,b}} + \left(r^p . \frac{L_{p:a,b}}{L_{a,b}}\right)^2 + \&c. \ldots\ldots \left(r^p . \frac{L_{p:a,b}}{L_{a,\text{[illegible]}}}\right)^m;$$

$$\overset{\overset{r}{\smile}}{\underset{m}{\overset{p}{p}}}\Big|_{\underline{a,b,c}} = r^p . \frac{L_{p:a,b,c}}{L_{a,b}} + r^p . \frac{L_{p:a,b,c}}{L_{a,b}} + \&c. \ldots\ldots \left(r^p . \frac{L_{p:a,b,c}}{L_{a,b,c}}\right)^m;$$

and also

$$\overset{\overset{r}{\smile}}{\underset{m}{\overset{p}{p}}}\Big|_{\underline{s}} = r^p . \frac{L_{p+s}}{L_s} + \left(r^p . \frac{L_{p+s}}{L_s}\right)^2 + \&c. \ldots\ldots \left(r^p . \frac{L_{p+s}}{L_2}\right)^m$$

Hence, if $\frac{L_{s+p}}{L_s} = \frac{L_{p:a,b}}{L_{a,b}}$, then $\overset{\overset{r}{\smile}}{\underset{m}{\overset{p}{p}}}\Big|_{\underline{s\quad}} = \overset{\overset{r}{\smile}}{\underset{m}{\overset{p}{p}}}\Big|_{\underline{a,b}}$; and also $\overset{\overset{r}{\smile}}{\underset{n}{\overset{p}{p}}}\Big|_{\underline{a,b,c}} = \overset{\overset{r}{\smile}}{\underset{m}{\overset{p}{p}}}\Big|_{\underline{s,c}}$; that is, according to this hypothesis, if the value of a periodic income of one pound to be paid at the expiration at every interval *p*, for a given number of intervals, on two joint lives, whose present ages are *a*, *b*, be equal to a similar income on the single life, whose present age is *s*, then the similar income on the three joint lives of the present ages *a*, *b*, *c*, will be equal to a similar income on the lives whose present ages are *s* and *c*. And it also follows, that a similar reduction may be made for more lives, and their value thus obtained according to the hypothesis. And this, I imagine, is the foundation of the practice for determining the value of annuities, on many joint lives, from two tables, of which the one is on single lives, and the other on two joint lives; and we discover, that if the same geometrical progression, or a proximity thereto, does not continue through the whole, or the certain portion in question of the lives, this method, which may be considered the general practice, must often lead to error.

It may be worth observing, that if L_{a+n}, L_{b+n}, L_{c+n}, &c. were respectively equal to $L_a . \pi^{nL'_a}$, $L_b . \pi^{nL'_b}$, $L_c \pi^{nL'_c}$, &c. whatever positive value n might be, that is, if the living from the respective ages a, b, c, &c., whilst the time increased in arithmetical progression form series in a geometrical progression, then would the present value of the periodic income on the joint lives of the ages a, b, c, &c. be the same for the same term, as on ages older than those by any number of years, either the same for each, or different. And hence we may have some reason to suspect that the value of annuities given by tables on old age, by assuming a necessary term to life, as is done in the adopted tables of mortality, is likely to be far from the truth.

Art. 5. As to the calculations of the values of $\left.\begin{array}{c} r \\ \smile \\ p \\ n \\ m \end{array}\right| \begin{array}{l} \underline{a, b, c, \&c.} \\ v \end{array}$, $\left.\begin{array}{c} r \\ \smile \\ p \\ n \\ m \end{array}\right| \begin{array}{ll} \underline{a, b, c, \&c. :} & \underline{a', b', c', \&c. : \&c.} \\ v & v' \end{array}$. See Art. 6 and 7. Sect. 1, and other expressions therein contained. Besides the usual mode of reducing them to their equivalents, a number of combinations of joint lives, for the purpose of working from calculated tables, which, when the lives are many, will be a laborious task, and subject to the errors of numerous interpolations and of the other approximative modes which will be necessary in many cases when the lives are numerous, it may be easier to work them directly without such reduction, or by reducing the terms to geometrical progressions for short periods, &c. and for all these purposes it is necessary to have the value of the contingencies referring to each payment. And in order to this, for the sake of brevity, let $E_{a,n}$ whatever a and n may be, represent the chance that a person of the age a, shall be living at the expiration of the time n, and $D_{a,n}$ be the chance of his being dead at the expiration of the time n. And consequently, $E_{a,n} + D_{a,n} = 1$; also $F_{a,n} = \frac{L_{a+n}}{L_a}$, and $D_{a,n} = 1 - \frac{L_{a+n}}{L_a}$. Moreover, if we introduce any letter x as a multiplier of $E_{a,n}$, if x be equal to unity, since $E_{a,n} = x \,.\, E_{a,n}$, we may under that idea of x being = unity, write $x \,.\, E_{a,n} + D_{a,n} = 1$, $x\,E_{a,n} + D_{c,n} = 1$, &c.; and also $(x\,E_{a,n} + D_{a,n}) \times (x\,E_{b,n} + D_{b,n}) \times (\overline{x\,E_{c,n} + D_{c,n}}) +$ &c. $= 1$; and if the left hand side be multiplied out at length, and there be μ persons or μ multipliers, then the coefficient of x^{μ} will be the chance that all the μ persons shall be living, the coefficient of $x^{\mu-1}$ will be the chance that there shall be $\mu-1$ and no more living, the coefficient of $x^{\mu-2}$ will be the chance that there shall be $\mu-2$ and no more living, and generally the coefficient of $x^{\mu-\pi}$ the chance that there shall be exactly $\mu-\pi$ of them living, and the sum of all the coefficients from that of x^{μ} to that of $x^{\mu-\pi}$ both included, will be the chance that there shall be $\mu-\pi$ or more of them living. If we write $1-E_{n,a}$ for $D_{a,n}$ our equation will stand $\overline{\overline{x-1}\,.\,E_{a,n}+1} \times \overline{\overline{x-1}\,.\,E_{b,n}+1} \times \overline{\overline{x-1}\,.\,E_{c,n}+1}$, &c. to μ terms $= 1$, x being supposed equal to unity. This is an identical equation, and if for the sake of brevity we put $P_{\mu,0} =$

the product of all the μ terms $E_{a,n}$, $E_{b,n}$, $E_{c,n}$, &c., $P_{\mu-1,1}=$ the sum of the products of every $\mu-1$ terms, $P_{\mu-2,2}$ the sum of the product of $\mu-2$ terms, &c. the equation will stand $\overline{x-1}|^{\mu}.P_{\mu,0}+\overline{x-1}|^{\mu-1}.P_{\mu-1,1}+\overline{x-1}|^{\mu-2}.P_{\mu-2,2}\&c.=1$, when $x=1$; that is

$$x^{\mu}P_{\mu,0}-\mu.x^{\mu-1}.P_{\mu,0}+\mu.\frac{\mu-1}{2}.x^{\mu-2}.P_{\mu,0}-\mu.\frac{\mu-1}{2}.\frac{\mu-2}{3}.x^{\mu-3}.P_{\mu,0},\&c.$$

$$+x^{\mu-1}.P_{\mu-1,1}-\frac{\mu-1}{1}.x^{\mu-2}.P_{\mu-1,1}+\frac{\mu-1}{1}.\frac{\mu-2}{2}.x^{\mu-3}P_{\mu-1,1},\&c.$$

$$+x^{\mu-2}P_{\mu-2,2}-\frac{\mu-2}{1}x^{\mu-3}P_{\mu-2,2},\&c.$$

$$+x^{\mu-3}P_{\mu-3,3}-\&c.$$

And the coefficient of $x^{\mu-\pi}$ or the chance of their being at the expiration of the time,

Just $\mu-\pi$ living is $P_{\mu-\pi,\pi}-\frac{\mu-\pi+1}{1}.P_{\mu-\pi+1,\pi-1}+\frac{\mu-\pi+2}{1},\frac{\mu-\pi+1}{2}.P_{\mu-\pi+2,\pi-2}-\&c.$
Ditto $\mu-\pi+1$. is $+\ P_{\mu-\pi+1,\pi-1}\quad-.\frac{\mu-\pi+2}{1}.P_{\mu-\pi+2,\pi-2}\&c.$
Ditto $\mu-\pi+2$. is - - - - - $P_{\mu-\pi+2,\pi-2}\&c.$
&c. &c.

and the sum of these, or the chance that there shall be $\mu-\pi$ or more living, is $=P_{\mu-\pi,\pi}-\overline{\mu-\pi}.P_{\mu-\pi+1,\pi-1}+\overline{\mu-\pi}.\frac{\mu-\pi+1}{2}.P_{\mu-\pi+2,\pi-2}-\&c.$ Similarly if x, x', x'', &c. be each unity, we have the identical equations $\overline{\overline{x-1}.E_{a,n}+1}\times\overline{\overline{x-1}.E_{b,n}+1}\times\overline{\overline{x-1}\,E_{c,n}+1}$, &c. $\times\overline{\overline{x'-1}.E_{a',n}+1}\times\overline{\overline{x'-1}.E_{b',n}+1}\times\overline{\overline{x'-1}.E_{c',n}+1}\,\&c.\times\overline{\overline{x''-1}.E_{a'',n}+1}\times\overline{\overline{x''-1}.E_{b'',n}+1}\,\&c.=1$, and the coefficient of $x^{\pi}\times x'^{\pi'}\times x''^{\pi''}$ will be the chance of there being exactly π living of the first set, π' living of the second set, π'' living of the third set, &c. We remark by the bye, that if a, b, c, &c. be equal to each other, then the equation $\overline{\overline{x-1}.E_{a,n}+1}\times\overline{\overline{x-1}.E_{b,n}+1}\times\&c.$ to μ terms $=1$, will be $\overline{\overline{x-1}.E_{a,n}+1}|^{\mu}=1$, and that $\overline{\overline{x-1}.E_{a,n}+1}|^{\mu}$ being

$= \overline{x-1}\rceil^{\mu} E^{\mu}_{a,n} + \mu . \overline{x-1}\rceil^{\mu-1} E^{\mu-1}_{a,n} + \mu . \frac{\mu-1}{2} . \overline{x-1}\rceil^{\mu-2} . E^{\mu-2}$ &c. therefore in this case $P_{\mu,o} = E^{\mu}_{a,n}$, $P_{\mu-1,1} = \mu . E^{\mu-1}_{a,n}$, $P_{\mu-2,2} = \mu . \frac{\mu-1}{2} E^{\mu-2}_{a,n}$, &c.

SECTION 3, being necessary Lemmata.

Art. 1, on the fluent of $L_{q+x} . \dot{L}_{r+x}$.

Suppose that between the intervals $x=n$ and $x=n+m$, the decrements of life of persons arrived at the ages $q+n$, $r+n$, each in proportion to the time elapsed from the commencement of the interval, or that they may be considered so with sufficient accuracy. See Sect. II. Art. 1. Put $x=n+t$, L_{q+x}, or $L_{q+n+t} = L_{q+n} - tL'_{q+n}$, $L_{r+x} = L_{r+n} - tL'_{r+n}$; and the fluent of $L_{q+x} . \dot{L}_{r+x}$ will be $=$ fluent of $(-L'_{r+n}\dot{t} \times \overline{L_{q+n} - tL'_{q+n}})$ $=$ correction $- tL'_{r+n} . L_{q+n} + \frac{t^2}{2} . L'_{r+n} . L'_{q+n}$; and as $\overset{x}{\overline{\begin{smallmatrix}o\\o\\n\end{smallmatrix}}}\Big| L_{q+y} . \dot{L}_{r+x}$ (Art. 3. Sect. I.) expresses the fluent of $L_{r+x} . \dot{L}_{r+x}$ from $x=o$ to $x=n$, this will be the correction, if the fluent be supposed to commence from $x=o$, and our fluent that is $\overset{x}{\overline{\begin{smallmatrix}o\\o\\n+t\end{smallmatrix}}}\Big| L_{q+x} . \dot{L}_{r+x}$ will stand $= \overset{x}{\overline{\begin{smallmatrix}o\\o\\n\end{smallmatrix}}}\Big| L_{q+x} . \dot{L}_{r+x} - tL'_{r+n} . L_{q+n} + \frac{t^2}{2} . L'_{r+n} . L'_{q+n} = \overset{x}{\overline{\begin{smallmatrix}o\\o\\n\end{smallmatrix}}}\Big| L_{q+x} . L_{r+x} - tL'_{r+n} . (L_{q+n} - \frac{1}{2} t'L_{q+n})$; but because according to hypothesis $tL'_{q+n} = L_{q+n} - L_{q+n+t}$; $L_{q+n+\frac{1}{2}t} = L_{q+n} - \frac{1}{2} tL'_{q+n}$; and also $tL'_{r+n} = L_{r+n} - L_{r+n+t}$; therefore $\overset{x}{\overline{\begin{smallmatrix}o\\o\\n+t\end{smallmatrix}}}\Big| L_{q+x} . \dot{L}_{r+x}$ the fluent in question may be written in either of the two forms $\overset{x}{\begin{smallmatrix}o\\o\\n\end{smallmatrix}}\Big| L_{q+x} . \dot{L}_{r\times x} - \frac{1}{2}(L_{r+n} - L_{r+n+t}) \times (L_{q+n} + L_{q+n+t})$, or $\overset{r}{\begin{smallmatrix}o\\o\\n\end{smallmatrix}}\Big| L_{q+x} . \dot{L}_{r+x} - (L_{r+n} - L_{r+n+t}) \times L_{q+n+\frac{1}{2}t}$, that is by reduction in either of the two forms $\overset{r}{\begin{smallmatrix}o\\o\\n\end{smallmatrix}}\Big| L_{q+x} . \dot{L}_{r+x} - \frac{1}{2} L_{r+n} . L_{q+n} + \frac{1}{2} L_{q+n} . L_{r+n+t} - \frac{1}{2} L_{q+n+t} . L_{r+n} + \frac{1}{2} L_{r+n+t} . L_{q+n+t}$ or $\overset{x}{\begin{smallmatrix}o\\o\\n\end{smallmatrix}}\Big| L_{q+x} : \dot{L}_{r+x} - L_{r+n} . L_{q+n+\frac{1}{2}t} + L_{r+n+t} . L_{q+n\frac{1}{2}t}$; that is by using the abbreviation of Art. 4. Section 1, $\overset{x}{\begin{smallmatrix}o\\o\\n\end{smallmatrix}}\Big| L_{q+x} . \dot{L}_{r+x} - \frac{1}{2} L_{r+n,\, q+n} + \frac{1}{2} L_{q+n,\, r+n+t} - \frac{1}{2} L_{q+n+t,\, r+n} + \frac{1}{2} L_{r+n+t,q+n+t}$ or $\overset{x}{\begin{smallmatrix}o\\o\\n\end{smallmatrix}}\Big| L_{q+x} . \dot{L}_{r+x} - L_{r+n,\, q+n+\frac{1}{2}t} + L_{r+n+t,\, q+n+\frac{1}{2}t}$.

Art. 2. Hence, if the deaths cannot be considered as the times, with sufficient accuracy, during the period x; but during any portion of the limits o and ε, ε and ε', ε' and ε'', ε'' and ε''', &c. the deaths produced in any portion of the same limits, are proportional to the portions of time in which they are produced, or may be considered so with sufficient accuracy, then as we shall have $\overset{x}{\begin{smallmatrix}o\\o\\o\end{smallmatrix}}\Big| L_{q+x} . \dot{L}_{r+x} = o$, we shall have $\overset{x}{\begin{smallmatrix}o\\o\\\varepsilon\end{smallmatrix}}\Big| L_{q+x} . \dot{L}_{r+x} = -\frac{1}{2} L_{r,q} + \frac{1}{2} L_{q,\, r+\varepsilon} - \frac{1}{2} L_{q+\varepsilon,\, r} + \frac{1}{2} L_{r+\varepsilon,\, q+\varepsilon}$;

and also $= -L_{r,q+\frac{1}{2}\epsilon} + L_{r+\epsilon,q+\frac{1}{2}\epsilon}$; this appears by merely writing o for n and ϵ for t in the above formulæ. And if we now write ϵ for n, and ϵ' for t in the forms of Art. 1, we shall have $\left.\begin{array}{c}x\\ \hline o\\ o\\ \epsilon+\epsilon'\end{array}\right| L_{q+x}\cdot\dot{L}_{r+x} = \left.\begin{array}{c}x\\ \hline o\\ o\\ \epsilon\end{array}\right| L_{q+x}\cdot\dot{L}_{r+x} - \frac{1}{2}L_{r+\epsilon,q+\epsilon} + \frac{1}{2}L_{q+\epsilon,r+\epsilon+\epsilon'} - \frac{1}{2}L_{q+\epsilon+\epsilon',r+\epsilon} + \frac{1}{2}L_{r+\epsilon+\epsilon',q+\epsilon+\epsilon'}$ and also $= \left.\begin{array}{c}x\\ \hline o\\ o\\ \epsilon\end{array}\right| L_{q+x}\cdot\dot{L}_{r+x} - L_{r+\epsilon,q+\epsilon+\frac{1}{2}\epsilon} + L_{r+\epsilon+\epsilon',q+\epsilon+\frac{1}{2}\epsilon'}$. And by continuing in this manner, we shall arrive at the value of $\left.\begin{array}{c}x\\ \hline o\\ o\\ n\end{array}\right| L_{q+x}\cdot\dot{L}_{r+1}$, where $n = \epsilon+\epsilon'+\epsilon''+$ &c.; and therefore by dividing the intervals into a sufficient number, we may find the fluent of $L_{q+x}\cdot\dot{L}_{r+x}$ to the utmost degree of accuracy, that the tables of mortality will admit of; and for most purposes, by dividing the time in but few intervals, the requisite accuracy will be obtained.

Art. 3. And considering the decrements also equal during the interval m at the expiration of the interval n, we have $\left.\begin{array}{c}x\\ \hline o\\ o\\ n+m\end{array}\right| L_{q+x}\cdot\dot{L}_{r+x}$, that is the fluent of $L_{q+x}\cdot\dot{L}_{r+x}$ from $x=o$ to $x=n+m$, $= \left.\begin{array}{c}x\\ o\\ o\\ n\end{array}\right| L_{q+x}\cdot\dot{L}_{r+x} - \frac{1}{2}L_{q+n,r+n} + \frac{1}{2}L_{q+n,r+n+m}, - \frac{1}{2}L_{q+n+m,r+n} + \frac{1}{2}L_{q+n+m,r+n+m}$, and also $\left.\begin{array}{c}o\\ n\end{array}\right| L_{q+x}\cdot\dot{L}_{r+x} - L_{r+n,q+n+\frac{1}{2}m} - L_{r+n+m,q+n+\frac{1}{2}m}$, these are obtained by writing m in the place of t in Art. 1.

Art. 4. And if ϵ, ϵ', ϵ'', &c. m, be each equal to, or less than the interval of the times between our tabulated numbers of living, these may generally be considered to give the utmost limit of accuracy the tables will afford; unless that there should appear sufficient regularity in the numbers of the tables to warrant a belief that an interpolation will offer a more accurate determination of the number of living for intermediate intervals than the first differences only will give; in which case our intervals ϵ, ϵ', &c. m, should, to obtain our confidence of their attaining the utmost accuracy, be taken smaller than the intervals of the tables, by taking their corresponding numbers by interpolations.

Art. 5. It may be observed with regard to Articles 1 and 2, that when the object is only the calculations of the value of $\left.\overset{x}{\overline{\begin{smallmatrix}0\\0\\n+t\end{smallmatrix}}}\right| \mathrm{L}_{q+x} \cdot \dot{\mathrm{L}}_{r+x}$, the less developed form $\left.\overset{x}{\overline{\begin{smallmatrix}0\\0\\n\end{smallmatrix}}}\right| \mathrm{L}_{q+x} \cdot \dot{\mathrm{L}}_{r+x} - (\mathrm{L}_{r+n} - \mathrm{L}_{r+n+t}) \cdot \mathrm{L}_{q+n+\frac{1}{2}t}$ of its value, may be more convenient than the form $\left.\overset{x}{\overline{\begin{smallmatrix}0\\0\\n\end{smallmatrix}}}\right| \mathrm{L}_{q+x} \cdot \dot{\mathrm{L}}_{r+x} - \mathrm{L}_{r+n,q+n+\frac{1}{2}t} + \mathrm{L}_{r+n+t,q+n+\frac{1}{2}t}$; and that n being $= \epsilon + \epsilon' + \epsilon'' +$ &c. we shall have $\left.\overset{x}{\overline{\begin{smallmatrix}0\\0\\n\end{smallmatrix}}}\right| \mathrm{L}_{q+x}, \dot{\mathrm{L}}_{r+x} = -(\mathrm{L}_{r} - \mathrm{L}_{r+\epsilon}) \cdot \mathrm{L}_{q+\frac{1}{2}\epsilon} - (\mathrm{L}_{r+\epsilon} - \mathrm{L}_{r+\epsilon+\epsilon'}) \cdot \mathrm{L}_{q+\epsilon+\frac{1}{2}\epsilon'} - (\mathrm{L}_{r+\epsilon+\epsilon'} - \mathrm{L}_{r+\epsilon+\epsilon'+\epsilon''}) \cdot \mathrm{L}_{q+\epsilon+\epsilon'+\frac{1}{2}\epsilon''}$, &c.

Art. 6. From Article 2 we have

$$\left.\overset{x}{\overline{\begin{smallmatrix}0\\0\\\epsilon+\epsilon'\end{smallmatrix}}}\right| \mathrm{L}_{q+x} \cdot \dot{\mathrm{L}}_{r+x} = \left.\overset{x}{\overline{\begin{smallmatrix}0\\0\\\epsilon\end{smallmatrix}}}\right| \mathrm{L}_{q+x} \cdot \dot{\mathrm{L}}_{r+x} - \frac{1}{2}\mathrm{L}_{r+\epsilon,\,q+\epsilon} + \frac{1}{2}\mathrm{L}_{q+\epsilon,\,r+\epsilon+\epsilon'} - \frac{1}{2}\mathrm{L}_{q+\epsilon+\epsilon\,,r+\epsilon'} + \frac{1}{2}\mathrm{L}_{q+\epsilon+\epsilon',\,r+\epsilon+\epsilon'};$$

but $\left.\overset{x}{\overline{\begin{smallmatrix}0\\0\\\epsilon\end{smallmatrix}}}\right| \mathrm{L}_{q+x} \cdot \dot{\mathrm{L}}_{r+x} = -$

$\frac{1}{2}L_{r,q}+\frac{1}{2}L_{q,r+\varepsilon}-\frac{1}{2}L_{q+\varepsilon,r}+\frac{1}{2}L_{q+\varepsilon,r+\varepsilon}$; and consequently

$$\left.\begin{array}{c}x\\\hline 0\\0\\\varepsilon+\varepsilon'\end{array}\right]L_{q+x}\cdot\dot{L}_{r+x}=-\frac{1}{2}L_{q,r}+\frac{1}{2}L_{q,r+\varepsilon}-\frac{1}{2}L_{q+\varepsilon,r}+\frac{1}{2}L_{q+\varepsilon,r+\varepsilon+\varepsilon'}-\frac{1}{2}L_{q+\varepsilon+\varepsilon',r+\varepsilon}+\frac{1}{2}L_{q+\varepsilon+\varepsilon',r+\varepsilon+\varepsilon'},$$

whereas if in Art. 1, we take $n=0$ and $t=\varepsilon+\varepsilon'$, we shall have

$$\left.\begin{array}{c}x\\\hline 0\\0\\\varepsilon+\varepsilon'\end{array}\right]L_{q+x}\,\dot{L}_{r+x}=-\frac{1}{2}L_{q,r}+\frac{1}{2}L_{q,r+\varepsilon+\varepsilon'}-\frac{1}{2}L_{q+\varepsilon+\varepsilon'}+\frac{1}{2}L_{q+\varepsilon+\varepsilon',r+\varepsilon+\varepsilon'}.$$

And we are here presented with two different forms for the value of $\left.\begin{array}{c}x\\\hline 0\\0\\\varepsilon+\varepsilon'\end{array}\right]L_{q+x}\cdot\dot{L}_{r+x}$; that is of the value of the fluent of L_{q+x}, $\cdot\,\dot{L}_{r+x}$, whilst x from 0 becomes $\varepsilon+\varepsilon'$; the latter form being on the supposition, that the decrements are proportional to the time throughout the interval $\varepsilon+\varepsilon'$, and the former, that they are first proportional to the time during the interval ε, and that after that they are again proportional to the time through the next interval ε'. The reason of adducing these two, which are commonly approximate values of $\left.\begin{array}{c}x\\\hline 0\\0\\\varepsilon+\varepsilon'\end{array}\right]L_{q+x}\cdot\dot{L}_{r+x}$, is that several ingenious authors in the solution of some of the problems hereafter to be considered, have taken as an approximation that if two persons, who were living at a particular period, are both dead at any nominated period after, within a certain limit, that it is an equal chance which of the two is the survivor; which as will appear is the natural consequence of supposing the decrements of the lives during every part of that period, to be in a constant proportion to the time elapsed from the commencement of the period. But the mode of analysis pursued by those gentlemen, have led them to divide their periods continually into two, the one a period generally of several years, and the other a period of one year only, and they have calculated for the two periods separately, and added the result of the two. This mode would be given from the first of the two forms of this article, which is extremely more complex than the second form, and apparently with very little, if any advantage; for if the decrements were accurately in a constant proportion to the time from the commencement, they would be perfectly of equal value; though the one possesses so much more simple a form than the other; and when they are each but approximations, there will be but little choice to be made between them, in point of proximity, in the use they are to be made of in those problems; though there will be a vast difference in the degree of simplicity in the resulting formula.

Art. 7. Hence the value of $L_{p+x} \times$ (fluent of $L_{q+x} \cdot \dot{L}_{r+x}$ commencing with x equal to o) will form Article 3, when

$x = n + \iota$, be $= L_{p+n+\iota} \times \left(\begin{array}{c|} x \\ \hline o \\ o \\ n \end{array}\, L_{q+x} \dot{L}_{r+x} - \frac{1}{2} L_{q+n,\, r+n} \right) +$
$\frac{1}{2} L_{p+n+\iota,\, q+n,\, r+n+\iota} - \frac{1}{2} L_{p+n+\iota,\, q+n+\iota,\, r+n} +$
$\frac{1}{2} L_{p+n+\iota,\, q+n+\iota,\, r+n+\iota}$; and when x becomes $n+\varepsilon+\varepsilon'$, supposing the uniform decrement to last through the whole period $\iota+\iota'$, the fluent will be $L_{p+n+\iota+\iota'} \times \left(\begin{array}{c|} x \\ \hline o \\ o \\ \iota+\iota' \end{array}\, L_{q+x} \dot{L}_{r+x} -\right.$
$\left.\frac{1}{2} L_{q+n,\, r+n}\right) + \frac{1}{2} L_{p+n+\iota+\iota',\, q+n,\, r+n+\iota+\iota'} -$
$\frac{1}{2} L_{p+n+\iota+\iota',\, q+n+\iota+\iota',\, r+n} + \frac{1}{2} L_{p+n+\iota+\iota',\, q+n+\iota+\iota',\, r+n+\iota+\iota'}$

and the excess of this above the other, that is $\left.\begin{array}{c}x\\ \hline o\\ o\\ n+\iota+\iota'\end{array}\right|(L_{p+n+\iota+\iota',\,q+x}\times\dot{L}_{r+x})-\left.\begin{array}{c}x\\ \hline o\\ o\\ n+\iota\end{array}\right|(L_{p+n+\iota,\,q+x}\times\dot{L}_{r+x})$ will be $\left(\left.\begin{array}{c}x\\ \hline o\\ o\\ n\end{array}\right|L_{q+x}\cdot\dot{L}_{q+x}-\frac{1}{2}L_{q+n,\,r+n}\right)\cdot(L_{p+n+\iota+\iota'}-L_{p+n+\iota})+$ $\frac{1}{2}L_{q+n}\cdot(L_{n+\iota+\iota':p,r}-L_{n+\iota:p,r})-\frac{1}{2}L_{r+n}(L_{n+\iota+\iota':p,q}-$ $L_{n+\iota:p,q})+\frac{1}{2}(L_{n+\iota+\iota':p,q,r}-L_{n+\iota:p,q,r})$ See the notation, Art. 4. Section 1.

Art. 8. If when $x=\nu$ and greater, $L_{q+x}\cdot\dot{L}_{r+x}$ is equal to o, and the fluent of $L_{q+x}\cdot\dot{L}_{r+x}$ generated from x equal to o becomes equal to ν, be γ, then L_{p+x} . fluent of ($L_{q+x}\cdot\dot{L}_{r+x}$ commencing with $x=o$), will when $x=\nu+m$ be $=\gamma L_{p+\nu+\mu}$, μ being positive.

Article 9. On the fluent of $L_{p+x}\cdot L_{q-x}\cdot\dot{L}_{r+x}$, that is, if $L_{x:p,q}\cdot\dot{L}_{r+x}$ by notation, Article 4. Section 1.

If between the limits $x=n$ and $x=n+m$, the decrements of each life be sufficiently nearly proportional to the times to admit of their being considered proportional, and x being put $=n+t$, we use the notation $L_{p+x}=L_{p+n}-tL'_{p+n}$, $L_{q+x}=L_{q+n}-tL'_{q+n}$, $L_{r+x}=L_{r+n}-tL'_{r+n}$, we shall have $L_{x:p,q}\cdot\dot{L}_{r+x}=-t\dot{L}'_{r+n}\times(L_{n:p,q}-t(L_{p+n}\cdot L'_{q+n})+$ $t^2L'_{p+n}\cdot L'_{q+n})$. Hence the fluent generated whilst x from o becomes $=n+m$ is $=\left.\begin{array}{c}x\\ \hline o\\ o\\ n\end{array}\right|L_{x:p,q}\cdot\dot{L}_{r+x}-mL'_{r+n}\cdot(L_{n:p,q}-$ $\frac{m}{2}(L_{p+n}\cdot L'_{q+n}+L_{q+n}\cdot L'_{p+n})+\frac{m^2}{3}L'_{p+n}:L'_{q+n})=$

$\left.\begin{smallmatrix}x\\ o\\ o\\ n\end{smallmatrix}\right| L_{x:p,q}\cdot\dot{L}_{r+x} - mL'_{r+n}\cdot\left(L_{n+\frac{1}{2}m:p,q}+\frac{1}{12}L'_{p+n}\cdot L'_{q+n}\right)$ since according to hypothesis $L_{p+n+\frac{1}{2}m}=L_{p+n}-\frac{1}{2}mL'_{p+n}$ and $L_{q+n+\frac{1}{2}m}=L_{q+n}-\frac{1}{2}mL'_{q+n}$; and by notations Art. 4. Section 1, $L_{n+\frac{1}{2}m:p,q}$ stands for $L_{p+n+\frac{1}{2}m}\times L_{p+n+\frac{1}{2}m}$. And we therefore have the fluent generated, whilst x from n becomes $=n+m$, that is $\left.\begin{smallmatrix}x\\ o\\ n\\ n+m\end{smallmatrix}\right| L_{x:p,q}\cdot\dot{L}_{r+x}=-mL'_{r+n}\cdot\left(L_{n+\frac{1}{2}m:p,q}+\frac{1}{12}L'_{p+n}\cdot L'_{q+n}\right)$ or its equal $-(L_{r+n}-L_{r+n+m})\cdot L_{n+\frac{1}{2}m:p,q}-\frac{1}{12}(L_{r+n}-L_{r+n+m})\cdot(L_{p+n}-{}_{p+n+m})\cdot(L_{q+n}-L_{q+n+m})$. And here it should be remarked, if m represents one year or less, as will be frequently the case in the application of this formula to questions of practice, that generally the part $\frac{1}{12}(L_{r+n}-L_{r+n+m})\cdot(L_{p+n}-L_{p+n+m})\cdot(L_{q+n}-L_{q+n+m})$ will be sufficiently small to be wholly neglected; but should greater accuracy be required, the case would be rare if it might not be considered constant throughout the possible duration of the joint lives.

Article 10. By writing $L_{q+n}-L_{q+n+m}$ for mL'_{q+n}, &c. we shall also have $\left.\begin{smallmatrix}x\\ o\\ o\\ n+m\end{smallmatrix}\right| L_{x:p,q}\cdot\dot{L}_{r+x}=\left.\begin{smallmatrix}x\\ o\\ o\\ n\end{smallmatrix}\right| L_{x:p,q}\cdot\dot{L}_{r+x}-(L_{r+n}-L_{r+n+m})\times\left\{L_{p+n,\,q+n}-\frac{1}{2}L_{p+n}(L_{q+n}-L_{q+n+m})-\frac{1}{2}L_{q+n}\cdot(L_{p+n}-L_{p+n+m})+\frac{1}{3}(L_{q+n}-L_{q+n+m})\cdot(L_{p+n}-L_{p+n+m})\right\}$ $=\left.\begin{smallmatrix}x\\ o\\ o\\ n\end{smallmatrix}\right| L_{x:p,q}\cdot\dot{L}_{r+x}-(L_{r+n}-L_{r+n+m})\times\{\frac{1}{3}L_{n:p,q}+\frac{1}{6}$

$$L_{n:p,q+m}+\tfrac{1}{6}L_{n:q,p+m}+\tfrac{1}{3}L_{n+m:q,p}\Big\}=\begin{smallmatrix}x\\ o\\ \cdot\\ n\end{smallmatrix}\Big\rceil L_{x:p,q}\cdot\dot{L}_{r+x}-$$
$$\tfrac{1}{3}L_{n:p,q,r}-\tfrac{1}{6}L_{n:r,p,q+m}-\tfrac{1}{6}L_{n:r,q,p+m}-\tfrac{1}{3}L_{n:r,q+m,p+m}+$$
$$\tfrac{1}{3}L_{n:p,q,r+m}+\tfrac{1}{6}L_{n:p,q+m,r+m}+\tfrac{1}{6}L_{n:p+m,q,r+m}+$$
$$\tfrac{1}{3}L_{n+m:p,q,r};$$

This form of developement would give the solution of some problems, to be considered in this paper, in the form which has already been given by mathematicians; but there are cases of application, in which the form in Article 9, will give a much less intricate solution.

Article 11. Because $L_{n:p,q}-\frac{m}{2}(L_{p+n}\cdot L'_{q+n}+L_{q+n}\cdot L'_{p+n})+\frac{m^2}{3}L'_{p+n}\cdot L'_{q+n}$, part of an expression of Article 9, is $=\frac{1}{2}(L_{p+n}-\frac{m}{2}\overline{1-\sqrt{\frac{1}{3}}}L'_{p+n})\cdot(L_{q+n}-\frac{m}{2}\cdot\overline{1-\sqrt{\frac{1}{3}}}\cdot L'_{p+n})+\frac{1}{2}(L_{p+n}-\frac{m}{2}\cdot\overline{1+\sqrt{\frac{1}{3}}}\cdot L'_{p+n})\cdot(L_{q+n}-\frac{m}{2}\cdot\overline{1+\frac{1}{3}}L'_{q+n})=$ agreeably to the hypothesis $\frac{1}{2}L_{n+bm:p,q}+L_{n+km:p,q}$, h being $=\frac{1}{2}-\frac{1}{2}\sqrt{\frac{1}{3}}$ and $k=\frac{1}{2}+\sqrt{\frac{1}{3}}$, and therefore the fluent of $L_{x:p,q}\cdot\dot{L}_{r+x}$ between the limits of $x=n$ and $x=n+m$ is $=\frac{1}{2}L'_{r+n}(L_{n+b:p,q}+L_{n+k:p,q})-L'_{r+n}\cdot L_{n:p,q}$.

Article 12. On the fluent of $L_{x:p,q,r,s,\&c.}\times\dot{L}_{x+x}$.

Suppose n and $n+m$ sufficiently near each other to admit the decrements to be considered with sufficient accuracy as proportional to the time t in which they are produced; t being greater than m, and $x=n+t$: and the fluent required using a similar notation to that hitherto used will be from

$t = 0$ to $t = m$, equal to $-mL'_{x+n} \times L_{n:p,q,r,\&c.} - \frac{1}{2}(L'_{p+n} . L_{n:q,r,s,\&c.} + L'_{q+n} L_{n,p,r,s,\&c.} + \&c.) + \&c. = -mL'_{x+n} \times L_{n+\frac{1}{2}m,\,p,q,r,s,\&c.}$, neglecting the remaining terms in which m^2, m^3 &c, not herein contained are concerned, as they will be small if m be small; though, if necessary, we may pursue similar means to those used in the last article.

Article 13. But if during short periods, instead of the arithmetical progressions, we use geometrical progressions, see Article 2, Section 2; and x being $= n + t$ we take $L_{p+x} = L_{p+n} . 10^{tL'_{p+n}}$, $L_{q+x} = L_{q+n} . \overline{10|}^{tL'_{q+n}}$ &c., $L_{x+x} = L_{x+n} . \overline{10|}^{tL'_{x+n}}$; and therefore putting π for the hyperbolical logarithm of 10, $\dot{L}_{x+x} = \pi L_{x+n} . L'_{x+n} \dot{t}\, \overline{10|}^{tL'_{x}}$, we shall have if for the sake of brevity we put $\mu = L'_{p+n} + L'_{q+n} + L'_{r+n} + \&c. + L'_{x+n}$, $L_{x:p,q,r,s,\&c.} \times \dot{L}_{x+x} = \pi L'_{x+n} . L_{n:p,q,r,s,\&c.,x} \times \dot{t} . \overline{10|}^{\mu t}$, and the fluent generated whilst x from n becomes $= n + m$ is $= L_{n:p,q,r,s,\&c.x} : \times \frac{L'_{x+n}}{\mu} \times (\overline{10|}^{\mu t} - 1) = \frac{1}{\mu} . L'_{x+n} \times L_{n+m:p,q,r,\&c.,x} - L_{n:p,q,r,\&c.,x}$.

Article 14. Because $L_{x:p,q,r,\&c.x} = L_{n:p,q,r,\&c.x} \times \overline{10|}^{\mu t}$ and $L_{x+x} = L_{x+n} . \overline{10|}^{tL'_{x+n}}$ it follows that $\frac{1}{\mu} L'_{x+n} =$

$$\frac{\text{Log : of } L_{r+x} - \text{Log of } L_{x+n}}{\text{Log : of } L_{x:p,q,r,\&c.,x} - \text{Log : of } L_{n:p,q,r,\&c.x}}$$

; x being equal to $n+t$, and t any positive quantity not greater than m; the same set of geometrical series being only supposed to take effect between the intervals n and $n+m$; but if the geometrical series between those limits only take effect proximatively, our fluent will be but an approximation, though as correct as we please by taking the intervals sufficiently small. When for the purpose of approximation a long interval is divided into smaller intervals, one of which, for instance, is from n to $n+m$, it will not be immaterial for obtaining the nearest approximation, what value we take for x between n and $n+m$, in the above value for $\frac{1}{\mu}L'_{x+n}$; an off-hand idea might be, that x should be taken somewhere about $n+\frac{1}{2}m$.

SECTION IV. Art. 1. Because $1-\frac{L_{b+x}}{L_b}$ is the chance that a person of the age b shall be dead in the time x, $-\frac{\dot{L}_{b+x}}{L}$ is the fluxion of the chance; that is the measure of the chance that he would have of dying during a finite time $\dot{x}$, on the consideration that that cause, if any should subsist, to make the deaths disproportionate to the time, should cease at the term x. And if this be multiplied by $\frac{L_{a+x}}{L_a}$, the product will represent the fluxions of the chance of the person of the present age b dying in the life time of the person of the present age a, and consequently, —fluent of $\frac{L_{a+x}\cdot\dot{L}_{b+x}}{L_{a,b}}$ is the chance that the person of the present age a has of surviving the person of the present age b. The calculation of this fluent between any limits is effected from the articles of Section 3, referring to the fluent of $L_{q+x}\cdot\dot{L}_{r+x}$. If for q and r we write a and b, and we wish to have the part of the contingency corresponding to the intervals between $x=n$ and $x=n+1$, taking $t=1$ in the first of the two forms we have it $=\frac{(L_{b+n}-L_{b+n+1})\cdot(L_{a+n}+L_{a+n+1})}{2L_{a,b}}$; and by taking n successively 0, 1, 2, 3, &c. and adding the results; making the division by $2\,L_{a,b}$ for the sake of convenience on the sum of numerators, we obtain in the same formula excepting the notation, with the ingenious Mr. MORGAN first, and Messrs. BAILY and MILNE after him, the value of the contingency for any part or the whole of life, that the age *a* shall survive *b*. If we use the second form, the part due to the interval between *n* and $n+1$, will stand $\frac{L_{b+n}-L_{b+n+1}}{L_{a,b}}$ $. L_{a+n+\frac{1}{2}}$; this form, at least if the tables of mortality have the number of living inserted for every half year, would be easier in practice than the other. If a less accurate solution would answer our purpose, we might take *t* much larger; if it were taken ten years we should soon get through the work, and frequently with sufficient accuracy, either from the article now quoted, or from Art. 13. of the same section; that due to the interval between *n* and $n+t$, is $\frac{(L_{b+n}-L_{b+n+t})\cdot(L_{a+n}+L_{a+n+t})}{2\,L_{a,b}}$. or $\frac{L_{b+n}-L_{b-n+nt}}{L_{a,b}} . L_{a+n+\frac{1}{2}t}$.

Art. 2. That A and B whose present ages are *a* and *b*, both die during the time; but that B dies last, is from a similar argument $=-$ fluent of $\left(1-\frac{L_{a+x}}{L_a}\right)\times\frac{\dot{L}_{b+x}}{L_b}=$ correction $-\frac{L_{b+x}}{L_b}+$ fluent of $\frac{L_{a+x}\cdot\dot{L}_{b+x}}{L_a\,.\,L_b}$; and this case may be therefore found immediately from the other; or the other case immediately from this. But if from the period $x=n$ to $x=n+m$, the decrements of each life be proportional to the times, however different the decrements of A's life and of B's life may be; then since $-\left(1-\frac{L_{a+x}}{L_a}\right)\cdot\frac{\dot{L}_{b+x}}{L_b}$ may if $x=n+t$, according to the notation we have used be written $\left(1-\frac{L_{a+x}}{L_a}+t\frac{L'_{a+n}}{L_a}\right)\dot{t}\cdot\frac{L'_{b+n}}{L_b}$, our fluent between the limit $x=n$ and $x=n+m$, will therefore become $\left(1-\frac{L_{a+n}}{L_a}\right)\cdot m\frac{L'_{b+n}}{L_b}+\frac{m^2}{2}\cdot\frac{L'_{a+n}\cdot L'_{b+n}}{L_a\cdot L_b}$; that is agreeably to the hypothesis putting $L_{a+n}-L_{a+n+m}=m\,.\,L'_{a+n}$, and $L_{b+n}-L_{b+n+m}=m\,.\,L'_{b+n}$, it becomes $\left(1-\frac{L_{a+n}}{L_a}\right)\cdot\frac{L_{b+n}-L_{b+n+m}}{L_b}+\frac{L_{a+n}-L_{a+n+m}}{L_a}\times\frac{L_{b+n}-L_{b+n+m}}{2L_b}$: this if n were equal to o would be reduced to $\frac{1}{2}\left(1-\frac{L_{a+m}}{L_a}\right)\times\left(1-\frac{L_{b+m}}{L_b}\right)$; which is just half the chance that they shall both have died in that time; an approximation frequently of service, and used in many cases by Mr. MORGAN, and the Gentlemen who have followed him. If there be a term of possible joint existence, and that be when $m=\mu$; and ν be some positive quantity, the formula $\frac{1}{2}\left(1-\frac{L_{a+n}}{L_a}\right)\cdot\left(1-\frac{L_{b+m}}{L_b}\right)$ will not answer when $m=\mu+\nu$, if A be the oldest; when $\nu=o$ it will answer and become $\frac{1}{2}\left(1-\frac{L_{b+m}}{L_b}\right)$, but when ν has a value, this must be increased by the chance that B has of dying beyond this time; that is, it must be increased by $\frac{L_{b+\mu}-L_{a+\mu+\nu}}{L_b}$; but if A be youngest, it becomes when

$m=\mu+\nu$, $\frac{1}{2}\left(1-\frac{L_{a+\nu}}{L_a}.\right)$ Moreover, though the chance of A's surviving B, being half the chance of their both being dead, is here derived from the hypothesis of the decrement's being proportional to the time in each original age; this is not the only hypothesis which will cause that relation of the contingencies; or, which is the same thing, provided they both are to be dead, that the contingency of A's surviving B shall be equal to the contingency of B's surviving A; for the fluxion of this equation is $-\left(1-\frac{L_{a+x}}{L_a}\right)\times\frac{\dot{L}_{b+x}}{L_b}=-\left(1-\frac{L_{b+x}}{L_b}\right)\times\frac{\dot{L}_{a+x}}{L_a}$; and therefore $\frac{\frac{-\dot{L}_{a+x}}{L_a}}{1-\frac{L_{a+x}}{L_a}}=\frac{\frac{-\dot{L}_{b+x}}{L_b}}{1-\frac{L_{b+x}}{L_b}}$, and consequently taking the correct fluent &c. we get $1-\frac{L_{a+x}}{L_a}=K.\left(1-\frac{L_{b+x}}{L_b}\right)$; k be a constant quantity; therefore the fluent of $\left(1-\frac{L_{a+x}}{L_a}\right)\cdot\frac{\dot{L}_{b+x}}{L_b}=-$ the fluent of $k\left(1-\frac{L_{b+x}}{L_b}\right)\cdot\frac{\dot{L}_{b+x}}{L_b}$; which taken to vanish when $x=o$ is the contingency according to the present hypothesis of B dying after A; and is $=k.\left(1-\frac{L_{b+x}}{L_b}-\frac{1}{2}+\frac{1}{2}\left(\frac{L_{b+x}}{L_b}\right)^2\right)=\frac{1}{2}k.\left(1-2\frac{L_{b+x}}{L_b}+\left(\frac{L_{b+x}}{L_b}\right)^2\right)=\frac{1}{2}k.\left(1-\frac{L_{b+x}}{L_b}\right)^2=\frac{1}{2}.\overline{1-\frac{L_{a+x}}{L_a}}.\overline{1-\frac{L_{b+x}}{L_b}}.$

Hence it appears, that whatever the decrements of life, or the constitution of the functions of life, it will be an equal chance if A and B are both to be dead, whether A dies after B, or B dies after A, if $1-\frac{L_{a+x}}{L_a}=k.(1-\frac{L_{b+x}}{L_b})$, k being constant.

“ *Note.* If the constitution of both functions of life is of one and the same continuous character, which is not necessarily the case, unless they be taken from one and the same continuous table; then it becomes an interesting problem to search, what is the constitution of the function of life, to admit of the aforesaid equality of contingency of A dying after B or B dying after A; on the condition of their both dying in that time; that is to say, to find the common characteristic L which shall be independent of a, b, and x, such that $1-\frac{L_{a+x}}{L_a}=k.(1-\frac{L_{b+x}}{L_b})$, the requisite equation above given. And as this is a problem which, from its mode of solution, may be equally interesting to the Analyst, as forming one of a rather novel species of problems, I shall give its solution for both purposes; in order to which I first observe, that unless k be unity, it must be expressible in the form $\frac{D_b}{D_a}$, so that the equation may be written in the form $D_a.\frac{L_{a+x}}{L_a}-D_a=D_b.\frac{L_{b+x}}{L_b}-D_b$; otherwise there would not be a perfect similarity of the character L, on both sides of the equation. For the sake of brevity, put $\frac{D_a}{L_a}=H_a$, and therefore $\frac{D_b}{L_b}=H_b$; and our equation will stand $H_a.(L_{a+x}-L_a)=H_b.(L_{b+x}-L_b)$; but by TAYLOR's

" theorem $L_{a+x} = L_a + x \cdot \frac{\dot{L}_a}{\dot{a}} + \frac{x^2}{2} \cdot \frac{\ddot{L}_a}{\dot{a}^2} + \&c.$ $\dot{a}$ being taken
" constant; and also $L_{b+x} = L_b + x \cdot \frac{\dot{L}_b}{\dot{b}} + \frac{x^2}{2} \cdot \frac{\ddot{L}_b}{\dot{b}^2} + \&c.$ $\dot{b}$
" being taken constant; therefore if these be substituted in
" the last equation, it is evident that as the thing is to re-
" main true, whatever x may be, the homologous powers of
" x must destroy each other; and consequently we must
" have, by making the comparison of the coefficients of those
" powers in the equation, $H_a \cdot \frac{\dot{L}_a}{\dot{a}} = H_b \cdot \frac{\dot{L}_b}{\dot{b}}$; $H_a \cdot \frac{\ddot{L}_a}{\dot{a}^2} = H_b$
" $\cdot \frac{\dot{L}_b}{\dot{b}^2}$; $H_a \cdot \frac{\dddot{L}_a}{\dot{a}^3} = H_b \cdot \frac{\dddot{L}_b}{\dot{b}^3}$, &c.; and as this is the case what-
" ever a and b are, it follows that each side of the equation
" must be constant; therefore putting $H_a \cdot \frac{\dot{L}_a}{\dot{a}} = g$, $H_a \cdot \frac{\ddot{L}_a}{L\dot{a}^2} = h$;
" g and h being constant quantities, we have $\frac{\ddot{L}_a}{\dot{L}_a} = \frac{b}{g} . \dot{a}$. And
" taking the fluent of this, we have hyp. log of $\frac{L_a}{p\dot{a}} = \frac{b}{g} a$;
" p being some constant quantity; and this may be reduced
" to the form $\dot{L}_a = p . e^a . a$, e standing for the number whose
" hyperbolical logarithm is $\frac{b}{g}$; and taking the fluent again,
" we have $L_a = e' - e'' . e^a$; e, e' and e'' being independent
" constant quantities, though e'' is $= -p . \frac{g}{b}$ the independence
remains because $p . \frac{g}{b}$ is arbitrary. And as the equation $\dot{L}_a =$
" $pe^a a$, makes $\ddot{L}_a = p . \frac{b}{g} . e^a \dot{a}^2 = \frac{b}{g} \dot{a} \dot{L}_a$; $\therefore \dddot{L}_a = \frac{b}{g} a \ddot{L}_a$;

" &c.; all the conditions are fulfilled by fulfilling the conditions that $H_a . \frac{\dot{L}_a}{\dot{a}}$, and $H_a . \frac{\dot{L}_a}{\dot{a}^2}$, are constant. Whence we " find the constitution is $L_a = d' - d'' . c^a$; where d', c, d'' are " any constants at pleasure. And if I mistake not, it is the " only function which will admit of this equality of chance " *continuously* between persons whose mortality is formed " from the *same function*; but if c be $= 1 + \varepsilon$, and ε be infinitely small, this will stand simply $L_a = d' - d'' (1 + a\varepsilon)$; " or writing g' for $d' - d''$ and g'' for $d''\varepsilon$, this becomes " $L_a = g' - g''a$, the formula of equal decrements in equal " times."

Moreover the same idea of $1 - \frac{L_{a+x}}{L_a} = k . \overline{1 - \frac{L_{b+x}}{L_b}}$, causes the fluent of $\frac{L_{a+x} . \dot{L}_{b+x}}{L_a . L_b}$ to be equal to the fluent of $k . \frac{L_{b+x}}{L_b}$ $\frac{L_{b+x}}{L_b}$, which if taken to commence with x equal to o, will be $-\frac{1}{2} k . \overline{1 - \left(\frac{L_{b+x}}{L_b}\right)^2} = -\frac{1}{2} k . \overline{1 - \frac{L_{b+x}}{L_b}} \times \overline{1 + \frac{L_{b+x}}{L_b}} = -\frac{1}{2} . \overline{1 - \frac{L_{a+x}}{L_a}} \times \overline{1 + \frac{L_{b+x}}{L_b}} = -\frac{1}{2} + \frac{1}{2} \frac{L_{a+x}}{L_a} - \frac{1}{2} \frac{L_{b+x}}{L_b} + \frac{1}{2} \frac{L_{x:a,b}}{L_{a,b}}$; and this is therefore not only the fluent in question in the hypothesis of constant decrements.

Article 3. If at the interest to be made for money, one pound discounted for one year were represented by r, and we wished to have the present value of one pound, to be received at the first of the equal periods p, after the time $n-p$ that shall happen after the death of B; provided he be survived by A; and that that event takes place between the periods $n-p$ and m; then if each separate period p, may throughout its duration, be considered within the limits of constant decrements, the value of that part due to the events happening between the periods π and $\pi+p$, (π being some one of the terms $n-p$, n, $n+p$, &c.) from Article 1 of this section will be $\frac{(L_{b+\pi}-L_{b+\pi+p})\,.\,L_{a+\pi+\frac{1}{2}p}}{L_{a,b}}\,.\,r^{\pi+p}=\frac{L_{a-\frac{1}{2}p,\,b-p}}{L_{a,b}}$ $\times\, r^{\pi+p}\,.\,\frac{L_{b+\pi,\,a+\pi+\frac{1}{2}p}}{L_{a-\frac{1}{2}p,\,b-p}}-\frac{L_{a-\frac{1}{2}p}}{L_{a}}\,.\,r^{\pi+p}\,.\,\frac{L_{a+\pi+\frac{1}{2}p,\,b+\pi+p}}{L_{a-\frac{1}{2}p,\,b}}$; and if π be interpreted by $n-p$, n, $n-2p$, &c. the sum of the whole will be $\frac{L_{a-\frac{1}{2}p,\,b-p}}{L_{a,b}}\,.\,\overset{r}{\overbrace{\substack{p\\n\\m}}}\Big|\underline{\,a-\tfrac{1}{2}p,\,b-p\,}-\frac{L_{a-\frac{1}{2}p}}{L_{a}}\,.\,\overset{r}{\overbrace{\substack{p\\n\\m}}}\Big|\underline{\,a+\tfrac{1}{2}p,b\,}.$

If p be one year or unity, this will be $\frac{L_{a-\frac{1}{2},\,b-1}}{L_{a,b}}\times$ (the annuity for the time on the ages $a-\frac{1}{2}$ and $b-1$) $-\frac{L_{a-\frac{1}{2}}}{L_{a}}\times$ annuity for the time on the ages $a+\frac{1}{2}$ and b.

If the value of the contingency, due to the intervals between π and $\pi+p$, be written $\frac{\frac{1}{2}L_{\pi:a,b}+\frac{1}{2}L_{\pi:a+p,b}-\frac{1}{2}L_{\pi.a,b+p}-\frac{1}{2}L_{\pi+p:a,b}}{L_{a,b}}\,.\,r^{\pi+p}$ we may obtain the value in the forms of Messrs. MORGAN, BAILY and MILNE; for by reducing the form in the shape $\frac{1}{2}\frac{L_{\pi:a,b}-L_{\pi+p:a,b}}{L_{a,b}}\,.\,r^{\pi+p}$ $+\frac{1}{2}\frac{L_{b-p}}{L_{b}}\times r^{\pi+p}\,.\,\frac{L_{\pi:a+p,b}}{L_{a,b-p}}-\frac{1}{2}\frac{L_{a-p}}{L_{a}}\,.\,r^{\pi+p}\,.\,\frac{L_{\pi:a,b+p}}{L_{a-p,b}}$, and interpreting π by $n-p$, n, $n+p$, &c. we have from Section 2, for the sum of them all $\frac{1}{2}\cdot\left.\begin{array}{c}\overset{r}{\smile}\\ p\\ n\\ m\end{array}\right|\underline{a,b}+\frac{1}{2}\frac{L_{b-p}}{L_b}\cdot\left.\begin{array}{c}\overset{r}{\smile}\\ p\\ n\\ m\end{array}\right|\underline{a,b-p}$ $-\frac{1}{2}\frac{L_{a-p}}{L_b}\cdot\left.\begin{array}{c}\overset{r}{\smile}\\ p\\ n\\ m\end{array}\right|\underline{a-p,b}$. This is the same in fact as Mr. MILNE's form, excepting something more general; in as much as it refers to the case if the assurance be temporary, and transferred at the same time, and that the interval p is not necessarily one year, and that it refers also to insurances when the contingencies are taken momentarily; but it must be remarked, that in this case p being infinitely small $\frac{1}{2}\frac{L_{b-p}}{L_b}\cdot\left.\begin{array}{c}\overset{r}{\smile}\\ p\\ n\\ m\end{array}\right|\underline{a,b-p}-\frac{1}{2}\frac{L_{a-p}}{L_a}\cdot\left.\begin{array}{c}\overset{r}{\smile}\\ p\\ n\\ m\end{array}\right|\underline{a-p,b}$ will have the appearance to some readers of being equal to o; whereas that is not necessarily the case; it is true that the ratio of $\frac{L_{b-p}}{L_b}\cdot\left.\begin{array}{c}\overset{r}{\smile}\\ p\\ n\\ m\end{array}\right|\underline{a,b-p}$ to $\frac{L_{a-p}}{L_a}\cdot\left.\begin{array}{c}\overset{r}{\smile}\\ p\\ n\\ m\end{array}\right|\underline{a-p,b}$ will differ infinitely little from the ratio of equality; but as they will be each of them infinite, their difference may be finite. See how to calculate this value in the Scholium Art. 3.

Article 4. A correct notation of the value in the last article is in conformity with our plan $-\left.\begin{array}{c}x\\ \hline p\\ n-p\\ m-p\end{array}\right]\left(\frac{r^{x+p}}{L_{a,b}}\times\left.\begin{array}{c}x\\ \hline o\\ x\\ x+p\end{array}\right]L_{a+x}\cdot\dot{L}_{b+x}\right)$

And if for L_{a+x} we write L_a-L_{a+x}, it will express the value of the assurance on the death of B, provided A be dead at that time, other things remaining the same. And because $\left.\begin{array}{c}x\\ \hline o\\ x\\ x+p\end{array}\right]$

$$\overline{L_a - L_{a+x}} \cdot \dot{L}_{b+x} = L_a (L_{b+n+x} - L_{b+n}) - \overset{x}{\overline{\begin{matrix}0\\ n\\ n+p\end{matrix}}}\Big|\, L_{a+x} \cdot \dot{L}_{b+x},$$

and $\overset{n}{\overline{\begin{matrix}k\\ n-p\\ m-p\end{matrix}}}\Big|\, r^{n+p} \frac{L_{b+n+p} - L_{b+p}}{L_b} = - \overset{r}{\smile}\!\!\begin{matrix}p\\ n\\ m\end{matrix}\Big|\underline{b}$; therefore the value of this contingency is $\overset{r}{\smile}\!\!\begin{matrix}p\\ n\\ m\end{matrix}\Big|\underline{b}$ — that of the last article.

Article 5. No. 1. If there may be a continuance of certain independent events, denominated for the sake of distinction K, K′, K″, K‴, &c.; and the chances of their continuing the times w, x, y, z, &c., be denoted by K_w, K'_x, K''_y, &c. respectively, then will $1 - K_w$, $1 - K'_x$, &c. denote the corresponding chances of their not continuing during those times; $\dot{K}_w$, $\dot{K}'_x$, $\dot{K}''_y$, &c. will express the fluxions of the chances of those events continuing; and if these fluxions be taken with regard to the respective under written letters w, x, y, &c., they will be the chances of continuance of those respective events due to the finite times $\dot{w}$, $\dot{x}$, $\dot{y}$, &c. respectively, on the consideration that at the terms w, x, y, &c. the causes, if any, perturbating the proportionality to the times of the discontinuance vanishes. And under the like hypothesis would $-\dot{K}_w$, $-\dot{K}'_x$, $-\dot{K}''_y$, which are the fluxions of the chances of discontinuance, express the chances of the discontinuance taking place during the times $\dot{w}$, $\dot{x}$, $\dot{y}$, &c.

No. 2. Hence we see that $-K''_x \dot{K}'''_x$ is the fluxion of the chance of the event K‴ discontinuing, whilst K″ continues; and $-\overset{x}{\overline{\begin{matrix}0\\ n''\\ m''\end{matrix}}}\Big|\, K''_x \cdot \dot{K}'''_x$ is the chance, that between the times n''

and m'', K''' discontinues during the time of K'''s continuing: And if y be either a constant quantity or a function of x, $-\overset{x}{\left.\begin{matrix}o\\n''\\m''\end{matrix}\right|}K''_y\,\dot{K}'''_x$ will denote the chance due to the time between n'' and m''', that is whilst x becomes from n'' to be equal to m''; that whatever x may be, at the time of K''''s discontinuance, that K'' shall be in continuance at the corresponding time y.

No. 3. $-K'_y\left(\overset{x}{\left.\begin{matrix}o\\n\\y\end{matrix}\right|}K''_x\,\dot{K}'''_x\right)$ will denote the chance that the event K' continues during the time y; with the proviso, that the event K'' shall fail some time between the times n and y; but on the condition that whenever that event shall take place, the event K'' shall not yet have discontinued.

No. 4. And $-K'_y\left(\overset{x}{\left.\begin{matrix}o\\n\\y\end{matrix}\right|}\overline{1-K''_x}\,.\,\dot{K}'''_x\right)$ will express the similar chance, except that the discontinuance instead of the continuance of K''' is to take place after the discontinuance of K''.

No. 5. $\overset{y}{\left.\begin{matrix}o\\n\\m\end{matrix}\right|}\left(\dot{K}_y\,\overset{x}{\left.\begin{matrix}o\\n'\\y\end{matrix}\right|}K'_x\,\dot{K}''_x\right)$ will denote the portion of the chance due to the time between n and m; of K discontinuing after the event has taken place of K'''s having discontinued during the continuance of K'; if that *super-continuance* of K' should not take place before the time n'. And if in the above expression $1-K_x$ be written for K'_x every thing will be the same; with the exception that that part which referred to the continuance of K', will now refer to the discontinuance of K' having taken place.

No. 6. $\overset{x}{\left.\begin{matrix}0\\n\\m\end{matrix}\right|}(\dot{K}_x K'_x \overset{y}{\left.\begin{matrix}0\\n'\\x\end{matrix}\right|} K''_y \dot{K}'''_y)$ will denote the portion of chance due to the time between n and m, that K shall discontinue during the continuance of K′, on the condition that previously to that event taking place, K‴ shall after the time n' have discontinued during the continuance of K″.

And thus we might proceed to an infinite variety of cases with regard to the limits in time, with regard to the number of events, &c.; and however compounded and numerous the signs of *fluentization* and summation may be, and if this mode of enunciation be duly considered, it will be found that the meaning of the more compound cases is much easier to express and to understand, in this analytical language, than by a more elaborate phraseology; and that this mode enables us as soon as the meaning of the question is understood, granting the theory of summations and *fluentizations*, to solve it.

No. 7. By way of illustration I shall only add here, that the nature of the events to which K, K′, K″ &c. may refer, is unlimited; they may refer to single lives only, to joint lives, to joint lives connected with other joint lives, to joint lives connected with deaths, &c. And that more compounded cases may be understood, I also mention that

$-\overset{n}{\left.\begin{matrix}p\\n-p\\m-p\end{matrix}\right|}(r^{x+p}. \overset{x}{\left.\begin{matrix}0\\n\\n+p\end{matrix}\right|} K''_x \dot{K}'''_x)$ expresses the assurance of one pound to be received at the first of the equal periods p, after the time $n-p$, that shall happen after the discontinuance of K‴; provided that that discontinuance happens during the continuance of K″; and provided also, it happens between the time $n-p$ and m; the present value of one pound certain to be received in one year being equal to r.

Article 6. As it is probable that the reader will be desirous to see the application of our theory to some of the problems which have been esteemed the most difficult, I shall consider the problems which have been solved by Mr. MORGAN, in the Philosophical Transactions; and in pursuing this object, I shall take the examples in the order in which they have been presented by Mr. BAILY, in his work on Assurances, from page 206, because that Gentleman's book was in my hand when I worked the examples. Note, that the object which I denote by p, has with those Gentlemen the particular value one year; and is supposed by them to be wholly within the limits of proportional decrement. Note also, that the present ages of A, B and C, I denote by a, b and c.

Example 1. The chance of A dying, the first of the three lives A, B, C, is by taking K″ of No. 2, in the last article $= \frac{L_{x:b,c}}{L_{b,c}}$, and $K''' = \frac{L_{a+x}}{L_a}$, the fluent of $\frac{L_{x:b,c}}{L_{a,b,c}}\dot{L}_{a+x}$.

If p be within the limits of proportional decrements (that is of decrements proportionate to the times) then by Section 3, Article 9, the chance of the events happening between the times π and $\pi+p$, that is $\left.\begin{matrix}x\\0\\\pi\\\pi+p\end{matrix}\right]\frac{L_{x:b,c}}{L_{a,b,c}}\cdot\dot{L}_{a+x}$ is $\frac{L_{a+\pi}-L_{a+\pi+p}}{L_{a,b,c}}\times$ $L_{\overline{\pi+\frac{p}{2}}:b,c}+\frac{1}{12}\frac{L_{a+\pi}-L_{a+\pi+p}}{L_a}\times\frac{L_{b+\pi}-L_{b+\pi+p}}{L_b}\times\frac{L_{c+\pi}-L_{c+\pi+p}}{L_c}$;

or if p be small; such for instance as one year, it will be simply $\frac{L_{a+\pi}-L_{a+\pi+p}}{L_{a,b,c}}\,.\,L_{\overline{\pi+\frac{p}{2}:b,c}}$ very nearly; as the other part will be comparatively with this extremely small; as an instance, if p answered to one year; $a+\pi$, $b+\pi$, $c+\pi$, each to 34 years, and the Northampton Tables be used: the error caused by neglecting the part in question, will not amount to the thirty three thousandth part of the real value. The error towards the very commencement of life, or towards the end, is certainly greater; thus if the values of $a+\pi$, $b+\pi$, $c+\pi$ were all 90; the error with the same tables, and on the same hypothesis of decrement, would bear a nearer proportion to the real value; but would not amount to the one hundred and thirtieth part of the real value; and it should be observed in these extreme cases, the hypothesis itself is defective. Moreover, the present value of one pound to be received at the end of the time $\pi+p$, if the event should take place between the times π and $\pi+p$, will be the said expression × $r^{\pi+p}$; and the sum of all the values produced by interpreting π, by $n-p$, n, $n+p$, &c. to $m-p$, will be the present value of the assurance of one pound on the contingency. If we wish to calculate this from tables of the values of periodic incomes; as the part due to the interval between π and $\pi+p$, may be expressed by neglecting the small part above alluded to,

$r^{\pi+p}\,.\,\frac{L_{\pi:a,b+\frac{1}{2}\overline{p,c+}\cdot\frac{1}{2}p}\;L_{\pi:a+p,b+\frac{1}{2}p,c+\frac{1}{2}p}}{L_{a,b,c}}$ or its equal

$r^{\pi+p}\,.\,\frac{L_{a-p,b-\frac{1}{2}p,c-\frac{1}{2}p}}{L_{a,b,c}}\times\frac{L_{\pi:a,b+\frac{1}{2}p,c+\frac{1}{2}p}}{L_{a-p,b-\frac{1}{2}p,c-\frac{1}{2}p}}-r^{\pi+p}\,.\,\frac{L_{b-\frac{1}{2}p,c-\frac{1}{2}p}}{L_{b,c}}$ $\times\frac{L_{\pi:a+p,b+\frac{1}{2}p,c+\frac{1}{2}p}}{L_{a,b-\frac{1}{2}p,c-\frac{1}{2}p}}$; and therefore we have the value of the assurance equal to

$$\frac{L_{a-p,\, b-\frac{1}{2}p,\, c-\frac{1}{2}p}}{L_{a,b,c}} \times \begin{array}{c} r \\ \smile \\ p \\ n \\ m \end{array}\Big|\underline{a-p,\, b-\tfrac{1}{2}p,\, c-\tfrac{1}{2}p} - \frac{L_{b-\frac{1}{2}p,\, c-\frac{1}{2}p}}{L_{b,c}} \begin{array}{c} r \\ \smile \\ p \\ n \\ m \end{array}\Big|\underline{a,\, b-\tfrac{1}{2}p,\, c-\tfrac{1}{2}p}.$$

This is on the supposition that we have neglected the value due to the part $\frac{1}{12} r^{n+p} \times \frac{L_{a+n} - L_{a+n+p}}{L_a} \times \frac{L_{b+n} - L_{b+n+p}}{L_b} \times \frac{L_{c+n} - L_{c+n+p}}{L_c}$, as being extremely small when p is small, such for instance as one year. But I observe that a near approximation of its value in this case is one twelfth of the income certain for the term multiplied by the chance of all three dying between the intervals n and $n+p$; and as an example of $p=1$ $n=1$, and the corresponding assurance for the whole possible joint existence be required from the Northampton Tables, for three lives at the age of 30, at the interest of 3 per cent.; considering the last income possibly to be received, to be at the age 95, the term will be 65, and the *approximative* value of this neglected part, if the assurance be on £1000. to be paid for in one payment; will be $1000 \times \frac{\overline{75}|^{3} \times 28{,}45}{12 \times \overline{4385}|^{3}}$ $=$ about $\frac{2845}{24.00.00}$ of one pound, which is less than three pence. If we use Article 10, Section 3, we shall obtain the same portions of the contingencies as has been used by Messrs. MORGAN and BAILY, whence &c.

" Note. I may observe that if the contingencies were as in " this example with the exception that A is to die before any " of the lives B, G, D, F, &c. the value would be

$$“\ \frac{L_{a-p,\, b-\frac{1}{2}p,\, c-\frac{1}{2}p,\, d-\frac{1}{2}p,\, \&c.}}{L_{a,b,c,d,\&c.}} \cdot \begin{array}{c} r \\ \smile \\ p \\ n \\ m \end{array}\Big|\underline{a-p,\, b-\tfrac{1}{2}p,\, c-\tfrac{1}{2}p,\, d-\tfrac{1}{2}p,\, \&c.} -$$

"$\frac{L_{b-\frac{1}{2}p,\,c-\frac{1}{2}p,\,d-\frac{1}{2}p,\,\&c.}}{L_{b,\,c,\,d,\,\&c.}} \times \overset{r}{\overbrace{p}}_{\substack{n\\m}}\Big|\underline{a,b-\tfrac{1}{2}p,c-\tfrac{1}{2}p,d-\tfrac{1}{2}p}$, &c. very nearly."

Note also, that if the lives be all equal, and there be q of them in number, the contingency of one in particular dying before the rest will be — fluent of $\frac{\overline{L_{a+x}}\rvert^{q-1}}{\overline{L_a}\rvert^{q}}\dot{L}_{a+x} =$ (if it commences with $x=0$) $\frac{1}{q}\left(1-\frac{\overline{L_{a+x}}\rvert^{q}}{L_a}\right)$ and the assurance on that contingency will be $\frac{1}{q}\times$ the assurance on the joint lives.

Example 2. In a similar manner from Art. 2. No. 5, of this section, writing $\frac{L_{c+x}}{L_c}.\overline{1-\frac{L_{b+x}}{L_b}}+\overline{1-\frac{L_{c+x}}{L_c}}.\frac{L_{b+x}}{L_b}$ for K″, and $\frac{L_{a+x}}{L_a}$ for K‴, we have the contingency that A is the second which fails of the three lives A, B, C $\doteq$ — fluent of $\left(\frac{L_{c+x}.\dot{L}_{a+x}}{L_{c,a}}+\frac{L_{b+x}.\dot{L}_{a+x}}{L_{b,a}}-\frac{2L_{x:b,c}.\dot{L}_{a+x}}{L_{a,b,c}}\right)=$ chance of A's dying before C + that of A's dying before B — twice the chance of A's dying before B and A; and the assurance is in a similar manner made up of the assurances on the like contingencies as Messrs. MORGAN, &c. have shown.

Example 3. If K″ be put $=\overline{1-\frac{L_{b+x}}{L_b}}\times\overline{1-\frac{L_{c+x}}{L_c}}$; that is $1-\frac{L_{c+x}}{L_c}-\frac{L_{b+x}}{L_b}+\frac{L_{b+x}.L_{c+x}}{L_{b,c}}$, and $K'''=\frac{L_{a+x}}{L_a}$, in the same article No. 2; we shall have the contingency of A's being the last which fails of the three lives = — fluent of $\left(\frac{\dot{L}_{a+x}}{L_a}\right.$

$- \frac{L_{b+x} . \dot{L}_{a+x}}{L_{b,a}} - \frac{L_{c+x} . \dot{L}_{a+x}}{L_{c,a}} + \frac{L_{x:b,c} . \dot{L}_{a+x}}{L_{a,b,c}} \Big)$; hence the assurance on this contingency is equal to the assurance on A's death, assurance on A's death if survived by B, — assurance on A's death if survived by C + assurance on A's death if survived by B and C, as Mr. Morgan, &c. makes it.

Example 4. If $K'' = 1 - \left(1 - \frac{L_{c+x}}{L_c}\right) . \left(1 - \frac{L_{b+x}}{L_b}\right)$ and $K''' = \frac{\dot{L}_{a+x}}{L_a}$, we shall see that the contingency of A's dying, and that he is the 1st or 2nd which dies, is equal to — the fluent $\frac{\dot{L}_{a+x}}{L_a} \times \left(\frac{L_{c+x}}{L_c} + \frac{L_{b+x}}{L_b} - \frac{L_{c+x} . L_{b+x}}{L_{c,b}}\right)$. Hence, &c.

Example 5. If A is to be the second or third which fails, by taking $K'' = 1 - \frac{L_{x:b,c}}{L_{b,c}}$, namely, the chance that B and C are not both living, we have the contingency of A's death $= -$ fluent $\overline{1 - \frac{L_{x:b,c}}{L_{b,c}}} . \frac{\dot{L}_{a+x}}{L_a}$. Hence, &c. And the assurance equal to the assurance of A's life absolutely — the assurance of his life on condition that he dies first.

Example 6. If A is either to be the first or last to die: then is the chance on his death equal to — fluent of $\frac{\dot{L}_{a+x}}{L_a} \times \left(\frac{L_{x:b,c}}{L_{b,c}} + \overline{1 - \frac{L_{b+x}}{L_b}} . \overline{1 - \frac{L_{c+x}}{L_c}}\right) = -$ fluent of $\dot{L}_{a+x} \times \left(1 - \frac{L_{b+x}}{L_b} - \frac{L_{c+x}}{L_c} + 2 \frac{L_{x:b,c}}{L_{b,c}}\right)$. Hence, &c.

Example 7. To find the contingency on the first of the two death A and B; provided that that be the first or last which dies of the three A, B, C. Here I first take $K'' = \frac{L_{x:b,c}}{L_{b,c}}$ and $K''' = \frac{L_{a+x}}{L_a}$; and then take $K'' = \frac{L_{x:a,c}}{L_{a,c}}$ and $K''' = \frac{L_{b+x}}{L_b}$; and I find the contingency $= -$ fluent of $\left(\frac{\dot{L}_{a+x}}{L_a} . \frac{L_{x:b,c}}{L_b} + \frac{\dot{L}_{b+x}}{L_c} . \frac{L_{x:a,c}}{L_{a,c}}\right) = -$ fluent of $\frac{L_{c+x}}{L_c} \left(\frac{L_{x:a,b}}{L_{a,b}}\right)^{\cdot} =$ correction $- \frac{L_{x:a,b,c}}{L_{a,b,c}}$ $+$ fluent of $\frac{L_{x:a,b}}{L_{a,b}} . \frac{\dot{L}_{c+x}}{L_c}$. Note, the word correction might have been omitted by considering it implicitly contained in the word fluent. Hence the assurance on the first of the deaths of A and B, provided it be the first to fail of the three A, B, C, is equal to the absolute assurance on the three joint lives, less the assurance on C's life, provided he dies first: the same as Mr. BAILY makes it in a note at page 240; by comparing the result of his solution with a former solution; but I should observe there is a typographical error in the note, by inserting + ABC instead of — ABC.

Example 8. If the first of the deaths of A and B, is to be the second of the three; in this case taking first $K'' = \overline{1 - \frac{L_{b+x}}{L_q}} . \frac{L_{c+x}}{L_c} + \overline{1 - \frac{L_{c+x}}{L_c}} . \frac{L_{b+x}}{L_b}$ and $K''' = \frac{L_{a+x}}{L_a}$, and then $K'' = \overline{1 - \frac{L_{a+x}}{L_a}} . \frac{L_{c+x}}{L_c} + \overline{1 - \frac{L_{c+x}}{L_c}} . \frac{L_{a+x}}{L_a}$ and $K''' = \frac{L_{b+x}}{L_b}$, we get the contingency $= -$ fluent of $\left\{ \frac{\dot{L}_{a+x}}{L_a} \left(\frac{L_{c+x}}{L_c} + \frac{L_{b+x}}{L_b} - \frac{2L_{x:b,c}}{L_{b,c}} \right) + \frac{\dot{L}_{b+x}}{L_b} \right.$

$\left.\left(\frac{\dot{L}_{c+x}}{L_c}+\frac{L_{a+x}}{L_a}-\frac{2L_{x:a,c}}{L_{a,c}}\right)\right\}=$ correction $-\frac{L_{x:a,b}}{L_{a,b}}-$ fluent of $\left(\frac{\dot{L}_{a+x}}{L_a}\cdot\frac{L_{c+x}}{L_c}+\frac{\dot{L}_{b+x}\cdot L_{c+x}}{L_b\cdot L_c}-\frac{2L_{x:b,c}}{L_{b,c}}\cdot\frac{\dot{L}_{a+x}}{L_a}-\frac{2L_{x:a,c}}{L_{a,c}}\cdot\frac{\dot{L}_{b+x}}{L_b}\right)$; and the assurance on the contingency may evidently bear the form in which Mr. Baily has put it.

Example 9. On the death of the last survivor of A, B and C; provided that should be either A or B: this contingency in a similar manner from the proper interpretations of K″ and K‴, is found immediately $=-$ fluent of $\left(\overline{1-\frac{L_{b+x}}{L_b}}\cdot\overline{1-\frac{L_{c+x}}{L_c}}\cdot\frac{\dot{L}_{a+x}}{L_a}+\overline{1-\frac{L_{a+x}}{L_a}}\cdot\overline{1-\frac{L_{c+x}}{L_c}}\cdot\frac{\dot{L}_{b+x}}{L_b}\right)=-$ fluent of $\overline{1-\frac{L_{c+x}}{L_c}}\times\left(\overline{1-\frac{L_{b+x}}{L_b}}\cdot\frac{\dot{L}_{a+x}}{L_a}+\overline{1-\frac{L_{a+x}}{L_a}}\cdot\frac{\dot{L}_{b+x}}{L_b}\right)=$ fluent of $\overline{1-\frac{L_{c+x}}{L_c}}\cdot\left(\overline{1-\frac{L_{a+b}}{L_a}}\right)^{\cdot}=\overline{1-\frac{L_{c+x}}{L_c}}\cdot\overline{1-\frac{L_{b+x}}{L_b}}\cdot\overline{1-\frac{L_{c+x}}{L_c}}+$ fluent of $\frac{\dot{L}_{c+x}}{L_c}\left(\overline{1-\frac{L_{b+x}}{L_b}}\cdot\overline{1-\frac{L_{a+x}}{L_a}}\right)$. Hence the assurance on this contingency, will be equal to the assurance on the longest of the three lives; — the assurance of C's life, provided that he be the last which fails. See Example 8. This in fact is almost evident at first sight.

Example 10. On the death of the first of the two A and B, provided it be the first or second which fails. This, as Mr. Morgan, &c. observe, is on the extinction of the joint lives A and B only, &c.

Article 7. I shall now offer in the same order as in Mr. BAILY's work, some other questions of Mr. MORGAN's papers, which are most of them of a nature in point of solution different; in as much as that they contain in my method double fluents; or, as we have reduced them, contain fluents multiplied by variable quantities.

Example 1. The contingency of the first of the deaths of A and B, which shall be the second or third which happens of the three A, B and C, will be — fluent of $\left(1-\frac{L_{c+x}}{L_c}\right)\left(\frac{L_{x.a,b}}{L_{a,b}}\right)^{\cdot}$ $+$ fluent of $\left(\frac{\dot{L}_{b+x}}{L_b}\right.$ fluent of $\left.\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{a+x}}{L_a}\right)$ $+$ fluent of $\frac{\dot{L}_{a+x}}{L_a}$. fluent of $\frac{L_{c+x}\cdot\dot{L}_{b+x}}{L_c\cdot L_b}$. Since the first part is by Article 5, No. 2, of this Section; the contingency of the joint lives A and B failing after C, and the second by No. 5, denotes the chance of B's dying after the event has taken place of A's dying in the life time of C: and the third denotes the chance of A's dying after the event has taken place of B dying in the life time of A; and independent of the correction, the first part is evidently $=\frac{-L_{x:a,b}}{L_{a,b}}$ $+$ fluent of $\frac{L_{c+x}\cdot(L_{a+x}\cdot L_{b+x})^{\cdot}}{L_{a,b,c}}$; the second part is $=\frac{L_{b+x}}{L_b}$ fluent of $\frac{L_{c+x}\cdot\dot{L}_{a+x}}{L_{c,a}}$ — fluent of $\frac{L_{x\;c,b}\cdot\dot{L}_{a+x}}{L_{a,b,c}}$ and the third part is equal to $\frac{L_{a+x}}{L_a}$ fluent of $\frac{L_{c+x}\cdot\dot{L}_{b+x}}{L_{c,b}}$ — fluent of $\frac{L_{x:a,c}\cdot\dot{L}_{b+x}}{L_{a,b,c}}$; and the sum of the three or the contingency in question is independent of the correction

$=-\frac{L_{x:a,b}}{L_{a,b}}+\frac{L_{b+x}}{L_b}$. fluent of $\frac{L_{c+x}\cdot\dot{L}_{a+x}}{L_c\quad L_a}+\frac{L_{a+x}}{L_a}$ fluent of $\frac{L_{c+x}\cdot\dot{L}_{b+x}}{L_c\cdot L_b}$; and here it is evident, that when the lives are all equal, the contingency will become constant $-\frac{L_{x:a,a}}{L_{a,a}}+\frac{L_{a+x}}{L_a}$ $\overline{\times\text{ constant}+\left(\frac{L_{a+x}}{L_a}\right)^2}$; or if all the fluents commence with $x=o$ it is $1-\left(\frac{L_{a+x}}{L_a}\right)^2-\frac{L_{a+x}}{L_a}+\left(\frac{L_{a+x}}{L_a}\right)^3$. But in other cases, if we are to have all the contingencies commence with $x=o$, and we are satisfied with the approximation that when two persons are dead, it is an equal chance which has died first, see note, Art. 2, Section 4, we shall have

$$\left.\begin{aligned}&\frac{L_{b+x}}{L_b}\text{ fluent of }\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{a+x}}{L_a}=-\tfrac{1}{2}\frac{L_{b+x}}{L_b}+\tfrac{1}{2}\frac{L_{x:a,b}}{L_{a,b}}-\tfrac{1}{2}\frac{L_{x:b,c}}{L_{b,c}}+\tfrac{1}{2}\frac{L_{x:a,b,c}}{L_{a,b,c}}\\ &\text{and }\frac{L_{a+x}}{L_a}\text{ fluent of }\frac{L_{c+x}\cdot\dot{L}_{b+x}}{L_c\quad L_b}=-\tfrac{1}{2}\frac{L_{a+x}}{L_a}+\tfrac{1}{2}\frac{L_{x:a,b}}{L_{a,b}}.-\tfrac{1}{2}\frac{L_{x:a,c}}{L_{a,c}}+\tfrac{1}{2}\frac{L_{x:a,b,c}}{L_{a,b,c}}\end{aligned}\right\}$$

and therefore the whole becomes $=-\frac{1}{2}\frac{L_{b+x}}{L_b}-\frac{1}{2}\frac{L_{a+x}}{L_a}$ $-\frac{1}{2}\frac{L_{x:b,c}}{L_{b,c}}-\frac{1}{2}\frac{L_{x:a,c}}{L_{a,c}}+\frac{L_{x:a,b,c}}{L_{a,b,c}}-1$, and the part of the contingency due to the interval between the times π and $\pi+p$ is $\frac{L_{b+\pi}-L_{b+\pi+p}}{2L_b}+\frac{L_{a+\pi}-L_{a+\pi+p}}{L_a}+\frac{L_{\pi:b,c}-L_{\pi+p:b,c}}{2L_{b,c}}+\frac{L_{\pi:a,c}-L_{\pi+p:a,c}}{2L_{a,c}}-\frac{L_{\pi:a,b,c}-L_{\pi+p,a,b,c}}{L_{a,b,c}}$. Hence the assurance of one pound on the contingency, in case the event should happen between the times $n-p$ and m; to be paid at the first of the equal periods p, from $n-p$, after the event, provided it be not beyond the time m is $\frac{1}{2} \cdot \overset{\overset{r}{\smile}}{\underset{\underline{m}}{\substack{p\\n}}}\left|\underline{\,b\,}\right. + \frac{1}{2} \overset{\overset{r}{\smile}}{\underset{\underline{m}}{\substack{p\\n}}}\left|\underline{\,a\,}\right. + \frac{1}{2} \overset{\overset{r}{\smile}}{\underset{\underline{m}}{\substack{p\\n}}}\left|\underline{\,b, c\,}\right.$ $+ \frac{1}{2} \overset{\overset{r}{\smile}}{\underset{\underline{m}}{\substack{p\\n}}}\left|\underline{\,a, c\,}\right. - \overset{\overset{r}{\smile}}{\underset{\underline{m}}{\substack{p\\n}}}\left|\underline{\,a, b, c\,}\right.$; that is half the sum of the assurances for the term on B's life, on A's life, on BC's joint life, and on AC's joint life;—the assurance on A, B, C's joint life for the term: on the supposition that if two persons are both to be dead, in a certain time less than their possible time of joint existence, it is an equal chance which is the survivor; our theorem therefore only goes to that term; after this, by consulting the formula, constant $- \frac{L_{x:a,b}}{L_{a,b}} + \frac{L_{b+x}}{L_b}$. fluent of $\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{a+\dot{x}}}{L_a} + \frac{L_{a+x}}{L_a}$ fluent of $\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{b+x}}{L_b}$; we find that if C be the one which must of necessity die before the extreme age of B or A; and if there is a possibility of his living as long as x is less than μ; we shall have $L_{c+\mu} = 0$: and the fluents of $\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{a+x}}{L_a}$ and $\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{b+x}}{L_b}$, after that time equal $-g$ and $-h$ if $-g$ and $-h$ be the values at that time; ∴ after that time, the contingency, is constant $- \frac{L_{x:a,b}}{L_{a,b}} - g \cdot \frac{L_{b+x}}{L_a} - h \cdot \frac{L_{a+x}}{L_a}$; ∴ if the whole assurance be required, find the whole value whilst m is not greater than μ, to which add $\overset{\overset{r}{\smile}}{\underset{\underline{m}}{p}}\left|\underline{\,a, b\,}\right. + g\, \overset{\overset{r}{\smile}}{\underset{\underline{m}}{p}}\left|\underline{\,b\,}\right. + h\, \overset{\overset{r}{\smile}}{\underset{\underline{m}}{p}}\left|\underline{\,b\,}\right.$.

But if A be the oldest, and there is a possibility of his living as long as x is less than μ, but not longer; use the theorem as long as m is less than μ; if the whole assurance be required: and observing that when x is equal to, or greater than μ, that $\frac{L_{a+x}}{L_a}$. fluent of $\frac{L_{c+x}}{L_c} \cdot \frac{L_{b+x}}{L_b}$ is $= o$; and that fluent of $\frac{L_{c+x}}{L_c} \cdot \frac{L_{a+x}}{L_a}$ will be $= -g$; if $-g$ is its value when $x = \mu$, we shall find the contingency after that term, that is when x is greater than μ, $=$ constant $- g \frac{L_{b+x}}{L_b}$; and $\therefore$ the remainder of the assurance will be $= g \,..\, \underset{\underline{m}}{\overset{\overset{r}{\smile}}{p}} \big| \underline{b}$ to be added. I do not state the case of B's being the oldest, because it is only necessary to write a for b in the last case to have this.

If we should not be satisfied with the approximation deduced, by assuming the equality of chance above named, during a long period, we have only to divide the period in shorter periods to attain any accuracy, being careful properly to correct the fluents.

The theorem I have just given for a solution to the problem in question, is so much more simple than the solutions I have seen to this problem, that I think it proper to inform the reader, that the cause will be understood from Art. 6, Section 3; and that no fear may be left in using the Theorem, I shall point out the connection between this solution and that given by Mr. BAILY, and I shall for the easier comparison denote L_a, L_b, L_c, by a, b, c; L_{a+n}, L_{b+n}, L_{c+n} by a', $b,'$ c'; and L_{a+n+1}, L_{b+n+1}, L_{c+n+1} by a'', b'', c''; and as the contingency, that the event takes place before the expiration of $n+1$, years will be found by writing n for x in the formula, constant $-\frac{L_{x:a,b}}{L_{a,b}}+\frac{L_{b+x}}{L_b}$ fluent of $\frac{L_{c+x}\cdot\dot{L}_{a+x}}{L_c\cdot L_a}+\frac{L_{a+x}}{L_a}\times$ fluent of $\frac{L_{c+x}\cdot\dot{L}_{b+x}}{L_c\cdot L_b}$; we find this value under the idea of constant decrements for the whole term to be $1-\frac{a'b'}{ab}+\frac{b'}{b}\cdot\left(-\frac{1}{2}\frac{ac}{ac}+\frac{1}{2}\frac{c.a'}{ac}-\frac{1}{2}\frac{c'a}{ac}+\frac{1}{2}\frac{a'c'}{ac}\right)+\frac{a'}{a}\left(-\frac{1}{2}\frac{bc}{bc}+\frac{1}{2}\frac{b'c}{bc}-\frac{1}{2}\frac{c'b}{bc}+\frac{1}{2}\frac{b'c'}{b.c}\right)=1+\frac{a'b'c'}{abc}-\frac{1}{2}\frac{b'}{b}-\frac{1}{2}\frac{a'}{a}-\frac{1}{2}\frac{b'c'}{bc}-\frac{1}{2}\frac{a'c'}{bc}$; and under the same hypothesis of the constant decrements, during the whole time $n+1$, we have the corresponding contingency $1+\frac{a'',b'',c''}{abc}-\frac{1}{2}\frac{b''}{b}-\frac{1}{2}\frac{a''}{a}-\frac{1}{2}\frac{b''c''}{bc}-\frac{1}{2}\frac{a''c''}{bc}$; and the excess of this above the contingency for the term n, is $\frac{1}{2}\frac{b'-b''}{b}+\frac{1}{2}\frac{a'-a''}{a}+\frac{b'c'-b''c''}{2bc}+\frac{a'c'-a''c''}{2ac}-\frac{a'b'c'-a''b''c''}{2ac}$; the same except in notation as before, as it should be; the difference of the operation only being in the notation. But if we only consider the uniform decrements to reach to the period n, and then during the next year take proportional decrements during the year, we shall have the fluent of $\frac{L_{c+x}\cdot\dot{L}_{a+x}}{L\cdot L_a}$ answering to $x=n+1$, $=-\frac{1}{2}\frac{ac}{ac}+\frac{1}{2}\frac{ca'}{ac}-\frac{1}{2}\frac{c'a}{ac}+\frac{1}{2}\frac{a''c'}{ac}-\frac{1}{2}\frac{a'c''}{ac}+\frac{c''a''}{2ac}$. See Art. 6, Section 3; consequently the value of $\frac{L_{b+x}}{L_b}\cdot$ fluent of $\frac{L_{c+x}\cdot L_{a+x}}{L_c\cdot L_b}$ answering to $x=n+1$ is $=-\frac{1}{2}\frac{b''}{b}+\frac{1}{2}\frac{a'b''}{ab}-\frac{1}{2}\frac{c'b''}{bc}+\frac{1}{2}\frac{b''c'a''}{abc}-\frac{1}{2}\frac{a'b''c''}{abc}+\frac{a''b''c''}{2abc}$; the corresponding value of $\frac{L_{a+x}}{L_a}$ fluent $\frac{L_{c+x}}{L_c}\cdot\frac{L_{b+x}}{L_b}$ will be

$= -\frac{1}{2}\frac{a''}{a} + \frac{1}{2}\frac{a''b'}{ab} - \frac{1}{2}\frac{c'a''}{ca} + \frac{1}{2}\frac{a''c'b''}{abc} - \frac{1}{2}\frac{a''b'c''}{abc} + \frac{a''b''c''}{2abc}$; and the corresponding value of $\frac{\dot{L}_{x:a,b}}{L_{a,b}} = \frac{a''b''}{ab}$. Hence we have the contingency on this hypothesis due to $n+1$ years $= 1 - \frac{a''b''}{ab} - \frac{1}{2}\frac{b''}{b} - \frac{1}{2}\frac{a''}{a} + \frac{1}{2}\frac{a'b''}{ab} + \frac{1}{2}\frac{a''b'}{ab} - \frac{1}{2}\frac{c'b''}{bc} - \frac{1}{2}\frac{c'a''}{ac} + \frac{a''b''c'}{abc} - \frac{a'b''c''}{2abc} - \frac{a''b'c''}{2abc} + \frac{a''b''c''}{abc}$; and if we take from this the chance of the events happening in n year, the remainder will be the chance of its happening during the interval between n and $n+1$ years; and will come out evidently $\frac{a'-a''}{2a} + \frac{b'-b''}{2b} + \frac{a'b''+a''b'-2a''b''}{2ab} + \frac{ab'-2a'b'+a'b-ab''-a''b+2a''b''}{2ab} \cdot \frac{c'}{c} - \frac{a'b''+a''b'-2a''b''}{2ab} \cdot \frac{c''}{c}$. And if we consider Mr. BAILY's a', b', c'; a'', b'', c'' to refer to the n^{th} and $n+1^{th}$ year, and collect his 1st, 2d, 3d, 5th, 6th, 8th, 9th, and 12th, contingencies, we shall find them amount to $\frac{a'-a''}{a}\left(\frac{b+b'}{2b} + \frac{c'}{c} \cdot \frac{b-b'-b''}{2b} - \frac{c''b''}{2bc}\right)$; and collecting the remaining terms which are 4th, 7th, 10th, 11th, and 13th, we obtain $\frac{b'-b''}{b}\left(\frac{a+a}{2a} + \frac{a-a'-a''}{2a} \cdot \frac{c'}{c} - \frac{a''c''}{ac}\right) - \frac{b'-b''}{b} \cdot \frac{a'-a''}{a}$; and if these two be added together we shall obtain the above expression.

Example 2. On the contingency of the first of the deaths of A and B, which shall be the first or last of the three A, B, C. This will evidently be the sum of the contingencies, of the joint lives discontinuing in C's life time; that A dies after B, C having died before B; and that B dies after A, C having died before A; therefore by Art 5 of this Section it is $= -$ fluent of $\frac{L_{c+x}}{L_c}\left(\frac{L_{x:a,b}}{L_{a,b}}\right)^{\cdot} +$ fluent of $\left(\frac{\dot{L}_{a+x}}{L_a}\right.$ fluent of $\left(1 - \frac{L_{c+x}}{L_c}\right)$

$\left.\frac{\dot{L}_{b+x}}{L_b}\right)$ + fluent of $\left(\frac{L_{b+x}}{L_b}\right.$ fluent of $\left.\left(1-\frac{L_{c+x}}{L_c}\right)\frac{\dot{L}_{a+x}}{L_a}\right)$; but considering all the fluents to commence with $x=o$, the second term is = fluent of $\left(\frac{\dot{L}_{a+x}}{L_a}\times\left(\frac{L_{b+x}-1}{L_b}-\right.\right.$ fluent of $\left.\left.\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}\right)\right)$ and the third term is = fluent of $\left(\frac{\dot{L}_{b+x}}{L_b}\times\left(\frac{L_{a+x}}{L_a}-1-\right.\right.$ fluent of $\left.\left.\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{a+x}}{L_a}\right)\right)$, and therefore the sum of the two = fluent of $\left(\frac{L_{b+x}}{L_b}\cdot\frac{L_{a+x}}{L_a}\right)^{\cdot}-$ fluent of $\left(\frac{\dot{L}_{a+x}}{L_a}\right.$ fluent of $\left.\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}\right)\Big)$ + fluent of $\frac{\dot{L}_{b+x}}{L_b}$ fluent of $\left.\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}\right)+1-\frac{L_{a+x}}{L_a}+1-\frac{\dot{L}_{b+x}}{L_b}$; and the whole by comparison with the commencement of the last example; if the fluents are to be corrected in a similar manner to vanish with $x=o$ shows immediately that $1-\frac{L_{a+x}}{L_a}+1-\frac{L_{b+x}}{L}-$ that value is equal to this; and therefore that the assurance of the contingency here, is the excess of the sum of the assurances on A's life and on B's life singly, above that: this agrees with the ingenious Mr. MILNE's observations on page 232 of his work on Assurances; and therefore, according to our solution of that case, we have during the possible joint existence the value

$$=\frac{1}{2}\cdot\begin{smallmatrix}\overset{r}{\smile}\\ p\\ n\\ \underline{m}\end{smallmatrix}\left|\underline{\,a\,}\right.+\frac{1}{2}\cdot\begin{smallmatrix}\overset{r}{\smile}\\ p\\ n\\ \underline{m}\end{smallmatrix}\left|\underline{\,b\,}\right.-\frac{1}{2}\cdot\begin{smallmatrix}\overset{r}{\smile}\\ p\\ n\\ \underline{m}\end{smallmatrix}\left|\underline{\,b,c\,}\right.-\frac{1}{2}\cdot\begin{smallmatrix}\overset{r}{\smile}\\ p\\ n\\ \underline{m}\end{smallmatrix}\left|\underline{\,a,c\,}\right.+\begin{smallmatrix}\overset{r}{\smile}\\ p\\ n\\ \underline{m}\end{smallmatrix}\left|\underline{\,a,b,c\,}\right.,$$ and

beyond that term the process as in the last example. This is according to the hypothesis of equality of chance, so often mentioned and used by Mr. MORGAN, &c.

Example 3. On the contingency of A's dying after C in the life time of A. This should come in the last article, as it does not involve the double fluent, but is given here not to interrupt the order in which I have taken my examples: the solution is — fluent of $\frac{L_{a+x}}{L_a} \cdot \overline{1 - \frac{L_{a+x}}{L_a}} \cdot \frac{L_{b+x}}{L_b}$, and is the chance of A's dying in the life time of B, — the chance of A's dying in the life time of both A and B. Hence the assurances is determined from those cases, as Mr. BAILY has done page 273.

Example 4. On the contingency of A's dying last; on the condition that C dies before B. This will evidently be fluent of $\frac{\dot{L}_{a+x}}{L_a}$ (fluent of $\left(1 - \frac{L_{c+x}}{L_c}\right) \frac{\dot{L}_{b+x}}{L_b}\Big) =$ if all the contingencies are supposed to commence with $x = o$, and the fluents be corrected to vanish with $x = o$, fluent of $\Big(\frac{\dot{L}_{a+x}}{L_a} \frac{L_{b+x}}{L_q} - \frac{\dot{L}_{a+x}}{L_a}$ $- \frac{L_{a+x}}{L_a}$. fluent of $\frac{\dot{L}_{c+x}}{L_c} \cdot \frac{\dot{L}_{b+x}}{L_b}\Big) = 1 - \frac{L_{a+x}}{L_a} +$ fluent of $\frac{\dot{L}_{a+x}}{L_a} \cdot \frac{L_{b+x}}{L_b} - \frac{L_{a+x}}{L_a}$ fluent of $\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{b+x}}{L_b} +$ fluent of $\frac{L_{x:a,c}}{L_{a,c}} \cdot \frac{\dot{L}_{b+x}}{L_b}$; and the manner of obtaining each of these fluents has been already delivered; and thence may the assurances be determined. If we originally in the expression fluent of $\overline{1 - \frac{L_{c+x}}{L}}$

$\cdot\frac{L_{b+x}}{L_b}$, which we will suppose to commence with x, write its approximate value $-\frac{1}{2}\left(1-\frac{L_{c+x}}{L_c}\times\right)\left(1-\frac{L_{b+x}}{L_b}\right)=-\frac{1}{2}+\frac{1}{2}\frac{L_{c+x}}{L_c}+\frac{1}{2}\frac{L_{b+x}}{L_b}-\frac{1}{2}\frac{L_{c+x}}{L_c}\cdot\frac{L_{b+x}}{L_b}$; we get the contingency $=-\frac{1}{2}\left(\frac{L_{a+x}}{L_a}-1\right)+\frac{1}{2}\,\text{fluent}\left(\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{a+x}}{L_a}\right)+\frac{1}{2}$ fluent of $\left(\frac{L_{b+x}}{L_b}\cdot\frac{L_{a+x}}{L_a}\right)-\frac{1}{2}$ fluent of $\frac{L_{c+x}\cdot L_{b+x}\cdot\dot{L}_{a+x}}{L_c\quad L_b\quad L_a}$ an approximation during the term of joint existence of B and C; and hence the method of finding the assurance for that term; for each part is evident from what has been shown; and how to proceed beyond that term, will be evident by considering the accurate fluent. But this last method throws the approximation on the whole value.

Example 5. On the death of A, provided he be the first or second of the three A, B, C; and provided C in the latter case dies before B. This contingency is $=-$ fluent of $\frac{\dot{L}_{a+x}}{L_a}\cdot\frac{L_{b+x}}{L_b}\cdot\frac{L_{c+x}}{L_c}-\text{fluent of}\left(1-\frac{L_{c+x}}{L_c}\right)\cdot\frac{L_{b+x}}{L_b}\cdot\frac{\dot{L}_{a+x}}{L_a}=-$ fluent of $\left(\frac{L_{b+x}}{L_b}\cdot\frac{\dot{L}_{a+x}}{L_a}\right)$; and the assurance is the same as the assurance of A's death, on condition that he dies before B; as Messrs. MORGAN, &c. makes it. This does not contain the double fluent or the variable quantity multiplied by a fluent, and therefore may be considered out of its place.

Example 6. On the death of A, provided he be the 2d or 3rd that fails of the three A, B, C; and provided C dies before B; this is $= -$ fluent of $\left(\frac{\dot{L}_{a+x}}{L_a} \times \left\{\left(1 - \frac{L_{c+x}}{L_c}\right)\frac{\dot{L}_{b+x}}{L_b}\right.\right.$ $-$ fluent of $\left.\left.\overline{1 - \frac{L_{c+x}}{L_c} \cdot \frac{L_{b+x}}{L_b}}\right\}\right) =$ fluent of $\left(\frac{\dot{L}_{a+x}}{L_a}\left(\text{fluent of } \frac{L_{b+x}}{L_b} \cdot \frac{\dot{L}_{c+x}}{L_c}\right)\right) = \frac{L_{a+x}}{L_a}$ fluent of $\frac{L_{b+x}}{L_b} \cdot \frac{\dot{L}_{c+x}}{L_c}$ $-$ fluent of $\frac{L_{x:a,b}}{L_{a,b}} \cdot \frac{\dot{L}_{c+x}}{L_c}$, and each part has been already treated on.

Example 7. On the death of A, provided he be the first or last which fails of the three lives A, B, C; and provided C in the latter case dies before B. This is evidently $= -$ fluent of $\left(\frac{\dot{L}_{a+x}}{L_a}\left\{\frac{L_{x:b,c}}{L_c} - \text{fluent}\left(1 - \frac{L_{c+x}}{L_c}\right)\frac{\dot{L}_{b+x}}{L_b}\right\}\right) = -$ fluent of $\frac{\dot{L}_{a+x}}{L_a}\left\{\frac{L_{x:b,c}}{L_{b,c}} - \frac{L_{b+x}}{L_b} + 1 + \text{fluent of } \frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{b+x}}{L_b}\right\} = 1 - \frac{L_{a+x}}{L_a}$ $-$ fluent of $\left(\frac{L_{x:b,c}}{L_{b;c}} \cdot \frac{\dot{L}_{a+x}}{L_a}\right) +$ fluent of $\left(\frac{\dot{L}_{a+x}}{L_a} \cdot \frac{L_{b+x}}{L_b}\right) - \frac{L_{a+x}}{L_a}$. fluent of $\frac{\dot{L}_{c+x}}{L_c} \cdot \frac{L_{b+x}}{L_b} +$ fluent of $\frac{\dot{L}_{x:a,c}}{L_{a,c}} \cdot \frac{L_{b+x}}{L_b}$, and each part has been already considered.

Example 8. On the death of the longest of the two lives A and B, provided they be the first which fail of the three A, B and C; this is evidently fluent of $\left(\frac{L_{c+x}}{L_c} \times \left\{\overline{1 - \frac{L_{a+x}}{L_a}} \cdot \overline{1 - \frac{L_{b+x}}{L_b}}\right\}\right) = -$ fluent of $\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{a+x}}{L_a} -$ fluent of $\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{b+x}}{L_b} +$ fluent of $\left(\frac{L_{a+x}}{L_a} \cdot \frac{L_{b+x}}{L_b}\right)\frac{\dot{L}_{c+x}}{L_c}$; and the assurance is the sum of the assurances on A if he survives C, and on B if he survives C — the assurance on C if he survives A or B. This agrees with Mr. BAILY's deduction. Note; for fluent of $\left(\frac{L_{a+x} \cdot L_{b+x}}{L_a \cdot L_b}\right) \frac{\dot{L}_{c+x}}{L_c}$ we may write $\frac{L_{x:a,b,c}}{L_{a,b,c}}$ — fluent of $\frac{L_{x:a,b}}{L_{a,b}} \cdot \frac{\dot{L}_{c+x}}{L_c}$, whence &c., and this not containing the fluent multiplied by a variable, may be considered out of its place.

Example 9. On the longest of the two lives A and B, provided they be the last that fail of the three A, B, C. This is fluent of $\left\{\frac{\dot{L}_{a+x}}{L_a}\right.$ fluent of$\left(1 - \overline{\frac{L_{c+x}}{L_c}} \cdot \frac{\dot{L}_{b+x}}{L_b} + \frac{\dot{L}_{b+x}}{L_b}\right.$ fluent of $\left.\overline{1 - \frac{L_{c+x}}{L_c}} \cdot \frac{\dot{L}_{a+x}}{L_a}\right\} =$ fluent of $\left\{\frac{\dot{L}_{a+x}}{L_a}\left(\frac{L_{b+x}}{L_b} - 1 -\right.\right.$ fluent of $\left.\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{b+x}}{L_b}\right) + \frac{\dot{L}_{b+x}}{L_b}\left(\frac{L_{a+x}}{L_a} - 1 -\right.$ fluent of $\left.\left.\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{a+x}}{L_a}\right)\right\}$ $=$ correction $- \frac{L_{a+x}}{L_a} - \frac{L_{b+x}}{L_b} + \frac{L_{x:a,b}}{L_{a,q}} -$ fluent of $\left(\frac{\dot{L}_{a+x}}{L_a}\right.$. fluent of$\left.\frac{L_{c+x} \cdot \dot{L}_{b+x}}{L_{c,b}}\right) -$ fluent of $\left(\frac{\dot{L}_{b+x}}{L_b}\right.$ fluent of $\left.\frac{L_{c+x} \cdot \dot{L}_{a+x}}{L_c \cdot L_a}\right)$ $=$ correction $- \frac{L_{a+x}}{L_a} - \frac{L_{b+x}}{L_b} + \frac{L_{x:a,b}}{L_{a,b}} - \frac{L_{a+x}}{L_a}$ fluent of $\left(\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{b+x}}{L_b}\right) - \frac{L_{b+x}}{L_b}$ (fluent of $\left.\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{a+x}}{L_a}\right) +$ fluent of $\frac{L_{c+x}}{L_c}\left(\frac{L_{x:a,b}}{L_{a,b}}\right)^{\cdot}$ and we may write for fluent of $\frac{L_{c+x}}{L_c}\left(\frac{L_{x:a,b}}{L_{a,b}}\right)^{\cdot}$ its equal $\frac{L_{x:a,b,c}}{L_{a,b,c}}$ — fluent of $\left(\frac{L_{x:a,b}}{L_{a,b}} \cdot \frac{\dot{L}_{x+c}}{L_c}\right)$ and each part, as well as the assurance on each, has been already considered. But if we use immediately for the approximation of the fluents of $\overline{1-\frac{L_{c+x}}{L_c}}\cdot\frac{\dot{L}_{b+x}}{L_b}$ and of $\overline{1-\frac{L_{c+x}}{L_c}}\cdot\frac{\dot{L}_{a+x}}{L_a}$, when those contingencies are meant to commence when $x=0$, and not to last longer than the possible joint continuance of life, the approximations so often named, which are respectively $-\frac{1}{2}+\frac{1}{2}L_{b+x}+\frac{1}{2}L_{c+x}-\frac{1}{2}L_{x:b,c}$ and $-\frac{1}{2}+\frac{1}{2}L_{a+x}+\frac{1}{2}L_{c+x}-\frac{1}{2}L_{x:a,c}$ our formula, independent of the above reduction, will become

$$\text{fluent of}\left\{-\frac{\dot{L}_{a+x}}{2L_a}+\frac{L_{b+x}}{2L_b}\cdot\frac{\dot{L}_{a+x}}{2L_a}+\frac{1}{2}\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{a+x}}{L_a}-\frac{1}{2}\frac{L_{x:b,c}}{L_{b,c}}\right.$$

$$\cdot\frac{\dot{L}_{a+x}}{L_a}-\frac{\dot{L}_{b+x}}{2L_b}+\frac{L_{a+x}}{2L_a}\cdot\frac{\dot{L}_{b+x}}{L_b}+\frac{1}{2}\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}-\frac{1}{2}\frac{L_{x:a,c}}{L_{a,c}}$$

$$\left.\cdot\frac{\dot{L}_{b+x}}{L_x}\right\}=\text{correction}-\frac{1}{2}\frac{L_{a+x}}{L_a}-\frac{1}{2}\frac{L_{b+x}}{2L_b}+\frac{1}{2}\frac{L_{x:a,b}}{L_{a,b}}+\frac{1}{2}$$

$$\text{fluent of}\left(\frac{L_{c+x}}{L_c}\cdot\frac{L_{a+x}}{L_a}\right)+\frac{1}{2}\,\text{fluent of}\left(\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}\right)-\frac{1}{2}\,\text{fluent}$$

of $\frac{L_{x+c}}{L_c}\left(\frac{L_{x:a,b}}{L_{a,b}}\right)$. But this last method throws the approximation on the whole term.

I may observe that the first method will resolve itself into

$$\text{correction}-\frac{L_{a+x}}{L_a}-\frac{L_{b+x}}{L_b}+\text{fluent of}\left(\frac{L_{c+x}}{L_c}\left(\frac{L_{x:a,b}}{L_{a,b}}\right)^{\cdot}\right)$$

— contingency of Example 1 Art. 7 of this Section, and the assurance will in consequence be the assurances on the single lives A and B together;—the assurance of that Example;—the assurance on the three joint lives A, B, C; + the assurance of C's life provided it fails before A and B. The last two expressions of this formula are derived from the resolution of fluent of $\frac{L_{c+x}}{L_c}\left(\frac{L_{x:a,b}}{L_{a,b}}\right)^{\cdot}$ into $\frac{L_{x:a,b,c}}{L_{a,b,c}}$ — fluent of $\frac{L_{x:a,b}}{L_{a,b}} \cdot \frac{\dot{L}_{c+x}}{L_c}$.

Example 10. On the death of the longest of the lives A and B; provided they be the first and last of the three A B and C. This is fluent of $\left\{\frac{\dot{L}_{a+x}}{L_a}\right.$ fluent of $\left(\overline{1-\frac{L_{b+x}}{L_b}} \cdot \frac{\dot{L}_{c+x}}{L_c}\right) + \frac{\dot{L}_{b+x}}{L_b}$ fluent of $\left.\left(\overline{1-\frac{L_{a+x}}{L_a}}\, \frac{\dot{L}_{c+x}}{L_c}\right)\right\} =$ fluent of $\frac{\dot{L}_{a+x}}{L_a}\left(\frac{L_{c+x}}{L_c} - 1 -\right.$ fluent of $\left.\frac{L_{b+x}}{L_b} \cdot \frac{\dot{L}_{c+x}}{L_c}\right) +$ fluent of $\frac{\dot{L}_{b+x}}{L_b}\left(\frac{L_{c+x}}{L_c} - 1\right.$ — fluent of $\left.\frac{L_{a+x}}{L_a} \cdot \frac{\dot{L}_{c+x}}{L_c}\right) =$ correction $- \frac{L_{a+x}}{L_a} - \frac{L_{b+x}}{L_b} +$ fluent of $\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{a+x}}{L_a} +$ fluent of $\frac{\dot{L}_{c+x}}{L_c} \cdot \frac{L_{b+x}}{L_b} - \frac{L_{a+x}}{L_a}$ fluent of $\frac{L_{b+x}}{L_b} \cdot \frac{\dot{L}_{c+x}}{L_c} - \frac{L_{b+x}}{L_b}$ fluent of $\frac{L_{a+x}}{L_a} \cdot \frac{\dot{L}_{c+x}}{L_c} - 2$ fluent of $\frac{L_{x:a,b}}{L_{a,b}} \cdot \frac{\dot{L}_{c+x}}{L_c}$.

Example 11. On the death of the longest of the lives A and B; provided B dies before C. This is fluent of $\left(\frac{\dot{L}_{a+x}}{L_a}\right.$ $\left(\right.$fluent of $\left.\left.\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{a+b}}{L_a}\right)\right)$ — fluent of $\frac{\dot{L}_{b+x}}{L_b}\left(1 - \frac{L_{a+x}}{L_a}\right)\frac{L_{c+x}}{L_c}$ $= \frac{L_{a+x}}{L_a}$ fluent of $\left(\frac{L_{c+x}}{L_c} \cdot \frac{\dot{L}_{b+x}}{L_b}\right)$ — fluent of $\left.\frac{L_{x:a,c}}{L_{a,c}} \cdot \frac{\dot{L}_{b+x}}{L_b}\right)$

$-$ fluent of $\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}+$ fluent of $\frac{L_{x:a,c}}{L_{a,c}}\cdot\frac{\dot{L}_{b+x}}{L_b}=\frac{L_{a+x}}{L_a}$. fluent of $\left(\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}\right)-$ fluent of $\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}$.

Note. This result may be obtained in rather a more simple manner; for the chance depends on the two events; namely, that A shall be dead, and that B dies whilst C is living; the former is $1-\frac{L_{a+x}}{L_a}$; and the latter is $-$ fluent of $\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}$ and the rectangle of the two gives the expression above.

Example 12. On the death of A and B, provided another life C, dies before B. This is fluent of $\Big(\frac{L_{a+x}}{L_a}$ fluent of $\Big(\overline{1-\frac{L_{c+x}}{L_c}}\cdot\frac{\dot{L}_{b+x}}{L_b}\Big)-$ fluent $\frac{\dot{L}_{b+x}}{L_b}\cdot\overline{1-\frac{L_{a+x}}{L_a}}\cdot\overline{1-\frac{L_{c+x}}{L_c}}=\frac{L_{a+x}}{L_a}$ fluent of $\Big(\overline{1-\frac{L_{c+x}}{L_c}}\cdot\frac{\dot{L}_{b+x}}{L_b}\Big)-$ fluent of $\Big(\overline{1-\frac{L_{c+x}}{L_c}}\cdot\frac{L_{a+x}}{L_a}\cdot\frac{\dot{L}_{b+x}}{L_b}\Big)$ $-$ fluent of $\frac{\dot{L}_{b+x}}{L_b}\cdot\overline{1-\frac{L_{a+x}}{L_a}}\cdot\overline{1-\frac{L_{b+x}}{L_c}}=1+\frac{L_{a+x}}{L_a}\cdot\frac{L_{b+x}}{L_b}-\frac{L_{a+x}}{L_a}-\frac{L_{a+x}}{L_a}$ fluent of $\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}-\frac{L_{b+x}}{L_b}+$ fluent $\frac{L_{c+x}}{L_c}\cdot\frac{\dot{L}_{b+x}}{L_b}$. If the contingencies begin with x.

There are some observations to be made on the manner we have corrected the fluents; and also on the manner of calculating without the usual tables, for which consult the Scholium.

Scholium.

As the Tables calculated for the valuation of annuities or yearly incomes may be serviceable for the valuation of incomes payable at less periods, it will be proper to show how the value of such incomes depend on each other; and also to show how to compare the value of assurances on lives, when the sum assured is to be received at some one of a number of periods which is to happen after the death, reckoning from a fixed period, with the value of the assurance, if the sum is to be received at a given distant time from the death; for instance, what is commonly called the assurance of £1. on a life, is the value of one pound to be received at the first anniversary from the payment of the premium, which shall happen after the death; but it is not the value of one pound to be received at the death; and it is, as will appear farther on, very nearly the value of one pound to be received a half year after the death shall happen.

Art. 1. Problem. Given $\begin{matrix}\overbrace{\;p\;}^{r}\\ n\\ m\end{matrix}\Big|\underline{a, b, c, \&c.}$ to find $\begin{matrix}\overbrace{\;q\;}^{r}\\ n\\ m\end{matrix}\Big|\underline{a, b, c, \&c.} \times \frac{q}{p}$ nearly, $\frac{p}{q}$ being a whole number, and p a small period?

Solution. We shall have $\begin{matrix}\overbrace{\;q\;}^{r}\\ n\\ m\end{matrix}\Big|\underline{a, b, c, \&c.} = \begin{matrix}\overbrace{\qquad q\qquad}^{r}\\ n\\ n+p-q\end{matrix}\Big|\underline{a, b, c, \&c.} + \begin{matrix}\overbrace{\qquad q\qquad}^{r}\\ n+p\\ n+2p-q\end{matrix}\Big|\underline{a, b, c, \&c.} + \begin{matrix}\overbrace{\qquad q\qquad}^{r}\\ n+2p\\ n+3p-q\end{matrix}\Big|\underline{a, b, c, \&c.} \ldots\ldots \begin{matrix}\overbrace{\quad q\quad}^{r}\\ m-p\\ m-q\end{matrix}\Big|\underline{a, b, c, \&c.} + r^{m} . \frac{L_{m : a, b, c, \&c.}}{L_{a, b, c, \&c.}}$. But considering that during the whole of any small interval p, that the living corresponding to time which are in arithmetical progression, are in geometrical progression. (See Section 3, Art. 2), $\overset{\overset{r}{\smile}}{\underset{\pi+p-q}{\overset{q}{\pi}}}\Big|\underline{a, b, c, \&c.}$ will be $= r^{\pi} \cdot \frac{\mathrm{L}_{\pi : a, b, c, \&c.}}{\mathrm{L}_{a, b, c, \&c.}} \times \frac{1 - r^{p} \cdot \frac{\mathrm{L}_{\pi+p : a, b, c, \&c.}}{\mathrm{L}_{\pi : a, b, c, \&c.}}}{1 - r^{q} \cdot \frac{\mathrm{L}_{\pi+q : a, b, c, \&c.}}{\mathrm{L}_{\pi : a, b, c, \&c.}}}$; but by the hypothesis $\overline{\frac{\mathrm{L}_{\pi+q : a, b, c, \&c.}}{\mathrm{L}_{\pi : a, b, c}}}\Big|^{\frac{p}{q}} = \frac{\mathrm{L}_{\pi+p : a, b, c, \&c.}}{\mathrm{L}_{\pi : a, b, c, \&c.}}$; consequently, if we put $r^{p} \cdot \frac{\mathrm{L}_{\pi+p : a, b, c, \&c.}}{\mathrm{L}_{\pi : a, b, c, \&c.}} = 1 - k$, we shall have $\overset{\overset{r}{\smile}}{\underset{\pi+p-q}{\overset{q}{\pi}}}\Big|\underline{a, b, c, \&c.}$ $= r^{\pi} \cdot \frac{\mathrm{L}_{\pi : a, b, c, \&c.}}{\mathrm{L}_{\pi : a, b, c, \&c.}} \times \frac{1-k}{1-(1-k)^{\frac{q}{p}}}$. Here k is generally very small; and if in the developement of $\overline{1-k}|^{\frac{q}{p}}$, we are satisfied with retaining only the first and second powers of k, we shall have $\overset{\overset{r}{\smile}}{\underset{\pi+p-q}{\overset{q}{\pi}}}\Big|\underline{a, b, c, \&c.} = r^{\pi} \cdot \frac{\mathrm{L}_{\pi : a, b, c, \&c.}}{\mathrm{L}_{a, b, c, \&c.}} \times \frac{1}{\frac{q}{p} - \frac{q}{p} \cdot \frac{q-p}{2p} \cdot k} = \frac{p}{q} \cdot r^{\pi} \cdot \frac{\mathrm{L}_{\pi : a, b, c, \&c.}}{\mathrm{L}_{a, b, c, \&c.}} \times \left(1 - \frac{p-q}{2p} \cdot k\right)$ nearly; $= \frac{p}{q} r^{\pi} \cdot \frac{\mathrm{L}_{\pi : a, b, c, \&c.}}{\mathrm{L}_{a, b, c, \&c.}} \times \left(\frac{p+q}{2p} + \frac{p-q}{2p} \cdot \overline{1-k}\right) = \frac{p+q}{2q} r^{\pi} \cdot \frac{\mathrm{L}_{\pi : a, b, c, \&c.}}{\mathrm{L}_{a, b, c, \&c.}} + \frac{p-q}{2q} \cdot r^{\pi+p} \cdot \frac{\mathrm{L}_{\pi+p : a, b, c, \&c.}}{\mathrm{L}_{a, b, c, \&c.}}$. Hence, if we interpret π successively by n, $n+p$, $n+2p$, &c. $m-p$, we shall get as an approximation from the above value of $\overset{\overset{r}{\smile}}{\underset{m}{\overset{q}{n}}}\Big|\underline{a, b, c, \&c.}$, $\overset{\overset{r}{\smile}}{\underset{m}{\overset{q}{n}}}\Big|\underline{a, b, c, \&c.} \cdot \frac{q}{p} = \Big(\frac{p+q}{2p}$

$$.r^{n}.\frac{L_{n:a,b,c,\&c.}}{L_{a,b,c,\&c.}}+\frac{p-q}{2p}.r^{n+p}.\frac{L_{n+p:a,bc,\&c.}}{L_{a,b,c,\&c.}}\Big)+\Big(\frac{p+q}{2p}.r^{n+p}.L_{n+p:a,b,c,\&c.}$$

$$+\frac{p-q}{2p}r^{n+2p}.\frac{L_{n+2p:a,b,c,\&c.}}{L_{a,b,c,\&c.}}\Big)\ldots\ldots\Big(\frac{p+q}{2p}r^{m-p}.\frac{L_{m-p:a,b,c,\&c.}}{L_{a,b,c,\&c.}}$$

$$+\frac{p-q}{2p}.r^{m}.\frac{L_{m:a,b,c,\&c.}}{L_{a,b,c,\&c.}}\Big)+\frac{p}{q}.\frac{L_{m:a,b,c,\&c.}}{L_{a,b,c,\&c.}}=\frac{p+q}{2p}.r^{n}.\frac{L_{n:a,b,c,\&c.}}{L_{a,b,c,\&c.}}$$

$$+\frac{1}{L_{a,b,c,\&c.}}\times\big(r^{n+p}.L_{n+p:a,b,c,\&c.}+r^{n+2p}.L_{n+2p:a,b,c,\&c.}$$

$$\ldots\ldots\ldots+r^{m}.L_{m:a,b,c,\&c.}\big)-r^{m}.\frac{p-q}{2p}.\frac{L_{m:a,b,c,\&c.}}{L_{a,b,c,\&c.}}$$

$$=\frac{r^{n}.\overline{p+q}.L_{n:a,b,c,\&c.}-r^{m}.\overline{p-q}.L_{m:a,b,c,\&c.}}{2p.L_{a,b,c,\&c.}}+\left.\overset{\overset{r}{\smile}}{\substack{p\\n+p\\m}}\right|\underline{a,b,c,\&c.},\text{ or}$$

$$=-\frac{p-q}{2p\,L_{a,b,c,\&c.}}\big(r^{n}.L_{n:a,b,c,\&c.}+r^{m}L_{m:a,b,c,\&c.}\big)+\left.\overset{\overset{r}{\smile}}{\substack{p\\n\\m}}\right|\underline{a,b,c.}$$

As a particular example, if we take $n=o$, m infinite, $p=1$, $q=\frac{1}{2}$, we have $\left.\overset{\overset{r}{\smile}}{\substack{\frac{1}{2}\\0}}\right|\underline{a,b,c.}\,.\,\frac{1}{2}=\frac{3}{4}+\left.\overset{\overset{r}{\smile}}{\substack{1\\1}}\right|\underline{a,b,c,\&c.}$ nearly, and therefore $\left.\overset{\overset{r}{\smile}}{\substack{\frac{1}{2}\\\frac{1}{2}}}\right|\underline{a,b,c,\&c.}\,.\,\frac{1}{2}=$ nearly $\frac{3}{4}-\frac{1}{2}\,r^{\frac{1}{2}}\frac{L_{\frac{1}{2}:a,b,c}}{L_{a,b,c}}+\left.\overset{\overset{r}{\smile}}{\substack{1\\1}}\right|\underline{a,b,c,\&c.}$; (or because $r^{\frac{1}{2}}\frac{L_{\frac{1}{2}:a,b,c,\&c.}}{L_{a,b,c,\&c.}}$ differs but little from unity) nearly equal to $\frac{1}{4}+\left.\overset{\overset{r}{\smile}}{\substack{1\\1}}\right|\underline{a,b,c,\&c.}$: that is an income of half a pound payable half yearly on the joint lives of the ages a, b, c, &c. is nearly equal to $\frac{1}{4}$ of a pound + the life annuity of one pound on the same lives. If $q=\frac{1}{4}$ we have $\left.\overset{\overset{r}{\smile}}{\substack{\frac{1}{4}\\0}}\right|\underline{a,b,c,\&c.}\cdot\frac{1}{4}=\frac{5}{8}+\left.\overset{\overset{r}{\smile}}{\substack{1\\1}}\right|\underline{a,b,c,\&c.}$ nearly; and $\left.\overset{\overset{r}{\smile}}{\substack{\frac{1}{4}\\\frac{1}{4}}}\right|\underline{a,b,c,\&c.}\cdot\frac{1}{4}=\frac{5}{8}-\frac{1}{4}r^{\frac{1}{2}}.\frac{L_{\frac{1}{2}:a,b,c,\&c.}}{L_{a,b,c,\&c.}}$

$+\left.\begin{matrix}\overbrace{1}^{r}\\ 1\end{matrix}\right|\underline{a,b,c,\&c.}=\frac{1}{2}+\left.\begin{matrix}\overbrace{1}^{r}\\ 1\end{matrix}\right|\underline{a,b,c,\&c.}$ nearly; and similarly is $\left.\begin{matrix}\overbrace{0}^{r}\\ 0\end{matrix}\right|\underline{a,b,c,\&c.}$ $.\,o=\frac{1}{2}+\left.\begin{matrix}\overbrace{1}^{r}\\ 1\end{matrix}\right|\underline{a,b,c,\&c.}$ nearly; that is a momentary income, which in a year certain without interest would amount to one pound, will, if it is to be received on the joint lives *a*, *b*, *c*, &c. reckoning interest, be worth ½ + the life annuity of one pound payable on the joint lives *a*, *b*, *c*, &c.

Art. 2. Moreover, because (Art. 3, Section 1) $\left.\begin{matrix}\overbrace{q}^{r}\\ n\\ \underline{m}\end{matrix}\right|\underline{a,b,c,\&c.}$

$$=\left.\begin{matrix}\overbrace{b}^{r}\\ n-q\\ m-q\end{matrix}\right|\underline{a,b,c,\&c.}\,.\,r^{q}-\left.\begin{matrix}\overset{r}{q}\\ n\\ m\end{matrix}\right|\underline{a,b,c,\&c.}=\left.\begin{matrix}\overbrace{q}^{r}\\ n\\ m\end{matrix}\right|\underline{a,b,c,\&c.}\,.\,r^{q}+r^{n}\,.\,\frac{L_{n-q:a,b,c,\&c.}}{L_{a,b,c,\&c.}}$$

$$-r^{m+q}\,.\,\frac{L_{m:a,b,c,\&c.}}{L_{a,b,c,\&c.}}-\left.\begin{matrix}\overbrace{q}^{r}\\ n\\ m\end{matrix}\right|\underline{a,b,c,\&c.}=r^{n}\,.\,\frac{L_{n-q:a,b,c,\&c.}-r^{m+q}\,.\,L_{m:a,b,c,\&c.}}{L_{a,b,c,\&c.}}$$

$-(1-r^{q})\times\left.\begin{matrix}\overbrace{q}^{r}\\ n\\ m\end{matrix}\right|\underline{a,b,c,\&c.}$, it is therefore from above $=$

$$r^{n}\,.\,\frac{L_{n-q:a,b,c,\&c.}-r^{m+q}\,.\,L_{m+q:a,b,c,\&c.}}{L_{a,b,c,\&c.}}-(1-r^{q})\frac{p}{q}\left\{\frac{r^{n}\,.\,\overline{p+q}\,.\,L_{n:a,b,c,\&c.}-r^{m+q}\,.\,\overline{p-q}\,L_{m:a,b,c,\&c.}}{2p\,.\,L_{a,b,c,\&c.}}\right.$$

$\left.+\left.\begin{matrix}\overbrace{p}^{r}\\ n+p\\ m\end{matrix}\right|\underline{a,b,c,\&c.}\right\}$. If $p=1$, $q=o$, $n=o$, m infinite, since $1-r^{q}$ will be equal to q hyp. log. of $\frac{1}{r}$ we shall have $\left.\begin{matrix}\overline{o}^{\,r}\\ \underline{o}\end{matrix}\right|\underline{a,b,c,\&c.}=$ nearly $1-\frac{1}{2}$ hyp. log. of $\frac{1}{r}-$ hyp. log. of $\frac{1}{r}\times\left.\begin{matrix}\overbrace{1}^{r}\\ \underline{1}\end{matrix}\right|\underline{a,b,c.\,\&c.}$; and if for $-$ hyp. log. of $\frac{1}{r}$ or its equal hyp. log. of $\overline{1-(1-r)}$ we write $-(1\quad r)-\frac{(1-r)^{2}}{2}-\frac{(1-r)^{3}}{3}$, &c. the expression will evidently, since r is near unity, be nearly equal to $1 - \frac{1-r}{2} - (1-r) \,.\, \overline{1 + \left(\frac{1-r}{2}\right)^2} \,.\, \underset{1}{\overset{\overset{r}{\smile}}{1}}\underline{\big|\,a, b, c, \&c.} =$ nearly $r^{\frac{1}{2}} - \frac{1-r}{r^{\frac{1}{2}}} \,.\, \underset{1}{\overset{\overset{r}{\smile}}{1}}\underline{\big|\,a, b, c, \&c.}$ but $\underset{\underline{1}}{\overset{\overset{r}{\smile}}{1}}\underline{\big|\,a, b, c, \&c.} = r - (1-r)\, \underset{1}{\overset{\overset{r}{\smile}}{1}}\underline{\big|\,a, b, c, \&c.}\,;\ \therefore \underset{\underline{0}}{\overset{\overset{r}{\smile}}{0}}\underline{\big|\,a, b, c, \&c.} = \frac{1}{r^{\frac{1}{2}}} \times \underset{\underline{1}}{\overset{\overset{r}{\smile}}{1}}\underline{\big|\,a, b, c, \&c.}$ nearly; that is the value of one pound to be received at the discontinuance of the joint lives of the ages $a, b, c,$ &c. is equal nearly to $\frac{1}{r^{\frac{1}{2}}}$ × the value of one pound to be received at the first anniversary from the present time, which shall happen after the discontinuance of the same lives. Also $\underset{\underline{0}}{\overset{\overset{r}{\smile}}{0}}\underline{\big|\,a, b, c, \&c.} \,.\, r^{\frac{1}{2}}$ or which is the same thing, the value of one pound to be received a half year after the discontinuance of the joint lives is nearly $= \underset{\underline{1}}{\overset{\overset{r}{\smile}}{1}}\underline{\big|\,a, b, c, \&c.}$, or the value of one pound to be received at the first anniversary from the present time, which shall happen after the discontinuance of the joint lives; and $\underset{\underline{0}}{\overset{\overset{r}{\smile}}{0}}\underline{\big|\,a, b, c, \&c.} \,.\, r$ is nearly $= \underset{\underline{1}}{\overset{\overset{r}{\smile}}{1}}\underline{\big|\,a, b, c, \&c.} \,.\, r^{\frac{1}{2}}$; that is the value of one pound to be received one year after the discontinuance of the joint lives of the ages $a, b, c,$ &c. is nearly equal to $r^{\frac{1}{2}} \times$ the value of the same contingency on one pound to be received at the first anniversary which shall happen after the discontinuance of the joint lives.

Art. 3. Again with respect to the calculation of the value of the expresion $\frac{L_{b-q}}{L_b} \,.\, \underset{m}{\overset{\overset{r}{\smile}}{\underset{n}{q}}}\underline{\big|\,a, b-q} - \frac{L_{a-q}}{L_a} \,.\, \underset{m}{\overset{\overset{r}{\smile}}{\underset{n}{q}}}\underline{\big|\,b, a-q}$ from the value of the expression $\frac{L_{b-p}}{L_b}\cdot\overset{\overbrace{}^{r}}{\substack{p\\n\\m}}\Big|\underline{a,\,b-p}-\frac{L_{a-p}}{L_a}\cdot\overset{\overbrace{}^{r}}{\substack{p\\n\\m}}\Big|\underline{a,\,b-p}$, and the like; see Art. 3. Section 4. I observe if $\frac{p}{q}$ be a whole number, and p small, from Art. 1 of this Scholium, that $\overset{\overbrace{}^{r}}{\substack{q\\n\\m}}\Big|\underline{g,\,b}$ $=-\frac{p-q}{2q}\cdot\frac{r^n.L_{n:g,b}+r^mL_{m:g,b}}{L_{g,b}}+\overset{\overbrace{}^{r}}{\substack{p\\n\\m}}\Big|\underline{g,\,b}\cdot\frac{p}{q}$; therefore $\frac{L_{b-q}}{L_b}$ $\cdot\overset{\overbrace{}^{r}}{\substack{q\\n\\m}}\Big|\underline{a,\,b-q}-\frac{L_{a-q}}{L_a}\cdot\overset{\overbrace{}^{r}}{\substack{q\\n\\m}}\Big|\underline{b,\,a-q}=-\frac{p-q}{2qL_{a,b}}\cdot(r^n\,.\,L_{n:a,b-q}+$ $r^m\,.\,L_{m:a,b-q}-r^n\,.\,L_{n:a-q,b}-r^m\,.\,L_{m:a-q,b})+\overset{\overbrace{}^{r}}{\substack{p\\n\\m}}\Big|\underline{a,\,b-q}$ $\cdot\frac{L_{b-q}}{L_b}\cdot\frac{p}{q}-\overset{\overbrace{}^{r}}{\substack{p\\n\\m}}\Big|\underline{a-q,\,b}\cdot\frac{L_{a-q}}{L_a}\cdot\frac{p}{q}$. But $L_{x:a,b-q}-L_{x:b,a-q}=$ $L_{x+a}\times(L_{x+b}+\frac{q}{p}(L_{x+b-p}-L_{x+b}))-L_{x+b}\times(L_{x+a}+$ $\frac{q}{p}(L_{x+a-p}-L_{x+a}))$ nearly under the hypothesis of q and p being small intervals; and this by an evident reduction is simply $\frac{q}{p}(L_{x:a,b-p}-L_{x:b,a-p})$; and in the same way it is shown that $L_{x:a,b-q}-L_{x:b,a-q}=\frac{q}{p}(L_{x:a,b-p}-L_{x:b,a-p})$ nearly. Moreover $\overset{\overbrace{}^{r}}{\substack{p\\n\\m}}\Big|\underline{a,\,b-q}\cdot\frac{L_{b-q}}{L_b}-\overset{\overbrace{}^{r}}{\substack{p\\n\\m}}\Big|\underline{a-q,\,b}\cdot\frac{L_{a-q}}{L_a}=$ $r^n\cdot\frac{L_{n:a,b-q}-L_{n:b,a-q}}{L_{a,b}}+r^{n+p}\cdot\frac{L_{n+p:a,b-q}-L_{n+p:a-q,b}}{L_{a,b}}+$ &c. $=$ from above, nearly $\frac{q}{p}\Big\{r^n\cdot\frac{L_{n:a,b-q}-L_{n:b,a-p}}{L_{a,b}}+$

$$r^{n+p}\cdot\frac{\mathrm{L}_{n+p:a,\,b-p}-\mathrm{L}_{n+p:b,\,a-p}}{\mathrm{L}_{a,\,b}}+\text{\&c.}\Big\}=\frac{q}{p}\Bigg\{\frac{\mathrm{L}_{b-p}}{\mathrm{L}_b}\cdot\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{{}_{a,\,b-p}}$$

$$-\frac{\mathrm{L}_{a-p}}{\mathrm{L}_a}\cdot\left.\begin{smallmatrix}r\\ \smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{{}_{b,\,a-p}}\Bigg\}.$$ Hence we get $\frac{\mathrm{L}_{b-q}}{\mathrm{L}_b}\cdot\left.\begin{smallmatrix}r\\ \smile\\ q\\ n\\ m\end{smallmatrix}\right|\underline{{}_{a,\,b-q}}-\frac{\mathrm{L}_{a-q}}{\mathrm{L}_a}$

$$\left.\begin{smallmatrix}r\\ \smile\\ q\\ n\\ m\end{smallmatrix}\right|\underline{{}_{b,\,a-q}}=\text{nearly}-\frac{p-q}{2p}\cdot\frac{r^{n}\cdot(\mathrm{L}_{n:a,\,b-p}-\mathrm{L}_{n:b,\,a-p})+r^{m}\cdot(\mathrm{L}_{m:a,\,b-p}-\mathrm{L}_{m:b,\,a-p})}{\mathrm{L}_{\;\;x}}$$

$$+\frac{\mathrm{L}_{b-p}}{\mathrm{L}_b}\cdot\left.\begin{smallmatrix}\smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{{}_{a,\,b-p}}-\frac{\mathrm{L}_{a-p}}{\mathrm{L}_a}\cdot\left.\begin{smallmatrix}\smile\\ p\\ n\\ m\end{smallmatrix}\right|\underline{{}_{b,\,a-p}},$$ when $\frac{p}{q}$ is a whole number.

Article 4. It is proper to observe, that what refers to the fluents of the expressions $\mathrm{L}_{q+x}\times\dot{\mathrm{L}}_{r+x}$, $\mathrm{L}_{p+x}\times\mathrm{L}_{b+x}\times\dot{\mathrm{L}}_{r+x}$, &c. of Section 3, equally apply whether L in the different expressions L_{q+x}, $\dot{\mathrm{L}}_{r+x}$, &c. is, *mutatis mutandis*, the same, or a different functional characteristic, whether when they refer to life contingencies, if L in the one part refers to one given constitution, and in the other part it refers to another constitution or not; for instance, if in the expression $\mathrm{L}_{q+x}\times\dot{\mathrm{L}}_{r+x}$, L_{q+x} refers to the Northampton lives, and $\dot{\mathrm{L}}_{r+x}$ to the Swedish lives, or whether they both refer to the same lives, &c. Whether they refer to the number of living at the ages $q+x$ and $r+x$, or whether they refer with respect to the variable time x to expressions compounded of the number of living and dead. But instead of resuming the characteristic L here, I shall, with a view to better distinction, consider the value $\overset{x}{\left.\begin{smallmatrix}0\\ w\end{smallmatrix}\right|}\,\mathrm{M}_x\,\dot{\mathrm{N}}_x$. And I observe, similarly to what is done in Section 3, that if x be put $=\varpi+t$, and that if when t be not greater than p, $M_{\varpi+t}$ be equal to $M_\varpi - tM'_\varpi$, and $N_{\varpi+t} = N_\varpi - tN'_\varpi$ sufficiently near, that we shall have $\overset{x}{\left.\begin{array}{c}0\\ \varpi\\ \varpi+p\end{array}\right]} M_x . \dot{N}_x = \overset{t}{\left.\begin{array}{c}0\\ 0\\ p\end{array}\right]} M_{\varpi+t} \times \dot{N}_{\varpi+t} = \overset{t}{\left.\begin{array}{c}0\\ 0\\ p\end{array}\right]} \overline{M_\varpi - M'_\varpi t} \times (-N'\dot{t}) = -pN'_\varpi M_\varpi + \frac{p^2}{2} . M'_\varpi N'_\varpi = -pN'_\varpi \times \overline{M_\varpi - \frac{p}{2} M'_\varpi} = -pN'_\varpi . M_{\varpi+\frac{p}{2}} = -(N_\varpi - N_{\varpi+p}) M_{\varpi+\frac{p}{2}}$. And therefore if M_x represent the chance that a certain circumstanc[illegible]hall exist at the time x, and N_x the chance that some other certain object shall exist at the time x; then the chance that the second circumstance shall fail during the existence of the first between the time $n-p$ and m is sufficiently nearly $= \overset{\varpi}{\left.\begin{array}{c}p\\ n-p\\ m-p\end{array}\right]} (\overline{N_\varpi - N_{\varpi+p}} \times M_{\varpi+\frac{1}{2}p})$. And the present value of one pound to be received at the first of the periods n, $n+p$, $n+2p$, &c. from the present time, which shall happen after such failure, provided it shall take place between the time $n-p$ and m, will be according to hypothesis with sufficient proximity $= \overset{\varpi}{\left.\begin{array}{c}p\\ n-p\end{array}\right]} (r^{\varpi+p} \times \overline{N_\varpi - N_{\varpi+p}} \times M_{\varpi+\frac{p}{2}}) = \overset{\varpi}{\left.\begin{array}{c}p\\ n-p\\ m-p\end{array}\right]} (r^{\varpi+p} \times N_\varpi . M_{\varpi+\frac{p}{2}}) - \overset{\varpi}{\left.\begin{array}{c}p\\ n-p\\ m-p\end{array}\right]} (r^{\varpi+p} \times N_{\varpi+p} \times M_{\varpi+\frac{p}{2}})$. And this mode of investigation will afford different modes of solution to ALL THE EXAMPLES OF SECTION 4. For instance: if this be applied to Example 8, Art. 7, Section 4, by

taking $N_x = \left(1-\frac{L_{a+x}}{L_a}\right).\left(1-\frac{L_{b+x}}{L_b}\right)$ and $M_x = \frac{L_{c+x}}{L_c}$, the assurance of one pound on the contingency of that example will become $\overset{\pi}{\left.\overline{\begin{matrix}p\\ n-p\\ m-p\end{matrix}}\right]}\left(r^{\pi+p}.\left(1-\frac{L_{a+\pi}}{L_a}\right).\left(1-\frac{L_{b+\pi}}{L_b}\right).\frac{L_{c+\pi+\frac{1}{2}p}}{L_c}\right)$ $-\overset{\pi}{\left.\overline{\begin{matrix}p\\ n-p\\ m-p\end{matrix}}\right]}\left(r^{\pi+p}.\left(1-\frac{L_{a+\pi+p}}{L_a}\right).\left(1-\frac{L_{b+\pi+p}}{L_b}\right).\frac{L_{c+\pi+\frac{1}{2}p}}{L_c}\right)=$ $r^p.\frac{L_{c+\frac{1}{2}p}}{L_c}$ × the income of one pound payable at every p interval, the first in the time $n-p$, and the last in the time $m-p$ on the life of the age $c+\frac{1}{2}p$, after the death of the two persons of the ages a and b, $-\frac{L_{c-\frac{1}{2}p}}{L_c}$ × the income of one pound payable at every p interval, the first in the time n, and the last in the time m on the life of the age $c-\frac{1}{2}p$, after the death of the ages a and b. " Because the first term of these two expres-" sions is $r^p.\frac{L_{c+\frac{1}{2}p}}{L_c}\times\overset{\pi}{\left.\overline{\begin{matrix}p\\ n-p\\ m-p\end{matrix}}\right]}\left(r^{\pi}.\overline{1-\frac{L_{a+\pi}}{L_a}}.\overline{1-\frac{L_{b+\pi}}{L_b}}.\frac{L_{a+\pi+\frac{1}{2}p}}{L_{c+\frac{1}{2}p}}\right)$
" $=\overset{\pi}{\left.\overline{\begin{matrix}p\\ n-p\\ m-p\end{matrix}}\right]}\left(r^{\pi+p}.\overline{1-\frac{L_{a+\pi}}{L_a}}.\overline{1-\frac{L_{b+\pi}}{L_b}}.\frac{L_{c+\pi+\frac{1}{2}p}}{L_c}\right)$ and the
" second term of the expression is $=\frac{L_{c+\frac{1}{2}p}}{L_c}\times\overset{\pi}{\left.\overline{\begin{matrix}p\\ n\\ m\end{matrix}}\right]}\left(r^{\pi}.\overline{1-\frac{L_{a+\pi}}{L_a}}.\overline{1-\frac{L_{b+\pi}}{L_b}}.\frac{L_{c+\pi-\frac{1}{2}p}}{L_{c-\frac{1}{2}p}}\right)=\overset{\pi}{\left.\overline{\begin{matrix}p\\ n-p\\ m-p\end{matrix}}\right]}\left(r^{n+p}.\overline{1-\frac{L_{a+\pi+p}}{L_a}}.\overline{1-\frac{L_{b+\pi+p}}{L_b}}.\frac{L_{c+\pi+\frac{1}{2}p}}{L_c}\right)$." Or we may develope the expression in the form

$$\overset{\pi}{\overline{\begin{array}{c}p\\n-p\\m-p\end{array}}}\Bigg|\Big(\Big(r^{\pi+p}\times\frac{L_{\pi:a,b,c+\frac{1}{2}p}}{L_{a,b,c}}-\frac{L_{\pi+p:a,b,c-\frac{1}{2}p}}{L_{a,b,c}}-\frac{L_{\pi:a,c+\frac{1}{2}p}}{L_{a,c}}-\frac{L_{\pi:b,c+\frac{1}{2}p}}{L_{b,a}}+\frac{L_{\pi+p:a,c-\frac{1}{2}p}}{L_{a,c}}+\frac{L_{\pi+p:b,c-\frac{1}{2}p}}{L_{b,c}}\Big)\Big)=r^{p}\cdot\frac{L_{c+\frac{1}{2}p}}{L_{c}}\times$$

$$\left(\overbrace{\begin{array}{c}p\\n-p\\m-p\end{array}}^{r}\Bigg|\underline{a,b,c,+\tfrac{1}{2}p}-\overbrace{\begin{array}{c}p\\n-p\\m-p\end{array}}^{r}\Bigg|\underline{a,c+\tfrac{1}{2}p}-\overbrace{\begin{array}{c}p\\n-p\\m-p\end{array}}^{r}\Bigg|\underline{b,c+\tfrac{1}{2}p}\right)-\frac{L_{c-\frac{1}{2}p}}{L_{c}}$$

$$\left(\overbrace{\begin{array}{c}p\\n\\m\end{array}}^{r}\Bigg|\underline{a,b,c-\tfrac{1}{2}p}-\overbrace{\begin{array}{c}p\\n\\m\end{array}}^{r}\Bigg|\underline{a,c-\tfrac{1}{2}p}-\overbrace{\begin{array}{c}p\\n\\m\end{array}}^{r}\Bigg|\underline{b,c-\tfrac{1}{2}p}\right).$$

When the term is not very long, especially when there are many lives concerned, it will be often preferable to calculate with the usual tables by calculating each term of the expression $\overset{\pi}{\overline{\begin{array}{c}p\\n-p\\m-p\end{array}}}\Bigg|\overline{N_{\pi}-N_{\pi+p}}$ $.\,M_{\pi+\frac{p}{2}}$, when put into the form $\overline{N_{n-p}-N_{n}}\,.\,M_{n-\frac{p}{2}}+\overline{N_{n}-N_{n+p}}$ $.\,M_{n+\frac{p}{2}}+$ &c. And I think when many lives are concerned, this will generally be the best way, if even the term should be long; but for the sake of preventing great error in such cases, or for near approximation, the following hint may be serviceable. Suppose we wished to calculate the value of $\overset{\pi}{\overline{\begin{array}{c}1\\1\\80\end{array}}}\Bigg|A_{\pi}$, that is the sum of the first 80 terms of the series whose π^{th} term is A_{π}; then since $\overset{\pi}{\overline{\begin{array}{c}1\\1\\80\end{array}}}\Bigg|A_{\pi}=\overset{\pi}{\overline{\begin{array}{c}8\\1\\73\end{array}}}\Bigg|A_{\pi}+\overset{\pi}{\overline{\begin{array}{c}8\\2\\74\end{array}}}\Bigg|A_{\pi}+\overset{\pi}{\overline{\begin{array}{c}8\\3\\75\end{array}}}\Bigg|A_{\pi}$ $+\overset{\pi}{\overline{\begin{array}{c}8\\4\\76\end{array}}}\Bigg|A_{\pi}\ldots$ &c. $\overset{\pi}{\overline{\begin{array}{c}8\\8\\80\end{array}}}\Bigg|A_{\pi}$, that is the sum of eight terms of a new series, whereof the first term is equal to the 1st + 9th + 17th &c. term of the original series; the second term equal to 2nd + 10th + &c. term of the original series, &c. it will follow if our original series be a gradually, but a very slowly converging series, such that $\frac{A_\varpi}{A_{\varpi+1}}$ differs very little from unity; since each term of the new series will be nearly equal to each other; that if this method be used, we shall have a means of detecting any great error, as it would be evinced by a too great difference produced in different terms. And if an approximation to the value be sufficient, we may avoid great labour by taking $8 \times \left.\begin{matrix}\varpi\\ 8\\ 5\\ 77\end{matrix}\right] A_\varpi$, or $8 \times \left.\begin{matrix}\varpi\\ 8\\ 6\\ 78\end{matrix}\right] A_\varpi$ for the value of the sum; and if r does not differ much from unity $\left.\begin{matrix}\varpi\\ 1\\ 1\\ 80\end{matrix}\right] A_\varpi\, r^\varpi$ will be nearly

$$= 8 \,.\, \frac{1-r^9}{1-r} \times \left.\begin{matrix}\varpi\\ 8\\ 5\\ 77\end{matrix}\right] r^{\varpi-4}\, A_\varpi \text{ or to } 8 \,.\, \frac{1-r^9}{1-r} \,.\, \left.\begin{matrix}\varpi\\ 8\\ 6\\ 78\end{matrix}\right] r^{\varpi-5}\, A_\varpi.$$

Art. 5. It may also be serviceable to observe, that if M_ϖ, M'_ϖ, M''_ϖ, do not contain x or y, that $\left.\begin{matrix}y\\ o\\ n'\\ z\end{matrix}\right] (\dot{M}''_y \,.\, \left.\begin{matrix}x\\ o\\ n\\ y\end{matrix}\right] M_x\, \dot{M}_x)$ by putting $\left.\begin{matrix}x\\ o\\ n\\ y\end{matrix}\right] M'_x\, \dot{M}_x = N_y$ will become $\left.\begin{matrix}y\\ o\\ n'\\ z\end{matrix}\right] \dot{M}''_y\, N_y = M''_z\, N''_z - M''_{n'}\, N_{n'} - \left.\begin{matrix}y\\ o\\ n'\\ z\end{matrix}\right] M_y\, \dot{N}_y = M''_z\, N_z - M''_{n'}\, N'_{n'} - \left.\begin{matrix}y\\ o\\ n'\\ z\end{matrix}\right] M''_y\, M'_y\, \dot{M}_y = M''_z \times \left.\begin{matrix}y\\ o\\ n\\ z\end{matrix}\right] M'_y\, \dot{M}_y - M''_{n'} \times \left.\begin{matrix}y\\ o\\ n\\ n'\end{matrix}\right] M'_y\, \dot{M}_y - \left.\begin{matrix}y\\ o\\ n\\ n'\end{matrix}\right] M''_y\, M'_y\, \dot{M}_y$; because when M_ϖ, M'_ϖ do not contain either x or y, $\overset{x}{\left.\begin{matrix}o\\n\\y\end{matrix}\right|} M'_x \dot{M}_x = \overset{y}{\left.\begin{matrix}o\\n\\y\end{matrix}\right|} M'_y \dot{M}_y$ and therefore $\dot{N}_y = M'_y \dot{M}_y$. This form is put down for the purpose of reducing double fluents at once, such as occur in Section 4. In a similar manner may the immediate reduction of analogous triple fluents be shown; and I may remark, that in the double fluents of Section 4, the case of the question may not always require them both to commence with the same value of x. The application of the symbol $\overset{r}{\left.\begin{matrix}o\\n\\m\end{matrix}\right|}$ to all cases of definite and indefinite fluents, and of simple, double &c. fluents, might be entered on perhaps with advantage to other branches of the mathematics; but this is not my present object.

Art. 6. If in the room of M_x of Art. 4, of this Scholium, we put $r^x M_x$, M_x still being put to represent the chance that the said certain circumstance shall exist in the time x, we shall have the present value of one pound to be received immediately on the failure of the second circumstance, provided it takes place during the existence of the first circumstance, and between the times $n-p$ and m, merely by making that substitution in the expression $\overset{\varpi}{\left.\begin{matrix}p\\n-p\\m-p\end{matrix}\right|} (\overline{N_\varpi - N_{\varpi+p}} \times M_{\varpi+\frac{1}{2}p})$ by which means it will become $\overset{\varpi}{\left.\begin{matrix}p\\n-p\\m-p\end{matrix}\right|} (\overline{N_\varpi - N_{\varpi+p}}\, r^{\varpi+\frac{1}{2}p} \times M_{\varpi+\frac{1}{2}p})$ or its equal $r^{-\frac{1}{2}p}$. Q; Q being put for the value of the contingency as in Article 4 of this Scholium; namely, when the payment is to be made at the first of the equal periods n, $n+p$,

&c. from the present time after the failure, provided it takes place between the intervals $n-p$ and m. And $\left.\overline{\begin{matrix}\varpi\\ p\\ n\\ m-p\end{matrix}}\right| (\overline{\mathrm{N}_{\varpi}-\mathrm{N}_{\varpi+p}} \,.\, r^{+\varpi+\frac{1}{2}p} \,.\, \mathrm{M}_{\varpi+\frac{1}{2}p})$ or its equal $r^{-\frac{p}{2}} \cdot \mathrm{Q}-\overline{\mathrm{N}_{n-p}-\mathrm{N}_{n}} \,.\, r^{n} \,.\, \mathrm{M}_{n-\frac{1}{2}p}$ is the value if it is to be paid immediately after the failure, provided it takes place between the intervals n and m. It is necessary to add, that the method pointed out in Art. 4 of this Scholium, for solving the problems of Section 4, will generally produce results which do not agree to absolute mathematical equality, with the results of that Section, except the interval p be infinitely small; but they will agree with each other as far as the first power of p is concerned; which when p is taken, the smallest interval of the tables will be as near the truth as any method should be considered to reach, as long as the real function of life is not known; except indeed there be sufficient regularity in the tables to induce the belief that we may approach nearer by interpolation, as hinted in Art. 4, Section 3; but if the interval p be not greater than one year, this will give, I think, sufficient accuracy for any useful purpose, except perhaps in very rare cases, and in which our tables (from more minute observation), should, I imagine, be divided into less periods than yearly interval, and then the same method would still apply by taking p smaller. The same observation will apply if a comparison is made with what is done in the present Article 6 of this Scholium, with the other articles of the Scholium; for instance, with Article 3.

As an annuity secured by land, only differs from common annuities, in as much as in case of death of the lives on which the annuity is determined, during the portion of a year, that that portion of the annuity is payable to the assignees of those persons; its value will be equal to $\left.\begin{matrix}\underset{\smile}{r}\\1\\n\\m\end{matrix}\right|\underline{a,b,c,\&c.}$ $+$ $\left.\begin{matrix}\varpi\\\overline{1}\\n-1\\m-1\end{matrix}\right|\left.\begin{matrix}t\\\overline{0}\\0\\1\end{matrix}\right| r^{\varpi+t}.t.\times\frac{-L_{\varpi+t:a,b,c,\&c.}}{L_{a,b,c,\&c.}}$, but if $L_{\varpi+t:a,b,c,\&c.}$ be considered sufficiently approximated by the expression $\frac{L_{\varpi:a,b,c,\&c.}}{L_{a,b,c,\&c.}}-\frac{L_{\varpi:a,b,c,\&c.}-L_{\varpi+1:a,b,c,\&c.}}{L_{a,b,c,\&c.}}$; and as a sufficient approximation for $r^{\varpi+t}$ we write $r^{\varpi+1}$, the expression will become $\left.\begin{matrix}\underline{r}\\1\\n\\m\end{matrix}\right|\underline{a,b,c,\&c.}+\left.\begin{matrix}\varpi\\\overline{1}\\n-1\\m-1\end{matrix}\right|.\frac{r}{2}^{\varpi+1}.\frac{L_{\varpi:a,b,c,}-L_{\varpi+1:a,b,c.\&c.}}{L_{a,b,c}}=$ $\left.\begin{matrix}\underline{r}\\1\\n\\m\end{matrix}\right|\underline{a,b,c,\&c.}+\frac{1}{2}\cdot\left.\begin{matrix}\underset{\smile}{r}\\1\\n\\m\end{matrix}\right|\underline{a,b,c,\&c.}$ and will agree with Mr. BAILY's observations on page 344 of his Doctrine of Life Annuities, as far as it goes. Note, we might with a similar proximity have omitted the t in the exponent of r; and as a nearer approximation have written $r^{\varpi+\frac{1}{2}}$ for $r^{\varpi+t}$ (not deeming it necessary to go nearer), and the expression will be $\left.\begin{matrix}\underline{r}\\1\\n\\m\end{matrix}\right|\underline{a,b,c,\&c.}+\frac{r}{2}^{\frac{1}{2}}.\times$ $\left.\begin{matrix}\underset{\smile}{r}\\1\\n\\m\end{matrix}\right|\underline{a,b,c,\&c.}$. I might make some farther observations on the comparison of the different methods pointed out, with respect to their proximity, but I fear that the length of this Paper has already caused me to occupy too large a portion of the present volume.

XVIII. *On the measurement of Snowdon, by the Thermometrical Barometer. By the Rev.* F. J. H. Wollaston, *B.D. F.R.S.*

Read June 29, 1820.

The Royal Society did me the honour to notice in their Transactions, 1817, p. 183, the description of a Thermometrical Barometer, by which it was conceived that accessible heights might with convenience be measured; and may therefore not be uninterested by the account of an actual measurement made with it. Having occasion last summer to visit Carnarvon, which would afford an opportunity of trying the instrument on the known height of Snowdon, and being aware that in 3550 feet the variations of the boiling temperature were not to be considered uniform, as they might in small elevations, on which alone I had before tried the experiment, I wished to provide myself previously with a table for making the necessary corrections; and from Dr. Ure's paper, Philosophical Transactions, 1818, p. 338, was supplied with data for the calculation. The law, which he discovers by approximation and lays down, is this: that, the elastic force of the air, or length of the column of mercury in the barometer being supposed 30 inches when water boils at 212° of Fahrenheit, $\frac{30}{1,23}$ will be the length of the column at 202°; $\frac{30}{1,23 \times 1,24}$ the length at 192°; and so on progressively, adding another increased divisor for every 10° of thermometrical temperature.

For my purpose it was necessary to calculate the variations at smaller intervals than ten degrees, and particularly to obtain those between 212° and 202°, by getting a series of divisors for each degree, instead of the simple divisor of 1,23 for the whole. By taking the tenth of the logarithm of 1,23, the uniform divisor appears to be 1,021; but to make the successive divisors increase at the lower temperatures, as Dr. Ure does, suppose them to be taken thus;

1,02059
1,02068
1,02077
1,02086
1,02095
1,02105
1,02114
1,02123
1,02132
1,02141
10,21000

The mean of which is 1.021.

The logarithms of these divisors, if their differences be equalized by a small change in the last figure, will stand thus:

88501
88892
89282
89673
90063
90454
90844
91234
91625
92105

the sum of these logarithms being 0,902583, the logarithm of 1,23 nearly.

By the use of these logarithms, the series of barometrical heights corresponding with each degree of Fahrenheit, calculated from 30 inches at 212°, downward to 202°, and carried upward to 214°, will be found in the following Table I. in the fifth column of which the difference of the logarithms of the barometric column, which corresponds by a known law with the number of fathoms of elevation, is multiplied by 6, to give the feet of elevation which produces the successive depression of the boiling point one degree of Fahrenheit; and the 6th column gives the total elevation, reckoning from the station where water boils at 212°, to any other station where the boiling temperature is at any degree between 202° and 214°. For the differences between each degree, the elevations may be taken with sufficient accuracy by simple proportion.

Table I.

1	2	3	4	5	6
Heat of boiling water.	Correspondent height of barometer.	Logarithm of height of barometer.	Difference of the Logarithms.	Difference of Logarithm multiplied by six equal feet.	Total feet from 212 degrees.
214	31,2395	4947044	87720	526,320	1054,986
213	30,6149	4859324	88111	528,666	528,666
212	30,0000	4771213	88501	531,006	0
211	29,3948	4682712	88892	533,352	531,006
210	28,7993	4593820	89282	535,692	1064,358
209	28,2133	4504538	89673	538,038	1600,050
208	27,6367	4414865	90063	540,378	2138,088
207	27,0695	4324802	90454	542,724	2678,466
206	26,5115	4234348	90844	545,064	3221,190
205	25,9627	4143504	91234	547,404	3766,254
204	25,4230	4052270	91625	549,750	4313,658
203	24,8923	3960645	92015	552,090	4863,408
202	24,3704	3868630			5415,498

In the application of this table to use, it must be adapted to the scale of the particular thermometrical barometer employed. In my former paper, I observed that a change of 1° Fahrenheit appeared to be produced by 0,589 of the common barometer. This was deduced from the comparison of my long thermometer of 3,98 inches to every degree with a common barometer. But having subsequently compared the same thermometer with a mountain barometer by Troughton, where I could ascertain the adjustment for the height of the mercury in the basin, I found that at a mean of 29,5 inches, 0,603 of barometer corrected was equal to 1° of Fahrenheit. This conclusion agrees very closely with the table deduced from Dr. Ure's observations, in which between 211° and 212°, the difference of the mercurial column is 0,605 nearly at the mean height of 29,7, and would be rather less at 29,5. On the scale of the instrument which I have now in use, the half inch is divided into ten parts, and by the Vernier into a hundred; the inch, therefore, into two hundred; and I find by observation, that 1,000 of the corrected barometer equals 372 parts on the scale of the thermometrical barometer, or 1,86 inch; consequently, 0,605 barometer or 1° Fahrenheit, = 225 parts on the scale, or $1\frac{1}{8}$ inch, which is a sensibility entirely sufficient. And those 225 parts being supposed, according to Table I, = 531 feet between 211° and 212°, 100 parts will equal 236 feet at that part of the scale. Between 202° and 203°, 225 parts = 552 feet, and, consequently, 100 parts = 246 feet nearly; and so of every other point; and the following table of heights, corresponding with the scale of my instrument, will be found in the nearest whole numbers.

TABLE II. *For converting observed differences on the Scale of the particular Thermometrical Barometer into feet of elevation.*

Fahrenheit.	Parts on the Scale of the Thermometrical Barometer.	Height in feet.	Correspondent point of Barometer nearly.
	300	702	30,8
213	225	528	,6
	200	469	,54
	100	235	,27
212	0	0	30,0
	100	236	29,73
	200	472	,46
211	225	531	,4
	300	709	29,2
	400	946	28,92
210	450	1064	,8
	500	1184	,65
	600	1422	,4
209	675	1600	,2
	700	1669	28,1
	800	1899	27,85
208	900	2138	
	1000	2378	
	1100	2618	
207	1125	2678	
	1200	2859	
	1300	3100	
206	1350	3221	
	1400	3341	
	1500	3583	
205	1575	3766	
	1600	3826	
	1700	4069	
204	1800	4313	
	1900	4556	
	2000	4800	
203	2025	4863	
	2100	5045	
	2200	5290	
202	2250	5415	24,37

A simple proportion applies this Table to the use of any other thermometrical barometer of greater or less sensibility.

As the instrument I was about to employ had a scale of only four inches, or 800 parts, and could therefore scarcely measure at its utmost limits 1900 feet, it was necessary, on a height of 3550 feet, to divide the whole measurement into two, or, for greater security, into three lengths, and it became therefore a fair trial of the practical use of the instrument in every respect.

On the 24th of August, having occasion to ride from Carnarvon to Gregory's New Inn, near Llanberis Lake, at the foot of Snowdon, for the purpose of securing accommodations for my party, I took the instrument in my pocket, having previously boiled it on the ground floor of the Vicarage house at Carnarvon, where it stood at 517,5, thermometer 66°, barometer 30,0. In the parlour at Gregory's it stood at 401, thermometer 66°, barometer not changed during my absence; difference at the two stations 116,5.

100 : 116,5 : : 236 : 275 feet, the approximate height:
add the correction 0,089 × .275 from General Roy's table following } 24,5
299,5 feet, the corrected height of Gregory's above the Vicarage.

The descent from the Vicarage to the north end of Carnarvon quay, was taken by levelling, and found 11,25 feet.

August 27. The instrument had been set at Gregory's at 768, before the party began to ascend the hill; but when we came to the place where the guide supposed we had ascended half way, it stood at about 300, showing that we had not gone far enough to take the whole in two lengths of the scale; and we therefore proceeded farther, to a point opposite a pass in the ridge of Snowdon, called Bwlch Cymbrwynog, by which the hill is ascended from Nant. In carrying the instrument incautiously from the former point while cooling, the mercury was drawn into the tube from the cap at the top, and the previous measure from Gregory's was therefore lost. I mention this, to show how the measure was afterwards recovered on descending.

At this point which I call H, the half height, the thermometrical barometer was set at 792, in the manner directed in my former paper; thermometer 62°. When boiled again on the summit it stood at 86, thermometer 66°. On descending again to H, it boiled at 785, thermometer 63°. From this point, my measurement to Gregory's downward, was to be made for recovering the accidental loss in the ascent; and for that purpose an operation was to be performed on the instrument, by no means so easy as that for taking a succeeding stage upward; for the mercury must in this case be expanded out of the tube into the cap over the flame of the lamp, until on boiling, it stands as near as may be requisite to the bottom of the scale. This was done at H, and the water boiled at 146, thermometer 63°. On carrying it down to the Inn it stood at 695, thermometer 65°. The barometer observed at Carnarvon about the time of boiling at H, on ascending, was at 29,91; and at the time of boiling there the second time, it stood at 29,88, where I found it on my return to Carnarvon.

Between Gregory's and H, the difference 695 — 146 = 549, mean thermometer 64°, barometer 29,9. Between H and the summit, the difference (from the mean of the two observations at H $\frac{792+785}{2}$=) 788,5 — 86 = 702,5, mean thermometer 64°. The mean thermometer being the same in both lengths, they may be calculated together, the total being 549 + 702,5 = 1251,5, thermometer 64°. The barometer being at 29,9, I begin from the corresponding point on Table II, and take out

1250 parts between 50 and 1300 = 3130 — 148 = 2982
1,5 - - - - - - = 3,5
1251,5 parts, giving the approximate height - 2985,5 feet.
to this add the correction from Table III, 0,0837 × 2985,5 = 250,0
the corrected height from Gregory's to the summit 3235,5 feet.
from Vicarage to Gregory's - - - 299,5
from Quay to Vicarage - - - 11,25
Total from north end of Quay to summit - 3546,25 feet.

General Roy's measurement trigonometrically makes it 3555,4 feet, and barometrically 3548,9 feet.

During the same visit to Carnarvon, I took the opportunity of trying the instrument on Moel Elio, which stands between that place and Snowdon, and has also been measured by General Roy. On September 6, at the Vicarage, the thermometrical barometer stood at 737, thermometer 59°, barometer 29,8. In the blacksmith's shop at Waen Fawr, at the foot of Moel Elio, water boiled at 550, thermometer 57°. The instrument was here set to 767, and carried to the summit, where by the pile of stones it stood at 25, thermometer 52°.

From Vicarage to Waen Fawr, the difference 737 — 550 = 187, mean therm. 58°.
187 parts - - - = 441 feet.
Add correction 0,0669 × 441 - - = 29,5
Corrected height from Vicarage to Waen Fawr - 470,5 feet.

From Waen Fawr to the top, the difference 767 — 25 = 742, mean therm. 55°.
742 parts taken from 258 to 1000 - - - = 1766
Add correction 0,0583 × 1766 - - - = 102,8
Corrected height from Waen Fawr to the top - - - 1868,8 feet.
From Vicarage to Waen Fawr - - - - - 470,5
From Quay to Vicarage - - - - - 11,25
Total from north end of Quay to top of Moel Elio - - 2350,55 feet.

General Roy's measurement trigonometrically makes it 2371 feet; barometrically 2391,8 feet. The three measurements in this instance, do not agree so well as at Snowdon: whether this arises in any degree from the different forms of the two mountains, rendering the point of observation less definite in the one case, I will not pretend to say; Snowdon terminates in a point, Moel Elio has a large bare summit.

To save the trouble of reference to General Roy's Paper, Philosophical Transactions, 1777, p. 771, I give here from that Paper, a part of the Table for correction on account of the expansion of the column of air between two stations at different temperatures, in thousandth parts of an observed height.

Table III.

	28,5		29		29,5		30		30,5	
		difference.		difference.		difference.		difference.		difference.
12°	44,7		45,6		46,6		47,5		48,4	
		2,19		2,24		2,28		2,33		2,37
22	22,8		23,3		23,7		24,2		24,7	
	—	2,28		2,33		2,38		2,42		2,47
32			Subtract above.			Add below this line.				
	+	2,37		2,42		2,47		2,52		2,57
42	23,7		24,2		24,7		25,2		25,7	
		2,46		2,51		2,56		2,61		2,66
52	48,3		49,3		50,3		51,3		52,3	
		2,55		2,60		2,66		2,71		2,76
62	73,8		75,4		76,9		78,4		79,9	
		2,51		2,56		2,61		2,66		2,71
72	98,9		100,9		103,0		105,0		107,1	
		2,46		2,51		2,56		2,61		2,66
82	123,5		126,1		128,6		131,2		133,7	

The instrument with which these experiments were made, had been improved from the original construction as described in the Philosophical Transactions, and I shall mention the particulars of difference. The thermometer itself is straight, and carried up the middle of the scale. The index moves by hand on a square rod on one side of the glass tube, with an adjustment by a screw at the top. The vernier is applied to the edge of the scale. I recommend that in making these thermometers, the bulb be blown on a piece of tube of $\frac{1}{8}$ inch bore, or nearly, so that the mercury expanded before boiling may be wholly contained in that tube without requiring an upper bulb; for if that bulb be made of a size to allow the mercury to separate in it, a globule may lodge in the upper part of it near the fine thread, and be attended with much inconvenience.

To protect the lamp from wind more effectually than was done by the small tent-stand formerly described, I have made the outside case of thin copper, hard soldered, so as to serve as a lanthorn to contain the lamp burning at bottom; while the boiler, which is made of a drawn tube, with the thermometer, slips down from above, and is also protected from the cold air during the experiment. The inverted boiler screws as before, over the scale of the thermometer, and packs in the centre of the case for carriage, the bulb of the thermometer going downwards into the chimney of the lamp; round the chimney in the lower part of the case, is space sufficient for stowing some matches of the oxymuriate of potash, a bottle of sulphuric acid, a wax candle, a tin bottle of water, a pair of scissors for trimming the lamp, a turnscrew and thermometer. In the upper part is also room for some tow or rag for cleaning, &c.; and the whole, when packed for use, weighs two pounds. Still farther to be guarded against wind, if necessary, I carried, as a walking-stick, a rod cut into three, and jointed at top, which belongs to a theodolite as its stand; and had in my pocket a conical bag of thin cotton, which would cover the whole as a tent, and might be pegged down to the ground if occasion required. This precaution, though it had been needless on Snowdon, was of use on Moel Elio, where I had a very high wind. The tent has of course an opening on one side, where the instrument, which hangs from a hook in the centre, is accessible.

S. Weald,
June, 1820.

XIX. *On Sounds inaudible by certain ears.* *By* WILLIAM HYDE WOLLASTON, *M. D. P. R. S.*

Read June 29, 1820.

IT is not my intention to occupy the time of this Society, with the consideration of that mere general dullness to the impression of all kinds of sound which constitutes ordinary deafness, but to request its attention to certain peculiarities that I have observed with respect to partial insensibility in different states of the ear, and in different individuals; for I have found that an ear, which would be considered as perfect with regard to the generality of sounds, may, at the same time, be completely insensible to such as are at one or the other extremity of the scale of musical notes, the hearing or not hearing of which seems to depend wholly on the pitch or frequency of vibration constituting the note, and not upon the intensity or loudness of the noise.

Indeed, although persons labouring under common deafness have an imperfect perception of all sounds, the degree of indistinctness of different sounds is commonly not the same; for it will be found upon examination, that they usually hear sharp sounds much better than low ones; they distinguish the voices of women and children better than the deeper tones in which men commonly speak; and it may be remarked, that the generality of persons accustomed to speak to those who are deaf, seem practically aware of this difference, and, even without reflecting upon the motives which guide them, acquire a habit of speaking to deaf persons in a shriller tone of voice, as a method by which they succeed in making them hear more effectually than by merely speaking louder.

In elucidation of this state of hearing, which casually occurs as a malady, I have observed, that other ears may for a time be reduced to the same condition of insensibility to low sounds. I was originally led to this observation, in endeavouring to investigate the cause of deafness in a friend, by trial of different modes of closing, or otherwise lessening the sensibility of my own ears. I remarked that, when the mouth and nose are shut, the tympanum may be so exhausted by forcible attempt to take breath by expansion of the chest, that the pressure of the external air is strongly felt upon the membrana tympani, and that, in this state of tension from external pressure, the ear becomes insensible to grave tones, without losing in any degree the perception of sharper sounds.

The state to which the ear is thus reduced by exhaustion, may even be preserved for a certain time without the continued effort of inspiration, and without even stopping the breath, since by sudden cessation of the effort, the internal passage to the ear becomes closed by the flexibility of the Eustachian tube, which acts as a valve, and prevents the return of air into the tympanum. As the defect thus occasioned is voluntary, so also is the remedy; for the unpleasant sensation of pressure on the drum, and the partial deafness which accompanies it, may at any instant be removed by the act of swallowing, which opens the tube, and by allowing the air to enter, restores the equilibrium of pressure necessary to the due performance of the functions of the ear.

In my endeavours to ascertain the extent to which this kind of deafness may be carried, some doubt has arisen, from the difficulty of finding sounds sufficiently pure for the purpose. The sound of stringed instruments are in this respect defective; for unless the notes produced are free from any intermixture of their sharper chords, some degree of deception is very liable to occur in the estimate of the lowest note really heard. I can, nevertheless, with considerable confidence, say, that my own ears may be rendered insensible to all sounds below F marked by the base cliff. But as I have been in the habit of making the experiment frequently, it is probable that other persons who may be inclined to repeat it, will not with equal facility effect so high a degree of exhaustion as I have done. To a moderate extent the experiment is not difficult, and well worth making. The effect is singularly striking, and may aptly be compared to the mechanical separation of larger and smaller bodies by a sieve. If I strike the table before me with the end of my finger, the whole board sounds with a deep dull note. If I strike it with my nail, there is also at the same time a sharp sound produced by quicker vibrations of parts around the point of contact. When the ear is exhausted it hears only the latter sound, without perceiving in any degree the deeper note of the whole table. In the same manner, in listening to the sound of a carriage, the deeper rumbling noise of the body is no longer heard by an exhausted ear; but the rattle of a chain or loose screw remains at least as audible as before exhaustion.

Although I cannot propose such an experiment as a means of improving the effect of good music, yet, as a source of amusement even from a defective performance, I have occasionally tried it at a concert with singular effect; since none of the sharper sounds are lost, but by the suppression of a great mass of louder sounds, the shriller ones are so much the more distinctly perceived, even to the rattling of the keys of a bad instrument, or scraping of catgut unskilfully touched.

Those who attempt exhaustion of the ear for the first time, rarely have any difficulty in making themselves sensible of external pressure on the tympanum; but it is not easy at first to relax the effort of inspiration with sufficient suddenness to close the Eustachian tube, and thus maintain the exhaustion; neither is it very easy to refrain long together from swallowing the saliva, which instantly puts an end to the experiment.

I may here remark, that this state of excessive tension of the tympanum is sometimes produced by sudden increase of external pressure, as well as by decrease of that within, as is often felt in the diving-bell as soon as it touches the water; the pressure of which upon the included air closes the Eustachian tube, and, in proportion to the descent, occasions a degree of tension on the tympanum, that becomes distressing to persons who have not learned to obviate this inconvenience. Those who are accustomed to descend, probably acquire the art of opening the Eustachian tube by swallowing, or incipient yawning, as soon as the diving-bell touches the water.

It seems highly probable that, in the state of artificial tension thus produced, a corresponding deafness to low tones is occasioned, but, as I never have been in that situation, I have not had an opportunity of ascertaining this point by direct experiment.

In the natural healthy state of the human ear, there does not seem to be any strict limit to our power of discerning low sounds. In listening to those pulsatory vibrations of the air of which sound consists, if they become less and less frequent, we may doubt at what point tones suited to produce any musical effect terminate; yet all persons but those whose organs are palpably defective continue sensible of vibratory motion, until it becomes a mere tremor, which may be felt and even almost counted.

On the contrary, if we turn our attention to the opposite extremity of the scale of audible sounds, and with a series of pipes exceeding each other in sharpness, if we examine the effects of them successively upon the ears of any considerable number of persons, we shall find (even within the range of those tones which are produced for their musical effects) a very distinct and striking difference between the powers of different individuals, whose organs of hearing are in other respects perfect, and shall have reason to infer, that human hearing in general is more confined than has been supposed with regard to its perception of very acute sounds, and has probably, in every instance, some definite limit, at no great distance beyond the sounds ordinarily heard.

It is now some years since I first had occasion to notice this species of partial deafness, which I at that time supposed to be peculiar to the individual in whom I observed it. While I was endeavouring to estimate the pitch of certain sharp sounds, I remarked in one of my friends a total insensibility to the sound of a small organ pipe, which, in respect to acuteness, was far within the limits of my own hearing, as well as of others of our acquaintance. By subsequent examination, we found that his sense of hearing terminated at a note four octaves above the middle E of the piano-forte. This note he seemed to hear rather imperfectly, but he could not hear the F next above it, although his hearing is in other respects as perfect, and his perception of musical pitch as correct as that of any ordinary ears.

The casual observation of this peculiarity in the organ of hearing, soon brought to my recollection a similar incapacity in a near relation of my own, whom I very well remember to have said, when I was a boy, that she never could hear the chirping that commonly occurs in hedges during a summer's evening, which I believe to be that of the gryllus campestris.

I have reason to think that a sister of the person last alluded to had the same peculiarity of hearing, although neither of them were in any degree deaf to common sounds.

The next case which came to my knowledge was in some degree more remarkable, in as much as the deafness in all probability extended a note or two lower than in the first instance. This information is derived from two ladies of my acquaintance, who agree that their father could never hear the chirping of the common house sparrow. This is the lowest limit to acute hearing that I have met with, and I believe it to be extremely rare. Deafness even to the chirping of the house cricket, which is several notes higher, is not common. Inability to hear the piercing squeak of the bat seems not very rare, as I have met with several instances of persons not aware of such a sound. The chirping, which I suppose to be that of the gryllus campestris, appears to be rather higher than that of the bat, and accordingly will approach the limit of a greater number of ears; for, as far as I am yet able to estimate, human hearing in general extends but a few notes above this pitch. I cannot, however, measure these sounds with precision; for it is difficult to make a pipe to sound such notes, and still more difficult to appreciate the degree of their acuteness.

The chirping of the sparrow will vary somewhat in its pitch, but seems to be about four octaves above E in the middle of the piano-forte.

The note of the bat may be stated at a full octave higher than the sparrow, and I believe that some insects may reach as far as one octave more; for there are sounds decidedly higher than that of a small pipe one-fourth of an inch in length, which cannot be far from six octaves above the middle E. But since this pipe is at the limit of my own hearing, I cannot judge how much the note to which I allude might exceed it in acuteness, as my knowledge of the existence of this sound is derived wholly from some young friends who were present, and heard a chirping, when I was not aware of any sound. I suppose it to have been the cry of some species of gryllus, and I imagine it to differ from the gryllus campestris, because I have often heard the cry of that insect perfectly.

From the numerous instances in which I have now witnessed the limit to acuteness of hearing, and from the distinct succession of steps that I might enumerate in the hearing of different friends, as the result of various trials that I have made among them, I am inclined to think, that at the limit of hearing, the interval of a single note between two sounds, may be sufficient to render the higher note inaudible, although the lower note is heard distinctly.

The suddenness of the transition from perfect hearing to total want of perception, occasions a degree of surprize, which renders an experiment on this subject with a series of small pipes among several persons rather amusing. It is curious to observe the change of feeling manifested by various individuals of the party, in succession, as the sounds approach and pass the limits of their hearing. Those who enjoy a temporary triumph, are often compelled, in their turn, to acknowledge to how short a distance their little superiority extends.

Though it has not yet occurred to me to observe a limit to the hearing of sharp sound in any person under 20 years of age, I am persuaded, by the account that I have received from others, that the youngest ears are liable to the same kind of insensibility. I have conversed with more than one person who never heard the cricket or the bat, and it appears far more likely that such sounds were always beyond their powers of perception, than that they never had been uttered in their presence.

The range of human hearing comprised between the lowest notes of the organ and the highest known cry of insects, includes more than nine octaves, the whole of which are distinctly perceptible by most ears, although the vibrations of a note at the higher extreme are six or seven hundred fold more frequent than those which constitute the gravest audible sound.

Since there is nothing in the constitution of the atmosphere to prevent the existence of vibrations incomparably more frequent than any of which we are conscious, we may imagine that animals like the grylli, whose powers appear to commence nearly where ours terminate, may have the faculty of hearing still sharper sounds, which at present we do not know to exist, and that there may be other insects hearing nothing in common with us, but endued with a power of exciting, and a sense that perceives vibrations of the same nature indeed as those which constitute our ordinary sounds, but so remote, that the animals who perceive them may be said to possess another sense, agreeing with our own solely in the medium by which it is excited, and possibly wholly unaffected by those slower vibrations of which we are sensible.

I should be always most unwilling to occupy the time of this Society with idle speculations on mere possible modes of existence, and should not have called its attention to this subject, had I not observed several curious facts which I thought might prove interesting, and may serve to justify some latitude of conjecture beyond the strict evidence of our senses.

XX. *Particulars respecting the anatomy of the Dugong, intended as a Supplement to Sir* T. S. RAFFLES' *Account of that animal. By Sir* EVERARD HOME, *Bart. F. R. S.*

Read June 29, 1820.

SOMETIME after Sir THOMAS STAMFORD RAFFLES' paper on the Dugong was laid before the Society, I received from him a young female of that animal, four feet six inches long, and the viscera of a male, eight feet long, preserved in spirit, with the bones dried.

From these materials I have selected the most interesting facts that came under my observation, not adverted to in Sir THOMAS STAMFORD RAFFLES' description, and now lay them before the Society, to render the account of this most extraordinary animal as complete as it is possible for me to do.

The external form of the animal was in sufficient preservation to enable Mr. CLIFT to give a representation of it, which is annexed, (Pl. XXV.)

Although the tusks, as well as the mode of shedding them, have been described in a former paper, the skull of the small dugong furnishes farther materials respecting the teeth of this animal. It has two incisors in the upper jaw immediately before the two milk tusks; these are more advanced in the gum than the tusks; and therefore, would appear before them. The gum covering the alveoli was very thick, and a ligamentous substance passed down from it into each separate opening, and attached itself to the teeth they contain, to serve as gubernacula, to guide the points of the incisors and tusks in the right direction to pierce the gum.

The first, or temporary set of molares which had been shed in the other skulls, was in the small one twenty in number, there being five on each side of each jaw.

In the anterior scabrous projection of the lower jaw, were four regular sockets on each side, filled with a ligamentous substance passing into them from the gum, forming gubernacula for the incisors not yet completely formed.

The tongue, which is represented in one of the annexed Plates (Pl. XXVII.) of the natural size, has two nipple-like processes at its base, one on each side; it is only loose for half an inch at the point, which is covered with long villi, showing that the sense of taste is very delicate.

The insides of the cheeks are covered with cuticle, in which are strong projecting bristles, as in the hare and rabbit.

The œsophagus is lined with cuticle, which terminates before the entrance into the stomach, where the œsophageal glands are situated.

The stomach differs from all the stomachs I have yet seen; for although it possesses peculiarities met with in the whale tribe, the pecari, the hippopotamus, and beaver, even these very peculiarities are differently arranged, as will be seen by referring to the annexed drawings.

The cardiac portion is small, for an animal living on vegetable food, and extends farther to the left side beyond the entrance of the œsophagus than is usual; its form is more globular than the human. On the upper or small curvature to the left of the entrance of the œsophagus, quite at the extremity, are situated the gastric glands, forming a rounded mass, as in the beaver. The orifices of these glands are small, and covered over with a membranous bag, which has only one large aperture. The glandular mass is divided into two portions. Their appearance is seen in Plate XXVII. and resembles more that of the same glands in the ardea argala, described in a former paper, than of any quadruped. The internal surface of this portion of the stomach is smooth, but not cuticular; the coats are thick near the cardia, but thin towards the pyloric portion. The communication between this and the pyloric portion, is by a round aperture three fourths of an inch in diameter, similar to what is met with between the different cavities of the stomach in the whale tribe. Immediately beyond this orifice there are two openings from the pyloric portion, one from the posterior side into a cul-de-sac six inches long, and one from the anterior only three inches in length. This portion is rather shorter than the cardiac, is thinner in its coats, has a smooth internal surface, and bends a little upon itself before it terminates in the pylorus, which is marked by a welt, or valve. The two appendages to the pyloric portion, differ from those in the hippopotamus and the pecari, in being smaller, and projecting farther, as well as in not belonging to the cardiac portion, as in these animals. All these cavities, including the appendages, were distended with fuci in a macerated state, undergoing trituration.

This complex stomach, belonging to an animal whose food is nearly the same as that of the hippopotamus, makes it very desirable that the internal cavities of the stomach of that animal should be examined and described. This, I believe, has never yet been done; and, whenever an opportunity offers of sending the stomach home preserved in spirit, the opportunity, we trust, will not be lost, as it will prove a most acceptable service to Comparative Anatomy. Such an examination would probably put us in possession of all the peculiarities in the structure of the organs of digestion, that are met with in nature, for digesting vegetable substances, as well those that grow upon land, as those that grow at the bottom of the sea, or of rivers of fresh water.

The duodenum receives the ducts of the liver and pancreas about four inches from its origin at the pylorus The coats are strong, the internal surface is honeycombed, having longitudinal ridges, and smaller ones in a transverse direction. The jejunum has, on that side attached to the mesentery, a row of orifices of glands, not in one line, but in a regular zig-zag. These were very distinct in the small dugong, but could not be seen in the large one. Similar orifices are met with in the colon of the ornithorhyncus paradoxus, ranged in ten separate dotted lines. These orifices extend to the cæcum. The mesenteric glands are large, flat, oval, and thinly scattered.

The cæcum is shown in Plate XXVII.; it is four times the size of the ilium, conical in its shape, and thick in its coats.

The colon has small lacunæ over its whole surface.

The whole intestinal canal is fourteen times the length of the animal, of which the small make five, the large nine. There are no valvulæ conniventes in any part of the intestines.

The spleen is made up of a fine beautiful reticulated network.

The external opening of the nose is that of the whale tribe in miniature; and the os hyoides has the same form as in that tribe. The epiglottis is long, and has a ligamentous edge; it stands up in the posterior nostrils, but does not form a tube with the glottis, as in the whale tribe. The glottis is very similar to the human, as well as the thyroide, cricoide, and arytænoide cartilages. There are no sacculi laryngæi; but two ligaments pass forwards from the base of the arytænoide cartilages to have an attachment to the concave surface of the thyroide, forming a rimula glottidis, which can be made wider and narrower, and the ligamentous bands tighter or looser, by the action of the arytænoide muscles. I have dwelt more upon the cartilages of the larynx, which are so different from those of the whale tribe, as this animal is said to be capable of making a noise not unlike the cry of a young child. The trachea is only two inches long before it divides into two; the rings are circular, although not regularly separate from each other.

The lungs in the larger dugong are two feet in length, that is one-fourth the length of the animal; they are exceedingly elastic, and the cells are very small, about the same size as in the whale: those nearest the surface are twice as large as the others, so that they cannot readily empty themselves entirely. The rings of the bronchiæ are very strong; they are oval in their form, and run into one another. This circumstance is shown in Plate XXIX.

The greatest peculiarity in the structure of this animal, is that of the ventricles of the heart being completely detached from one another. This is not met with in any other animal, and it is at first difficult to account for it, as the circulation of the blood does not differ from that of the whale tribe and quadrupeds in general.

It was natural to look to the heart of the whale for something at least analogous to it. In the whale, the right ventricle has its apex on the same line with that of the left, and the muscular fibres appear to be more nearly of the same length, than in the hearts of quadrupeds in general. This circumstance, of the muscular coats of the two ventricles of the whale's heart being more equal in power than is usual, renders it probable, that in the dugong, where the lungs have such uncommon length, it was necessary that the approach to equality between the two ventricles should be still greater; and this equality could in no way be so well effected as by giving the muscles of the right all the superior mechanical advantages which, in other animals, belong to the left, as was explained to the Society in the year 1790.

The ventricles, although similar in structure, are not of the same size or thickness; the left in the larger dugong, is five inches long, and thicker than the right, which is only four and a half.

The auricles have transverse elastic bands, passing from one side to the other, as in the whale. The orifice of the foramen ovale was completely closed, although the part where it had been, was distinctly marked. The valvulæ mitrales and tricuspides had nothing particular in their appearance; nor had the semilunar valves of the aorta and pulmonary artery.

The relative size of these two great trunks is the same as in the elephant.

The kidneys are long narrow conglobate, and have eleven mammæ on each side.

The form of the penis is represented in Plate XXX.; it has no bone in it, as in the hippopotamus and sea otter. This representation is taken from that of the dugong eight feet long.

The vagina is four inches long. The os tincæ takes on the form of a rose. The uterus is about three inches long. The horns go off at right angles, as in the engraving, (Pl. XXX.) which was taken from a drawing of the dugong four feet six inches long.

The cribriform plate of the ethmoide bone in the small skull, had three distinct foramina on each side, for the branches of the olfactory nerve.

The eye had a membrana nictitans. The cornea was prominent; the lens double convex, the coats thin, the nigrum pigmentum very black.

The sternum in the small dugong was ligamentous in the middle, where the cartilages of the ribs joined it. This peculiar structure, which appeared to be for the purpose of allowing the body to bend forward, was not met with in the large one, the whole being formed into one bone. The appearance is so remarkable, that it is shown in Plate XXXI.

The two bones in the flesh, substitutes for a pelvis, are shown in Plate XXXI.; they were situated opposite the fourth vertebra of the loins.

There is a bony canal in the anterior part of the spine, from the anus to the tail, for the great blood vessels, the spinous processes of which are only attached to the vertebræ by ligament.

There is only one bone corresponding to the phalanges of the thumb, and it is the same with respect to the little finger;

that belonging to the little finger is the broadest, so that the thumb (if the term may be used) is in the situation commonly occupied by the little finger.

EXPLANATION OF THE PLATES.

PLATE XXV.

The external appearance of a young dugong, upon a scale of two inches to a foot.

PLATE XXVI.

The stomach distended, to show the prominent part of the cardiac portion containing the gastric glands, the contraction between the cardiac and pyloric portion, and the two cæca that communicate with the pyloric portion; the thickening at the pylorus, and the termination of the ducts of the liver in the duodenum; on a scale of half an inch to one inch.

PLATE XXVII.

Consists of three figures.

Fig. 1. The gastric glands in one mass, and the œsophageal glands, with the orifices exposed; of the natural size.

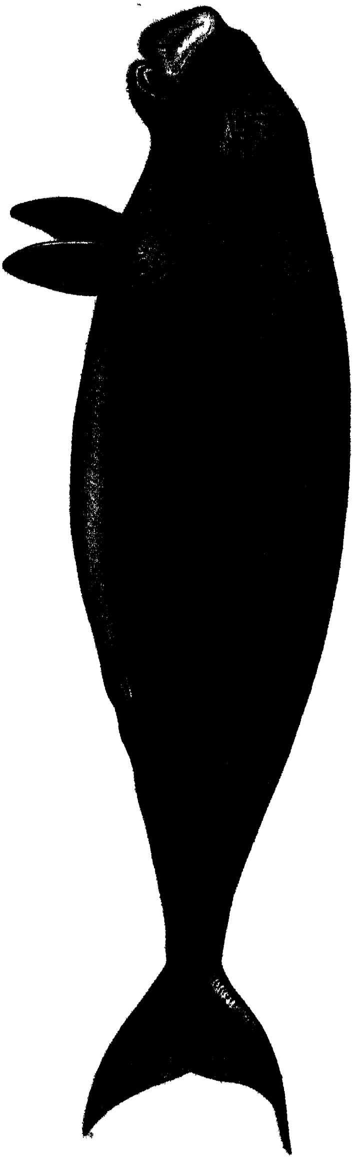
Fig. 2. The tongue in situ, only loose for half an inch at the point; two nipple like processes at the root; the papillæ at the tip for the sense of taste; natural size.

Fig. 3. The cæcum; on a scale of half an inch to one inch.

PLATE XXVIII.

The heart, with the aorta and pulmonary artery; of the natural size; from the dugong, eight feet long. The great peculiarity of the ventricles being separate, is distinctly shown.

Philos Trans MDCCCXX. Plate XXV. p. 32



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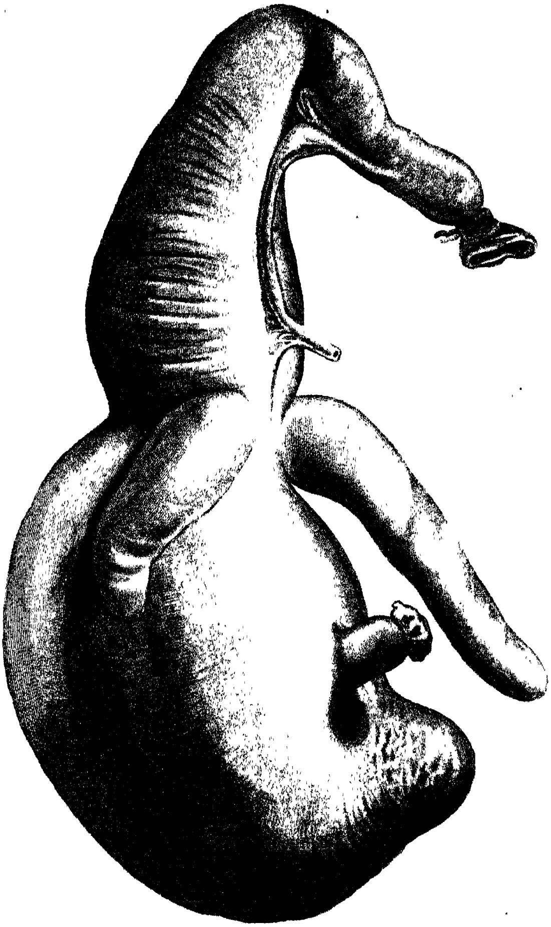

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Fig. 1

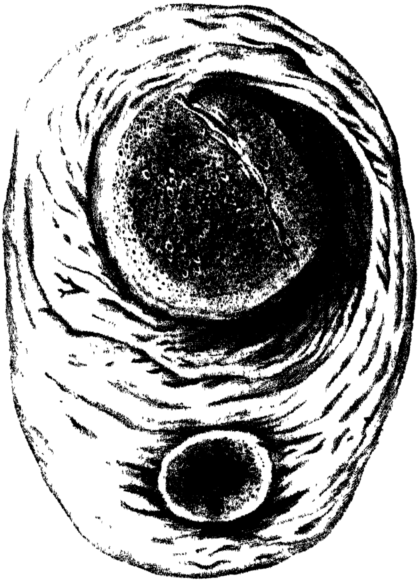


Fig. 2

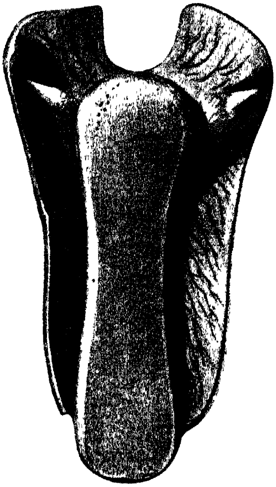
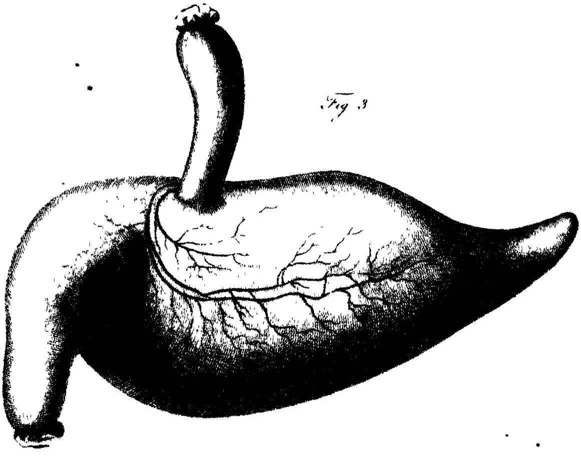


Fig. 3



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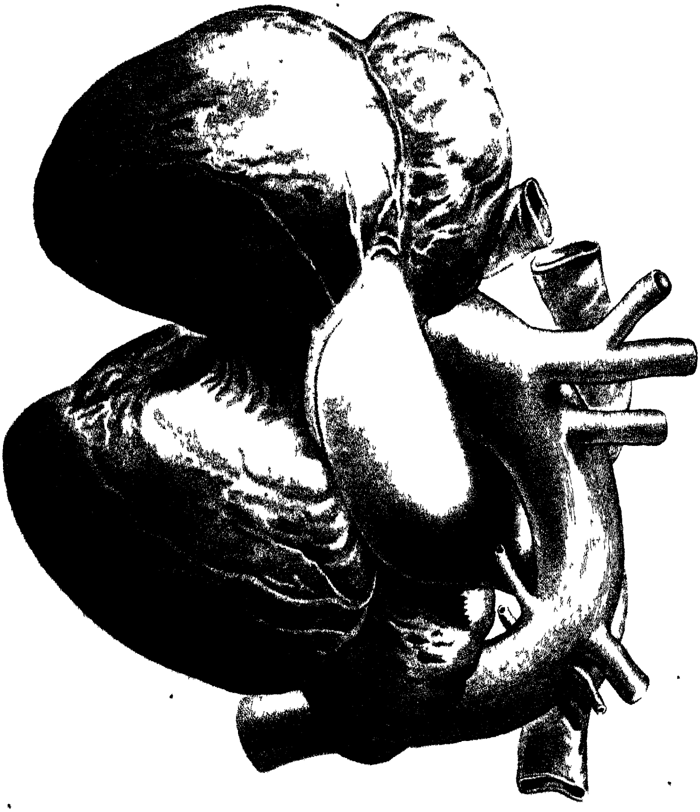

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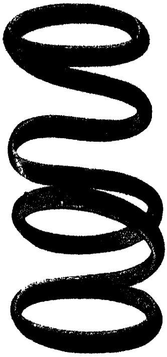

Fig 2

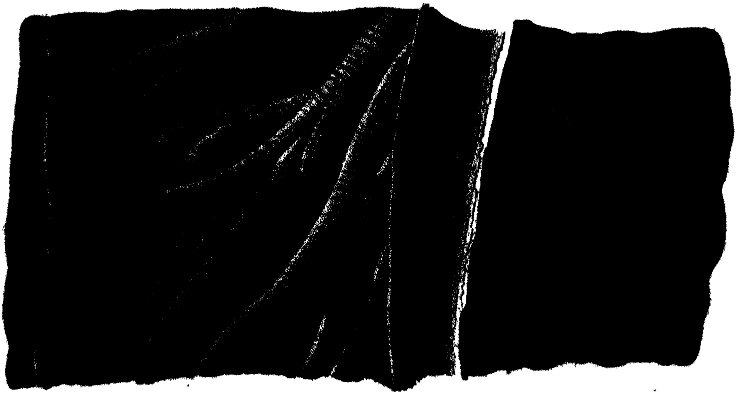

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Fig. 1

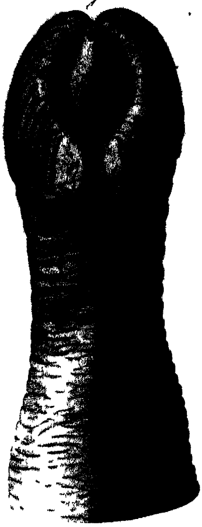

Fig. 2

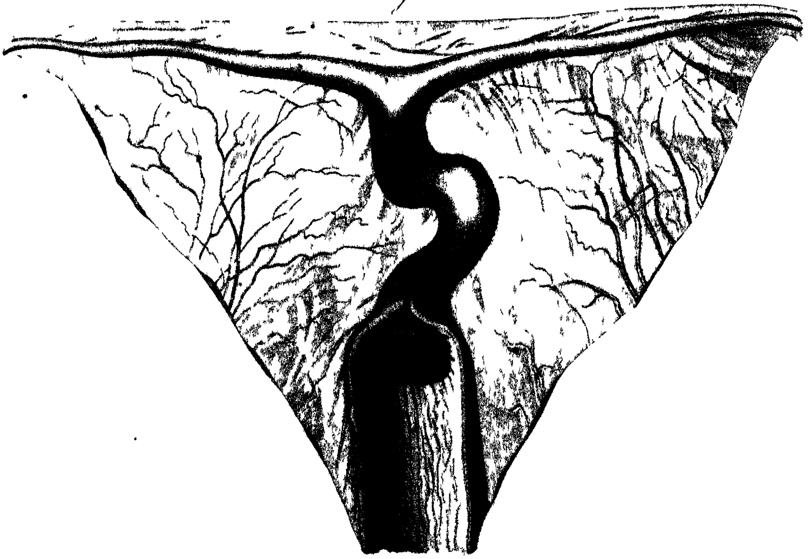

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Fig. 1

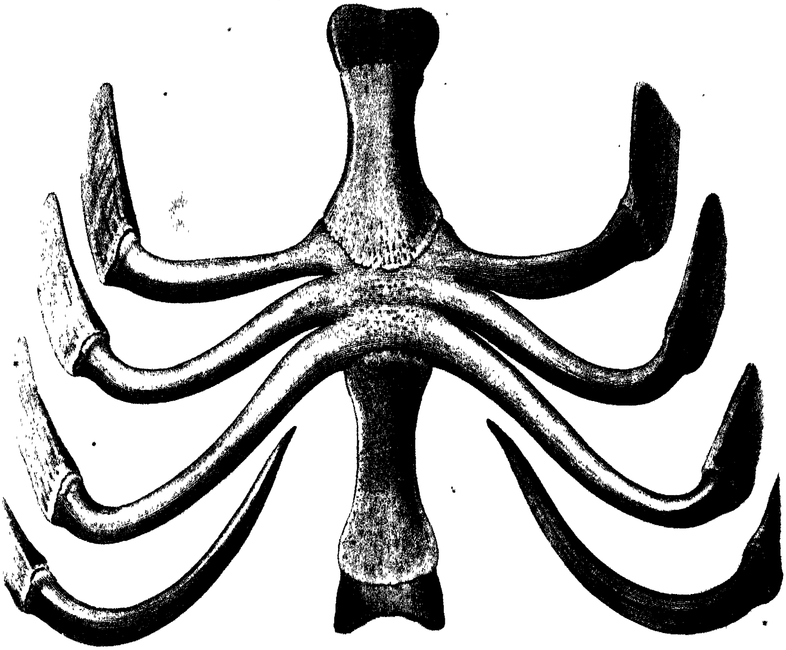

Fig. 2

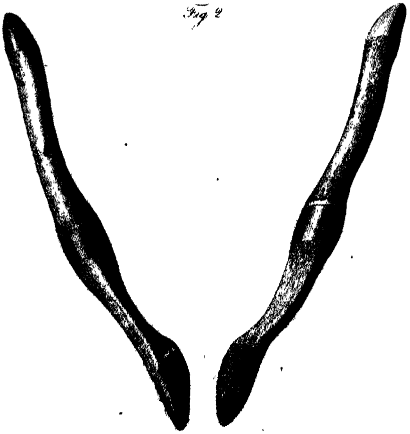

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Fig 2.

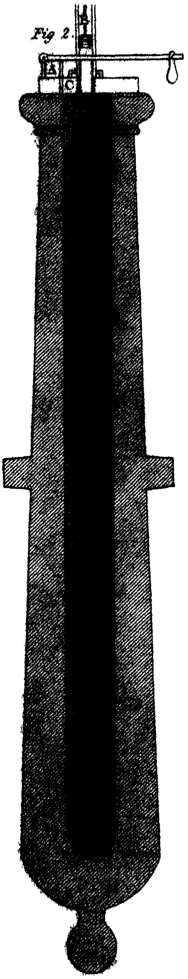


Fig 1.

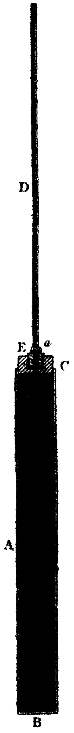


Fig 3.

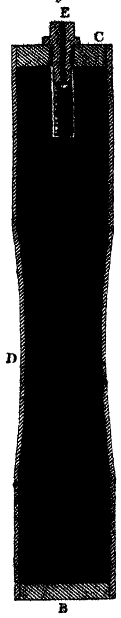
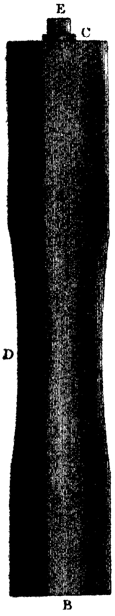


Fig 3



Js. Basire sc.

PLATE XXIX.

Consists of two figures of the natural size; from the dugong eight feet long.

Fig. 1. A portion of trachea below the bifurcation, to show the connection of the rings.

Fig. 2. A portion of the lungs of the same dugong, to show the appearance of the cells, and the internal surface of the bronchia.

PLATE XXX.

Consists of two figures.

Fig. 1. The glans penis of the natural size; from the dugong eight feet long.

Fig. 2. Vagina and uterus of the natural size; from the dugong, four feet six inches long.

PLATE XXXI.

Consists of two figures.

Fig. 1. The sternum in the young dugong, to show that the ribs are attached to a ligament between two portions of bone; but in the older, all these parts are ossified. The parts are of the natural size. This peculiarity must render the body more flexible at that ag e.

Fig. 2. The bones forming a substitute for the pelvis in the female dugong, four feet six inches long; of the natural size.

XXI. *On the compressibility of Water.* By Jacob Perkins, *Esq. Communicated by the late Right Hon. Sir* Joseph Banks, *Bart. G. C. B. P. R. S.*

Read June 29, 1820.

Having believed for many years that water was an elastic fluid, I was induced to make some experiments to ascertain the fact. This was done by constructing an instrument which I call a piezometer, and which is represented in Plate XXXII, Fig. 1. The cylinder, A, was three inches diameter, and eighteen inches long. The end, B, was made water tight by means of a plate which was soldered firmly to it. At the other end, C, a cap was made to screw on and off at pleasure; being also made water tight. The rod or plunger, D, which was five-sixteenths of an inch in diameter, was made to pass through a tight stuffing box, E. On the rod immediately above the stuffing box, was fixed a flexible ring, *a.* A cannon, Fig. 2, of a sufficient size to contain the piezometer, was fixed vertically in the earth, the muzzle being left about eighteen inches above ground, and the touch-hole plugged tight. At the mouth a strong cap, A, was firmly screwed on. In the centre of this cap a small forcing pump, B, was tightly screwed, the piston of which was five-eighths of an inch in diameter. There was an aperture, C, in the cap, to introduce a valve for the purpose of ascertaining the degree of pressure. One pound pressure on this valve indicated an atmosphere. The piezometer was introduced into the cannon, and the water forced in until the cap showed signs of leakage; the valve at the same time indicating a pressure of one hundred atmospheres. The piezometer was then taken out of the cannon, and the flexible ring found to be eight inches up the rod, evidently proving the rod to have been forced into the cylinder that distance, showing also a compression of about one per cent. We have seen by repeated experiments, that to be able to produce this degree of compression, three per cent must be pumped into the gun. This fact proves, either that the gun expands, or that the water enters the pores of the cast iron; it is probable both these circumstances contribute to produce this effect.

This experiment was made in America in the year 1819, and before I had time to strengthen my apparatus for the purpose of making farther experiments, I was obliged to embark for this country. On my passage, however, I had frequent opportunities of repeating those I had already made, and of making others by a natural pressure. They were as follows. The piezometer, by the assistance of fifty-four pounds of lead attached to it, was sunk in the ocean to the depth of five hundred fathoms, which is about equal to the pressure of one hundred atmospheres. When drawn in, the gauge or ring was found removed eight inches up the rod, indicating, as in the before-mentioned experiment, a compression of one per cent. This experiment was several times repeated, and with the same result.

The next experiment was that of sinking a strong empty porter bottle to the depth of one hundred and fifty fathoms, having first tightly corked and sealed it, in the following manner. Six coverings of cotton cloth, saturated with a composition of sealing wax and tar, were strongly fastened over the cork, by a cord wound round them, directly under the projection at the neck of the bottle. After the bottle had been suffered to remain at the depth mentioned a few minutes, it was drawn up. No water was found to have been forced into it, neither was there any visible change at the mouth.

The same bottle was again sunk, and at the increased depth of two hundred and twenty fathoms: when drawn in, it was found to contain about a gill of water; but not the slightest visible change had taken place in the sealing.

The same bottle was now sunk, for the third time, to the still greater depth of three hundred fathoms, and when drawn up, only a small part of the neck was found attached to the line. Its appearance was truly interesting. The bottle was not broken by external pressure, but evidently by the expansion of the condensed sea-water, which had found its way through the sealing. Upon examination, it was found that the cork had been compressed into half its length, making folds of about one eighth of an inch; and that the coverings, consisting of six layers of cloth and cement, had been torn up on one side before the bottle burst. The effect produced upon the cork cannot, we imagine, be accounted for but in one way, viz. that the water, divided into very minute particles, must, by the surrounding pressure of water, have been forced through the coverings, and filled the bottle; that the water thus forced in and condensed, to a great degree, expanded as the pressure was removed by drawing it towards the surface, not only so as to press the cork back into the neck, and, owing to the resistance of the coverings, to compress it half its size, but to separate the neck from the body of the bottle.

Experiment 4. An empty porter bottle, the strongest that could be found, was stopped in the following manner. A cork with a large head was firmly driven into the neck; it was then covered with six layers of fine linen, saturated with a composition of tar and wax; over them was applied a covering of leather, and all perfectly secured by being well bound at the neck. The bottle thus prepared was sunk two hundred and seventy fathoms. When drawn in, it was found perfectly sound, and the sealing unchanged; but filled with water to within an inch of the cork. The coverings were taken off, layer after layer, but no signs of moisture were visible. Had the bottle remained down a sufficient length of time to have completely filled, it would undoubtedly have been broken by the expansion of the water upon being drawn towards the surface, as was the case in the former experiment. It is worthy of remark, that when the water from this bottle was poured into a tumbler, it effervesced like mineral water.

Experiment 5. In this experiment two strong bottles were sunk to the depth of five hundred fathoms. One of them was stopped with a ground glass stopper, and well cemented, then placed in a strong canvass bag. When the bag was drawn in, it was found that the bottle had been crushed into many thousand pieces. The other bottle was very tightly corked, but not having been left down a sufficient length of time, it came up whole, having filled to within one and a half inch. The cork had been driven in and remained so; but the cementation was unaltered, excepting at the surface, where it had become a little concave.

Being satisfied that the piezometer as first constructed, would not show all the compression, I determined to make one differently modified. The object was to avoid the friction occasioned by the collapsing of the leather upon the rod under such great pressure. The drawing in Plate XXXII, Fig. 3, shows another modification of the piezometer, made since I have been in this country. This proves my suspicions to have been correct; since, under the same pressure, it indicated nearly double the compression shown by the former.

This instrument is constructed as follows, Fig. 3 being a section of it. It is simply a small tube, A, closed at the end, B, and water-tight. At the upper end, C, the water is allowed to enter through a small aperture, E, closed by a very sensible valve opening inwards. The tube is flattened at D, in order that it may yield to the expansion of the water when taken out of the press.

The experiment with this instrument was made at Mr. KIER's manufactory, in the presence of many scientific gentlemen. The piezometer being perfectly filled with water (the weight of which was accurately known) was put into an hydraulic press, and subjected to a pressure of about three hundred and twenty six atmospheres. When it was taken out and weighed, there was found an increase of water amounting to three and a half per cent. This water had been previously boiled, and cooled down to a temperature of forty eight degrees, and kept at the same temperature during the experiment.

A machine calculated to avoid loss of pressure from destruction of the materials of which it is composed, will be made with all convenient speed. This machine being constructed with metallic stuffings and flexible metallic pistons, will effect a much greater pressure than the hydraulic press, the power of which is limited by the animal stuffing now used. It is probable, a pressure of from two to three thousand atmospheres may be obtained before the metallic piston is destroyed.

It is expected that this machine will be sufficiently accurate to give the exact ratio of the compressibility of water with much greater precision than has hitherto been obtained; but the results of farther experiments must be the subject of a future communication.

29, *Austin Friars,*
June 6, 1820.

XXII. *Astronomical Observations.* By Stephen Groombridge, *Esq. F. R. S.*

Read June 29, 1820.

Observations of the Solstices in the years 1818 and 1819.

Observed zenith distance of the Sun's centre on the meridian.

1818		Barometer.	Therm. within.	Therm. without.	Refraction.	1819		Barometer.	Therm. within.	Therm. without.	Refraction.
	° ′ ″				′ ″		° ′ ″				′ ″
June 13	28.14 53.08	29.79	72.5	81	0.29.77	June 9	28.33.30.54	29.49	61.5	68	0.30.54
14	28.11 38.72	29.87	68	66	0 30.10	16	28. 6.40.00	29.84	58	62	0.30.62
23	28. 0. 5 55	29 76	62	66	0.30.15	19	28. 1.20.52	30.00	62	68	0.30.38
27	28. 5.39.66	29.75	68	79	0.29.84	20	28. 0.21.51	30.11	62.5	66	0.30.45
28	28. 8. 9.04	29 69	66	73	0.29.97	21	27.59.47.37	30.04	62.5	68	0.30.37
29	28 10.55 40	30.08	67	72	0.30.37						
July 1	28 17 51.98	30.00	69.5	77	0.30.27						
Dec. 12	74 29 34.34	30.06	44	40	3.31.92	Dec. 13	74.32.57.59	29.70	36	32.5	3.33.32
14	74 37.55.16	30.15	43.5	41	3.34.10	26	74 48.56.96	29.44	39	32	3.35.50
19	74 50.26.17	30.21	38	34	3.40.54	27	74.46.54.68	29.39	39	34	3.33.78
22	74 52.23.59	30.45	42.5	35	3.43.15						
23	74 52. 3.00	30.31	38.5	31	3 42.94						
27	74.46.16.87	30.06	39	37	3.37.14						

The above observations computed by Mr. Groombridge's Table of Refraction.

1818	Zenith distance.	Equation.	Solstitial distance.
	° ′ ″	′ ″	° ′ ″
June 13	28.15.22.85	15.15.31	28. 0. 7.54
14	12. 8.82	11.55.34	13.48
23	0.35.70	0.24.45	11.25
27	6. 9.50	6. 1.15	8.35
28	8.39.01	8.27.31	11.70
29	11.25.77	11.17.91	7.86
July 1	18.22.25	18.12.52	9.73
		″	28. 0. 9.99
	Nutation	+ 7.40	
	Parallax	—4.11	+ 2.77
	Sun's Latitude	—0.52	
			28. 0.12.76
Dec. 12	74.33. 6.26	22 57.04	74.56. 3.30
14	41.29.26	14.38.20	7.46
19	54. 6.71	1.58.11	4.82
22	56. 6.74	0. 0.18	6.92
23	55.45 94	0.17.56	3.50
27	49.54.01	6.10.39	4.40
		″	74.56. 5.07
	Nutation	— 8.64	
	Parallax	— 8.45	— 17.58
	Sun's Latitude	—0.49	
			74.55.47.49
			28. 0.12.76
			2)46.55.34.73
	Mean obliquity, 1818		23.27.47.36

1819	Zenith distance.	Equation.	Solstitial distance.
	° ′ ″	′ ″	° ′ ″
June 9	28.34. 1.08	33.54 65	28. 0. 6.43
16	7.10.62	7. 1.93	8 69
19	1.50.90	1.39.50	11.40
20	0.51.96	0.41.67	10.29
21	0.17.74	0. 8.60	9.14
		″	28. 0 9 19
	Nutation	+ 9.23	
	Parallax	—4.11	+ 4.62
	Sun's Latitude	—0.50	
			28. 0.13.81
Dec. 13	74.36.30.91	19.36 30	74.56. 7.21
26	52.32.46	3.32.75	5.21
27	50.28.46	5.36.53	4.99
		″	74.56. 5.80
	Nutation	—9.60	
	Parallax	—8.45	— 18.58
	Sun's Latitude	—0.53	
			74.55.47.22
			28. 0 13.81
			2)46.55 33.41
1819	Mean Obliquity	- - - -	23.27.46 70
1818	- - - - -	- - - -	23 27.47.36
	Mean obliquity at the vernal equinox, 1819.		23.27.47.03

Oppositions of the New Planets.

	Mean Time.	Right Ascension.	Declination.	Longitude.	Latitude.
	h ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
1818.	Vesta.		Par. + 5,3		
April 3	12.47.39	203.38.54.3	3.55.53.3 N	200.23.41.8	12.50.31.8 N
4	12.42.50	203.25.47.4	4. 2. 6.0	200. 8.52.1	12.51.23.5
12	12. 4. 9	201.36.56.4	4.46.53.1	198. 8.26.5	12.51.55.3
15	11.49.35	200.55.18.7	5. 0.54.2	197.23.37.9	12.49. 6.0
17	11.39.54	200.27.48.1	5. 9.18.5	196.54.22.4	12.46.24.1
	Pallas.		Par. 3.0 3.2		
Sept. 1	12.20.21	345.38.31.6	2.10.22 4 N	347.38.10.3	7.40.11.7 N
2	12.15.40	345.27.15.1	1.57.46.6	347.22.45.1	7.32.58.6
6	11.56.56	344.42. 4.6	1. 6.25.2	346.20.42.6	7. 3. 9.5
9	11.42.53	344. 8. 3.1	0.27. 5.2	345.33 48.0	6.40. 1.3
11	11.33.31	343.45.24.6	0. 0.29 5 N	345. 2.27.3	6.24.12.1
12	11.28.50	343.34. 9.1	0.12.50.9 S	344.46.50.3	6.16.13.0
13	11.24. 9	343.22.52.8	0.26.15.1	344.31.11.5	6. 8.10.1
1820.	Pallas.		Par. —6.1 —5.7		
Feb. 1	9.42.57	96.45. 6.6	25.32.13.7 S	99.15.54.7	48.47. 4.0 S
4	9.30.22	96.33.10.2	24.30.48.1	98.53.18.1	47.46.32.0
6	9.22. 9	96.27.45.0	23.49.10.4	98.41.59.8	47. 5.57.5
10	9. 6. 5	96.22.35.4	22.19.58.0	98 26.41.6	45.36.37.1
16	8.43. 1	96.30.28.2	20. 1.36.7	98 24 49.8	43.18. 4.7
17	8.39 16	96.33.26.7	19.37.58.7	98.26.38.8	42.54.19.0
	Ceres.		Par. +1.9 +1.7		
Feb. 1	11.49.38	128 30.18.9	31.29. 3.2 N	122.55. 5.1	12 19 4.7 N
4	11.34.57	127.47. 5.7	31 44.49.4	122.14.33.3	12 25. 4.3
10	11. 5.54	126.24.49 8	32.11. 3.4	120.58.39.7	12.33.19.8
16	10.37.26	125.11.27 3	32 29.41.0	119 52 25.4	12.36.37.2
17	10.32.46	125. 0.23.1	32.32. 1.8	119.42.33.6	12.36.42.6

ERRATA.

Page 220, lines 11 and 20, for $\mathrm{L}_{n:a,b,c,\&c}$, read $\mathrm{L}_{a,b,c}$. Page 221, line 21, for $\left.\begin{matrix}\underbrace{r}_{p}\\ n\\ m\end{matrix}\right|\begin{matrix}a, b, c, \&c.\\ \overline{}\,\overline{}\end{matrix}$ read $\left.\begin{matrix}\underbrace{r}_{p}\\ n\\ m\end{matrix}\right|\underline{a, b, c, \&c.}$ Page 224, lines 2 and 5, insert a comma before the second a; line 15, insert : before a''. Page 226, bottom line for $\left.\begin{matrix}\underbrace{r}_{p}\\ n\\ m\end{matrix}\right|$ read $\left.\begin{matrix}\underbrace{r}_{p}\\ n\\ m\end{matrix}\right|\overline{c}$. Page 227, line 14, for $\left.\begin{matrix}\underbrace{r}_{p}\\ n\\ \underline{m}\end{matrix}\right|\begin{matrix}a, b, c, \&c.\\ \overline{}\,\overline{}\end{matrix}$ read $\left.\begin{matrix}\underbrace{r}_{p}\\ n\\ \underline{m}\end{matrix}\right|\begin{matrix}a, b, c, \&c.\\ \overline{}\\ ,\end{matrix}$ Page 228, lines 1, 3 and 4, for $\left.\begin{matrix}\underbrace{r}_{o}\\ n\\ \underline{m}\end{matrix}\right|\begin{matrix}a, b, c, \&c.\\ \overline{}\end{matrix}$ read $\left.\begin{matrix}\underbrace{r}_{o}\\ n\\ \underline{m}\end{matrix}\right|\begin{matrix}a, b, c, \&c.\\ \overline{}\,\overline{}\end{matrix}$; line 11, for $\left.\begin{matrix}\underbrace{r}_{o}\\ n\\ m\end{matrix}\right|\begin{matrix}a, b, c, \&c.\\ \overline{}\end{matrix}$ read $\left.\begin{matrix}\underbrace{r}_{o}\\ n\\ \underline{m}\end{matrix}\right|\begin{matrix}a, b, c, \&c.\\ \overline{}\,\overline{}\end{matrix}$ Page 240, line 8, for $r+1$, read $r+x$. Page 247, line 16, after $\frac{1}{\mu}\,.\,\mathrm{L}'_{x+n}$ insert (, line 17, before the period insert). Page 255, line 12, for $+\frac{1}{2}p$, read $-\frac{1}{2}p$; line 15, for $+\frac{1}{2}$, read $-\frac{1}{2}$. Page 290, line 6, for with, read without; line 14, for $\overset{\pi}{\left.\begin{matrix}\overline{1}\\ 1\\ 0\end{matrix}\right|}$ read $\overset{\pi}{\left.\begin{matrix}\overline{1}\\ 1\\ 80\end{matrix}\right|}$ Page 291, for N″ and N′, read N. Page 294, line 5, insert t, before ; .

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	Del Proteo Anguino di Laurenti. Monografia publicata da P. Configliachi e M. Rusconi. Pavia, 1819. 4°	Professor Configliachi and Dr. Rusconi.
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	Memorie Medico-Clinichi per servire d' Interpretazione ai Prospetti Clinici. Padova, 1816. 8°	————————

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Nov. 4	Jordani Ruffi Calabriensis Hippiatria. Patavii, 1818. 8°	Dr. Jerome Molin.
	Adversaria Zoologica, Auctore Gotthelf Fischer. Fasciculus primus. 4°	Dr. G. Fischer.
	Essai sur la Pellegrina, ou la Perle incomparable des Frères Zozima, par G. Fischer. Moscow, 1818. 8°	———————
	Mémoire sur la Construction des Instrumens à Cordes et à Archet, par Felix Savart, M. D. Paris, 1819. 8°	Mr. Felix Savart.
	Rotuli Scotiæ in Turri Londinensi et in Domo Capitulari Westmonasteriensi asservati. Vol. 2, 1819. F°	The Commissioners of Public Records.
	The Transactions of the Linnean Society of London, Vol. 12, Part 2. London, 1818. 4°	The Linnean Society.
	A Memoir on the Birds of Greenland, with Descriptions and Notes on the Species observed in the late Voyage of Discovery in Davis' Straits and Baffin's Bay, by Captain Edward Sabine, R. A. London, 1819. 4°	Captain Edward Sabine.
	Observations Introductory to a Work on English Etymology, by John Thomson, M. A. S. &c. London, 1819. 4°	John Thomson, Esq.
	An Inquiry into the Nature and Properties of the Blood, as existent in Health and Disease, by C. Turner Thackrah. London, 1819. 8°	C. Turner Thackrah, Esq.
	Lectures on the Blood and on the Anatomy, Physiology, and surgical Pathology of the vascular System of the human Body, by James Wilson, F. R. S. London, 1819. 8°	James Wilson, Esq.
	An Essay on Chemical Analysis, chiefly translated from the 4th Vol. of the last Edition of the Traité de Chimie Elémentaire of L. J. Thenard, with Additions, comprehending the latest Discoveries and Improvements in this Branch of the Science, by John George Children, F.R.S. London, 1819. 8°	John Geo. Children, Esq.
	A Treatise on the Nature and Cure of Gout and Rheumatism, including general Considerations on morbid States of the digestive Organs, some Remarks on Regimen, and practical Observations on Gravel, by Charles Scudamore, M. D. 3d Edition. London, 1819. 8°	Dr. Charles Scudamore.
	Medico-Chirurgical Transactions, Vol. 10, Part 1. London, 1819. 8°	The Medical and Chirurgical Society of London.
	Speech of Lieut. General William Thornton in the House of Commons, on Thursday, 7th May, on his Motion to Repeal the Declarations against the Belief of Transubstantiation, and asserting the worship of the Church of Rome to be Idolatrous, with Authorities and Illustrations, Deduction and Conclusion. London, 1818. 8°	Lieut. General William Thornton.

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Nov. 4.	A History of the high Operation for the Stone by Incision above the Pubis, with Observations on the Advantages attending it, and an Account of the various Methods of Lithotomy, from the earliest Periods to the present Time, by J. C. Carpue, Esq. London, 1819. 8°	J. C. Carpue, Esq.
	The Quarterly Journal of Literature Science and the Arts, No. 15. 8°	The Managers of the Royal Institution.
	Annals of Philosophy, No. 80-83. 8°	Dr. Thomas Thomson.
	The Philosophical Magazine, No. 255 to 258. 8°	Mr. Alexander Tilloch.
	The Monthly Review for July, August, September, and October, and Appendix to Vol. 89. 8°	Mr. George Edward Griffiths.
	The European Magazine for July, August, September, and October. 8°	The Editor.
11.	Surgical Essays by Astley Cooper, F. R. S. &c. and Benjamin Travers, F. R. S. &c. Part 2. London, 1819. 8°	Astley Cooper and Benjamin Travers, Esq.
25.	The Statutes of the Realm, printed by Command of H. M. King George III. in pursuance of an Address of the House of Commons of Great Britain, Volumes 4, 5 and 6. 1819. F°	The Commissioners of Public Records.
	A Sketch (analytical) of the History and Cure of Contagious Fever, by Robert Jackson, M. D. London, 1819. 8°	Dr. Robert Jackson.
	Medical Topography of Upper Canada, by John Douglas, Assistant Surgeon, 8th Regiment. 8°	John Douglas, Esq.
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	De l'Ausculation mediate, ou Traité du Diagnostic des Maladies des Poumons et du Cœur, fondée principalement sur ce nouveau Moyen d'Exploration, par R. T. H. Laennec. Paris, 1819. 8°	M. R. T. H. Laennec.
Dec. 9.	The Ordnance Survey of Great Britain, Part 1 to 7, comprising Essex, Devonshire, Cornwall, Dorsetshire, Sussex, Surry, and Wiltshire.	Major General Mudge.
	Esprit, Origine, et Progrés des Institutions des principaux Pays de l'Europe, par J. D. Meyer. Londres, 1819. 2 Vols. 8°	Chev. J. D. Meyer.
	Elements of a Plan for the Liquidation of the public Debt of the United Kingdom, by Richard Heathfield, Gent. London, 1819. 8°	Richard Heathfield, Esq.
	Annals of Philosophy. No. 84.	Dr. Thomas Thomson.
	The Philosophical Magazine. No. 259.	Mr. Alexander Tilloch.
	The Monthly Review for November.	Mr. George Edward Griffiths.
	The European Magazine for November.	The Editor.
16.	Mémoires de l'Academie Royale des Sciences de l'Institut de France, année 1817, Tome 2. Paris, 1819. 4°	The Royal Institute of France.

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Dec. 23.	Ensaio Historico sobre a Origem e Progressos das Mathematicas em Portugal, por Francisco de Borja Garçao Stockler. Paris, 1819. 8°	D. Francis de Borja Garçao Stockler.
	Leçao XII. dos Elementos de Geometria Philosophica escritos por Francisco de Borja Stockler, a qual tem por titolo das Correlaçoes que existem entre as Operaçoes elementares da Technia Geometrica e da Technia Algebraica. Lisboa, 1819. 8°	———
	Account of a Voyage of Discovery to the West Coast of Corea and the Great Loo Choo Island, with an Appendix, &c. by Captain Basil Hall, R. N. London, 1818. 4°	Captain Basil Hall, R.N.
	Voyage to Corea and the Island of Loo Choo, by Captain Basil Hall, R. N. a New Edition. London, 1820. 12°	———
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Jan. 13.	Traité complette de Mécanique appliquée aux Arts, &c. Vol. 6 des Machines employées dans divers Fabrications, par M. J. A. Borgnis. Paris, 1819 4°	M. J. A. Borgnis.
	Medico-Chirurgical Transactions published by the Medical and Chirurgical Society of London, Vol. 10, Part 2. London, 1820. 8°	The Medical and Chirurgical Society of London.
	Nine Charts, viz. of Gaspar Straits, Carimata Passage, West part of Kilo Leet, Lucepar Channel, Romania Channel, Singapore New Settlement, Samangca Bay, Coloombyan Harbour, and Survey of the entrance of Hoogley River, by James Horsburgh, Esq.	James Horsburgh, Esq.
	Mémoire sur l'Utilité des Pièces d'Anatomie artificielle chirurgicale, par J. F. Ameline. Paris, 1819. 8°	M. J. F. Ameline.
	The Quarterly Journal of Literature Science and the Arts, No. 16. 8°	The Managers of the Royal Institution.
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	The Monthly Review for December. 8°	Mr. George Edward Griffiths.
	The European Magazine for December. 8°	The Editor.
	The London Magazine, No. 1. 8°	The Editors.
	A Treatise on Nervous Diseases, by John Cooke, M.D. Vol. 1. London, 1820. 8°	Dr. John Cooke.
	A Dissertation on the Disorder of Death, or that State of the Frame under the Signs of Death called suspended Animation, &c. by the Rev. Walter Whiter. Norwich, 1819. 8°	The Rev. Walter Whiter.
20.	Esperienze ed Osservazione sul Galvanismo, Memoria del Sig. Carlo Francesco Bellingeri, M.D. 4°	Dr. Carlo Francesco Bellingeri.

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	Réglement de la Société d'Instruction Medicale, par Le Chev. J. J. Leroux. Paris, 1818. 4°	M. Leroux, Doyen de la Faculté de Medecine à Paris.
	Travels in Nubia by the late John Lewis Burckhardt, published by the Association for Promoting the Discovery of the Interior parts of Africa. London, 1819. 4°	The Committee of the Association for Promoting the Discovery of the Interior Parts of Africa.
Feb. 17.	Flora Batava, No. 55. 4°	H. M. The King of the Netherlands.
	Zoognosia Tabulis Synopticis illustrata in usum Praelectionum Academiæ Imper Med Chirurg. Mosquensis edita. Auct. Gotthelf Fischer. Mosquiæ, 1813 et 1814 3 vols. 4° and 8°	Dr. Gotthelf Fischer.
	Notizen über die Mineralien Sammlung S. Excellenz des Herren Drs. Alexanders von Crichton, von Joseph Frederick Wagner. Moskwa, 1818. 4°	Mr. Joseph Frederick Wagner.
	An Account of the variolid Epidemic which has lately prevailed in Edinburgh and other Parts of Scotland, with Observations on the Identity of Chicken Pox with modified Small Pox; in a Letter to Sir James M'Grigor, by John Thomson, M. D. London, 1820. 8°	Dr. John Thomson.
	Annals of Philosophy, No. 86. 8°	Dr. Thomas Thomson.
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	The Monthly Review for January, and Appendix to Vol. 90. 8°	Mr. George Edward Griffiths.
	The European Magazine for January. 8°	The Editor.
	The London Magazine, No. 2. 8°	The Editors.
	An Introduction to Solid Geometry, and to the Study of Crystallography, containing an Investigation of some of the Properties belonging to the Platonic Bodies independent of the Sphere, by N. J. Larkin, M. G. S. London, 1820. 8°	Mr. N. J. Larkin.
	A View of the Lead Mines of the Missouri, including some Observations on the Mineralogy, Geology, Geography, Antiquities, Soil, Climate, Population, and Productions of Missouri and Arkansaw, and other Sections of the Western Country, by Henry R. Schoolcraft. New York, 1819. 8°	Dr. David Hosac.
	A Biographical Memoir of Hugh Williamson, M. D. LL. D., by David Hosac, M. D. New York, 1820. 8°	———
	Bibliotheca MS. Stowensis. A Descriptive Catalogue of the Manuscripts in the Stow Library, by the Rev. Charles O'Connor, D. D. Buckingham, 1818. 2 Vols. 4°	The Marquis of Buckingham.

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	A Catalogue of the Lansdown Manuscripts in the British Museum, with Indexes of Persons, Places, and Matters, printed by command of H. M. King George III. Parts 1 and 2. 1812, 1819. F°	The Commissioners of Public Records.
	Annals of Philosophy, No. 87. 8°	Dr. Thomas Thomson.
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	The Monthly Review for February.	Mr. George Edward Griffiths.
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	Meteorological Journal kept on board the Marine Society's Ship off Deptford Creek in the years 1816, 1817, 1818, 1819. MS.	The Committee of Governors of the Marine Society.
	Transactions of the Society instituted at London for the Encouragement of Arts Manufactures and Commerce, Vol. 37. London, 1820. 8°	The Society for Encouragement of Arts, &c.
9.	Annales des Mines ou Recueil de Mémoires sur l'Exploitation des Mines et sur les Sciences qui s'y rapportent. Année 1818, Liv. 2, 3, et 4, et Année 1819, Liv. 1, 2, 3. Paris. 8°	Conseil Général des Mines de France.
	De' Contagi e de la cura di loro Effetti, Lezione Medica Practiche del Cav. Valeriano Luigi Brera, M. D. Padova, 1819. 2 Vols. 8°	Il Cav. Valeriano Luigi Brera.
	Prospetti de' Risultamente ottenuti nella Clinica Medica dell' I. R. Università di Padova nel Anni Scolasticè, 1809—1816 del Sig. V. L. Brera, M.D. Padova, 1816 e 1817. 2 Vols. 8°	———
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	Notice sur l'Archipel de Jean Potocki situé dans la Partie septentrionale de la Mer Jaune, par Jules Klaproth, avec une Carte. Paris, 1820. 4°	M. Jules Klaproth.

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	Jo. Frid. Blumenbachii decas sexta Collectionis suae Craniorum diversarum Gentium illustrata. Gottingæ, 1820. 4°	Professor J. F. Blumenbach.
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	The Philosophical Magazine, No. 264. 8°	Mr. Alexander Tilloch.
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11.	Transactions of the Horticultural Society, Vol. 4, Part 1. London, 1820. 4°	The Horticultural Society.
	Geological Map of England in 6 Sheets and Memoir of a Geological Map of England, to which are added an Alphabetical Index to the Hills, and a List of the Hills arranged according to Counties, by G. B. Greenough, Pres. Geol. Soc. &c. London, 1820. 4°	G. B. Greenough, Esq.
	The Modern London Catalogue of Books, with their Sizes, Prices, and Publishers. London, 1818. 8°	Mr. William Bent.
	Vindiciæ Geologicæ, or the Connection of Geology with Religion explained, in an Inaugural Lecture delivered before the University of Oxford, May 15, 1819, on the Endowment of a Readership in Geology, by H. R. H. the Prince Regent; by the Rev. W. Buckland, B. D. &c. Oxford, 1820. 4°	Rev. William Buckland.
18.	Mémoire sur la Digue de Cherbourg comparée au Breakwater ou Jetée de Plymouth, par J. M. F. Cachin. Paris, 1820. 4°	M. J. M. F. Cachin.
	Su la Falsità dell' Origine Scandinava data ai Popoli detti Barbari che distrussero l' Impero di Roma, Dissertazione Istorica di Jacopo Gråberg di Hemsö. Pisa, 1815. 8°	Sig. Jacopo Gråberg di Hemsö.
	Appellation au Parlement de la Grande Bretagne, by Hoëne Wronski. London, 1820. 12°	M. Hoëne Wronsky.
	A Journey in Carniola, Italy, and France, in the years 1817 and 1818, containing remarks relating to Language, Geography, History, An-	W. A. Cadell, Esq.

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June 1.	Astronomical Observations made at the Radcliffe Observatory at Oxford, from May 1, 1819, to May 1, 1820, by and under the direction of the Rev. Abram Robertson, D. D. F. R. S. Sav. Prof. of Astron. and Radcliffian Observer. MS. F°	The Trustees under the Will of the late Dr. Radcliffe.
	Transactions of the Geological Society, established Nov. 13, 1807, Vol. 5, Part 1. London, 1819 4°	The Geological Society.
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	Essays on the Combinatorial Analysis, showing its Application to some of the most useful and interesting Problems of Algebra, by Peter Nicholson. London, 1818. 8°	Mr. Peter Nicholson.
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	Report of the Lords Committees appointed to search the Journals of the House Rolls of Parliament and other Records and Documents for all Matters touching the Dignity of a Peer of the Realm, &c. May 25, 1820. F°	Lord Redesdale.

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	On Circular Polarisation, as exhibited in the Optical Structure of the Amethyst, with Remarks on the Distribution of the Colouring Matter in that Mineral, by David Brewster, LL. D. Edinburgh, 1820. 4°	Dr. David Brewster.
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